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Prediction of the longitudinal dispersion coefficient for small watercourses

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ABSTRACT. Longitudinal dispersion coefficient (D_L) is considered an essential physical parameter to water quality modeling in rivers. Therefore, the estimation of this parameter with high accuracy guarantees the reliability of the results of a water quality model. In this study, the observed values of longitudinal dispersion coefficient are determined for natural streams (with discharge less than 2.84 $m^3 s^{-1}$), based on sets of measured data from stimulus-response tests using sodium chloride as a tracer. Additionally, a semi-empirical equation for prediction of D_L is derived using dimensional analysis and multiple linear regression technique. The performance of the produced equation was compared to five empirical prediction equations of D_L selected from literature. It presented correlation coefficient r^2 = 0.87, suggesting that this equation is suitable for the estimation of D_L in streams. It also presented better results for predicting the D_L than the five equations from literature, showing an accuracy of 71%.

Keywords: longitudinal dispersion coefficient, small watercourses, sodium chloride tracer.

Predição do coeficiente de dispersão longitudinal para pequenos cursos d´água

RESUMO. O coeficiente de dispersão longitudinal (D_L) é considerado um parâmetro físico essencial para modelagem de qualidade da água em rios. Por isso, a estimativa desse parâmetro, com elevada acurácia, garante a confiabilidade dos resultados de um modelo matemático de qualidade de água. No presente trabalho, o coeficiente de dispersão longitudinal é determinado para dois cursos d´água, de pequeno porte (vazões inferiores a 2,84 m^3 s^{-1}), a partir de ensaios de campo de estímulo-resposta com traçador salino (cloreto de sódio) e do método da propagação (*routing procedure*), corrigido para considerar a perda do traçador. Adicionalmente, uma equação semiempírica de previsão de D_L foi desenvolvida a partir da análise dimensional e da técnica de regressão linear múltipla. A equação desenvolvida foi comparada com cinco equações empíricas de predição de D_L existentes na literatura. O r^2 da equação gerada foi de 0,87, o que sugere que esta equação é adequada para a estimativa de D_L para os cursos d'água estudados. A equação produzida gerou melhores resultados do que as cinco equações retiradas da literatura, apresentando uma acurácia de 71%.

Palavras-chave: coeficiente de dispersão longitudinal, rios, traçador salino.

Introduction

The mathematical models used for the simulation of water quality are useful in predicting environmental impacts from pollutants discharged into rivers (Sardinha et al., 2008; Pasquini, Formica, & Sacchi, 2012; Gonçalves & Giorgetti, 2013). These models are made up of parameters that, if poorly estimated, can reduce the accuracy of the produced results. An important parameter that represents the fluvial system capacity to disperse pollutants is the longitudinal dispersion coefficient (D_L), especially when pollutant discharges are accidental or not permanent.

The most reliable way of obtaining D_L is through direct methods, which requires knowledge of temporal distributions of tracer concentrations thrown into the river upstream of the sampling stations. Most existing direct methods are derived from the one-dimensional advection-dispersion equation that is shown in Equation 1:

$$\frac{\partial C}{\partial t} + U\frac{\partial C}{\partial t} = D_L\frac{\partial^2 C}{\partial x^2} \quad (1)$$

where:

U and C are the cross-sectional average velocity of the flow and the cross-sectional average concentration, respectively;

t is the time in which the process develops;

x is the direction of mean flow.

Important used direct methods are: moment method, routing procedure, Krenkel and Chatwin graphics methods, peak concentration method, reference concentration or crown of concentration method, and method of adjustment (Fischer, List, Koh, Imberger, & Brooks, 1979; French, 1985; Rutherford, 1994). Although the direct methods are the most reliable for prediction of D_L, the costs and the need of a qualified technical team to run the field tests required for these methods encourage the use of empirical and semi-empirical equations. With these equations, D_L can be easily estimated using hydraulic and geometrical parameters of the stream, as channel depth and width, average velocity and slope (Devens, Barbosa, & Silva, 2006). In this context, empirical and semi-empirical equations are very useful in D_L prediction, once they facilitate the process of obtaining D_L from few parameters related to the characteristics of the water flow (Ribeiro, Silva, Soares, & Guedes, 2010). Currently, there are several equations (Seo & Cheong, 1998; Kashefipour & Falconer, 2002; Rieckermann, Neumann, Ort, & Gujer, 2005; Barbosa Jr., Silva, Neves, & Devens, 2005; Devens et al., 2006; Ribeiro et al., 2010; Devens, Barbosa, Silva, & Giorgetti, 2010). However, these equations have been deduced for specific conditions of flow, limiting their applicability to similar geometric and hydraulic conditions.

A study was carried out to measure the longitudinal dispersion coefficient in the Grande River watershed, which resulted in the creation of a semi-empirical equation that can be used in watercourses with similar features to those of this basin. This area was chosen because of the occurrence of a serious railway accident near the banks of Alegria Creek, 15 km from the pumping station for public water supply of Uberaba city. The machinery was composed of wagons loaded with chemicals, including methanol, octanol, butanol and grained potassium chloride, which collided in the derailment, dumping about 700 m^3 of the chemicals in the soil and in the creek. The accident caused environmental damage to the region of the creek (a tributary of the Grande River) and the interruption of water supply service to the population of Uberaba during eight days.

The semi-empirical equation developed in this study is a tool that will enable decision-making on different management options in the case of accidental pollutant discharges reaching watercourses in the region.

Empirical and Semi-empirical Equations for Predicting D_L

One of the first studies on this subject was conducted by Elder (1959). Using the analysis of Taylor (1954), Elder derived an equation to estimate D_L, assuming uniform flow, infinite width channel, and logarithmic velocity profile shown in Equation 2:

$$D_L = 5.93 H U_* \tag{2}$$

where: H and U_* are the depth of flow and the shear velocity, respectively.

McQuivey and Keefer (1974) proposed a simple method for prediction of D_L from correlations with field data obtained in 18 natural watercourses in 14 different stages.

Based on an analogy between one-dimensional linear flow equations and the one-dimensional linear dispersion equation, Equation 3 was obtained:

$$D_L = 0.058 \frac{HU}{S} \tag{3}$$

where: S is the slope of the channel.

Field tests carried out with this model estimates an average standard error of approximately 30%, reaching a margin of 100% for isolated predicted values of D_L.

Based on the results of the Equation 3 and giving some additional considerations, Fischer (1975) presented the following Equation 4:

$$D_L = 0.011 \frac{U^2 B^2}{U_* H} \tag{4}$$

where: B is the width of the channel.

Seo and Cheong (1998) analyzed data sets from 59 concentration profiles measured from tests with tracers carried out at 26 watercourses in the United States, and using dimensional analysis and nonlinear multiregression methods developed Equation 5.

$$\frac{D_L}{HU_*} = 5.915 \left(\frac{B}{H}\right)^{0.620} \left(\frac{U}{U_*}\right)^{1.428} \tag{5}$$

Deng, Singh, & Bengtsson (2001), using the same database as Seo and Cheong (1998), developed an analytical method to determine the longitudinal dispersion coefficient in Fischer's triple integral expression for natural rivers, emphasizing the importance of turbulent cross mix in addition to other variables of Fischer's triple integral, and obtaining Equation 6:

$$D_L = 0.15 \left(\frac{HU_*}{8\varepsilon_t}\right) \left(\frac{B}{H}\right)^{5/3} \left(\frac{U}{U_*}\right)^2 \tag{6}$$

where: ε_t is the cross-sectional dispersion coefficient.

Based on dimensional analysis and regression analysis, Kashefipour and Falconer (2002) have developed another empirical relationship to estimate D_L, using data obtained from 30 rivers in the USA (Equation 7). The range of variation of the mean velocity of the flow is 0.14 to 1.55 m s^{-1} and the range of variation of mean depth is 0.26 to 4.75 m.

$$D_L = 10.612 HU \left(\frac{U}{U_\star}\right) \quad (7)$$

Equation 7 showed good results when compared with the equations of Fischer (1975) and Seo and Cheong (1998). The fit between the measured values and the estimated by the equation was proven to be better when the analysis was performed in large rivers.

Devens et al. (2006) developed Equation 8 from dimensional analysis and regression analysis for small watercourses with discharges between 0.00521 and 0.173 m^3 s^{-1}. The used calculation method of D_L was the routing procedure. The range of variation of mean velocity is 0.08 to 0.34 m s^{-1}, and the range of variation of depth is 0.02 to 0.10 m.

$$D_L = 3.55\ 10^{-4} \frac{U^{-0.793} B^{0.739}}{H^{1.610} S^{0.026}} \quad (8)$$

Ribeiro et al. (2010), also using dimensional analysis and regression analysis, developed Equation 9 for medium-sized rivers with discharge rates from 16.2 to 98 m^3 s^{-1} using fluorescent tracers. The range of variation of mean velocity is between 0.50 and 0.92 m s^{-1}, and the range of variation of average depth is between 1.17 and 2.42 m.

$$D_L = 7.326 U_\star^{0.303} H^{1.316} B^{0.445} U^{1.458} \quad (9)$$

Material and methods

Field Experiments

The longitudinal dispersion coefficient was determined from 15 field tests in two tributaries of Grande River: Jaú Stream, with UTM coordinates 186187E and 7819128N, and Lageado Stream, 206500E and 7811966N, zone 23 (Figure 1). The climatic regimes in the watershed are two: a cold and dry winter and a hot and rainy summer. The rainfall regime is characterized by a rainy season of six months, from October to March, and a dry period of four months, from June to September; April and May can be considered transitional months. Regarding the thermal regime, the average annual temperature varies between 20 and 24°C. October to February are the warmest months of the year, with mean temperatures ranging between 21 and 25°C, and July is the coldest month, with temperatures ranging from 16 to 22°C.

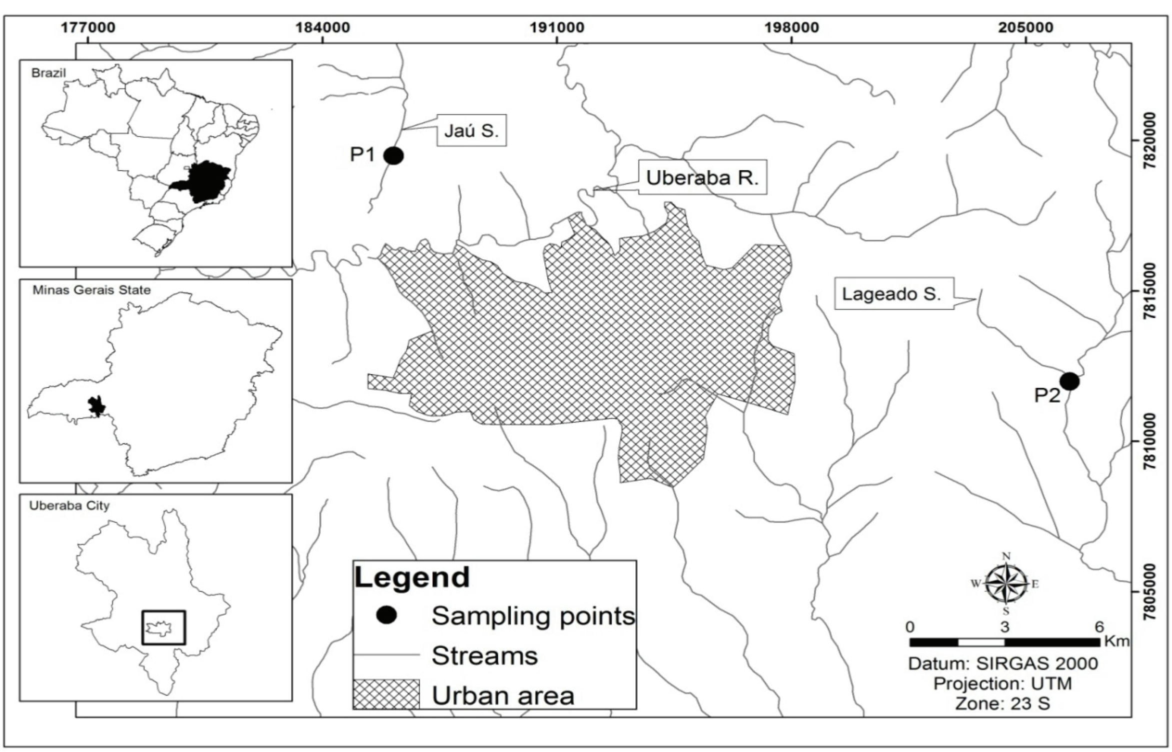

Figure 1. Location of sampling sections.

Concentration versus time curves (response curves) were produced from instantaneous saline tracer injection (sodium chloride), which was released on the axis of the channel so the mixing length would be reduced (Fischer, 1967; 1968). The salt as a tracer has the following advantages: low cost, easily measurable in small watercourses, relatively conservative, non-toxic to the aquatic ecosystem (in low concentrations, such as those achieved in this study). The sampling of the tracer dispersion cloud was done with a conductivity probe in two measuring stations downstream of the tracer injection point. The location of the sampling points was determined based on preliminary calculations, described by Rutherford (1994), to ensure that: (1) the maximum concentration of tracer is over the accuracy limit of the probe (0.05 mg L^{-1}); (2) there is enough time to measure the concentration profile in each sampling section; and (3) there is significant changes in the profile concentration of the sections, ensuring the determination of flow velocity and longitudinal dispersion coefficient. To verify if the first sampling section was downstream of the advective zone, Equation 10 (Fischer et al., 1979) was used to estimate L_x.

$$L_x = 0.0532\frac{UB^2}{H^{1.5}S^{0.5}} \qquad (10)$$

where: L_x is the length of the advective zone.

Subsequently, this condition has been validated in the field, since the ratio between the minimum and maximum concentrations along the channel cross section width was greater than 0.9, as suggested by Rutherford (1994).

To obtain the geometrical characteristics of the channel, bathymetric and altimetric surveys of the test sections were performed. The flow velocity (U) was determined by measuring the distance between the sampling sections and the average time of passage of tracer cloud on each sampling section (Equation 11).

$$U = \frac{x_2 - x_1}{\bar{t}_2 - \bar{t}_1} \qquad (11)$$

where:

X_2 and X_1 are the sampling sections;

$\bar{t}_2$ and $\bar{t}_1$ are average times of passage of tracer cloud relative to downstream and upstream sections, respectively.

The flow rate was calculated by integration method. The following assumptions were made (Barbosa Jr., Silva, & Giorgetti, 1999): (1) sampling section downstream of the advective zone; (2) steady-state flow; and (3) conservative tracer.

Calculation Procedure for the Determination of D_L

In this study, the measurement of longitudinal dispersion coefficients was calculated using the routing procedure, which is currently the most used direct method for determination of D_L. This method, developed by Fischer (1968), uses the response concentration curve from two sampling sections. The response curve measured at the upstream section, C(x1, τ), is used as initial distribution of the tracer cloud to generate the concentration distribution for the downstream section, C(x2, t), using preselected values of D_L. Then, this generated concentration distribution for the downstream section is compared with the actually measured concentration at downstream section, $\widehat{C}$(x2, t), using a convolution process. Mathematically, the method consists of applying a convolution integral of the upstream concentration distribution, with a one-dimensional linear response function, written in the form:

$$C(x_2,t) = \int_{-\infty}^{+\infty} \frac{UC(x_1,\tau)}{\sqrt{4\pi D_L(\bar{t}_2-\bar{t}_1)}} \exp\left\{\frac{[U(\bar{t}_2-\bar{t}_1-t+\tau)]^2}{4D_L(\bar{t}_2-\bar{t}_1)}\right\} d\tau \qquad (12)$$

where: τ is the integration variable of time.

The comparison between the measured $\widehat{C}$(x2, t) and the estimated C(x2, t) concentration profiles of the downstream section was held following the premise that the value of D_L sought is one that minimizes the mean square of the differences between measured and estimated values (mean square error), defined as shown in Equation 13:

$$MSE = \frac{1}{n}\sum_{i=1}^{i=n}\left[\widehat{C}(x_2,t) - C(x_2,t)\right]^2 \qquad (13)$$

where: n is the number of concentration measurements in downstream section.

The routing procedure assumes that the tracer is conservative. However, it is known that there are losses due to adsorption of the tracer in the hydraulic perimeter of the channel. To eliminate the error in the determination of D_L that comes from these losses, the data were normalized by dividing the concentration C(x, t) by the area under the response curve of concentration A(x). This division (Equation 14) defines the normalized variable y(x, t) which replaces the concentration variable in Equation 12.

$$y(x,t) = \frac{C(x,t)}{A(x)} = \frac{C(x,t)}{\int_{-\infty}^{+\infty} C dt} \qquad (14)$$

Results and discussion

Figure 2 presents the normalized response curves from both sections 1 and 2 of test 3 (Table 1) measured at Jaú Stream. It is shown that there was a great fit between the measured values and the estimated values by the routing procedure, producing a mean square error (MSE) equal to 3.9 x 10^{-9} s^{-2}. Figure 3 presents the values of D_L used as initial attempts. Using polynomial regression, the chosen value of D_L was a value for which d (MSE) $d(D_L)^{-1} = 0$.

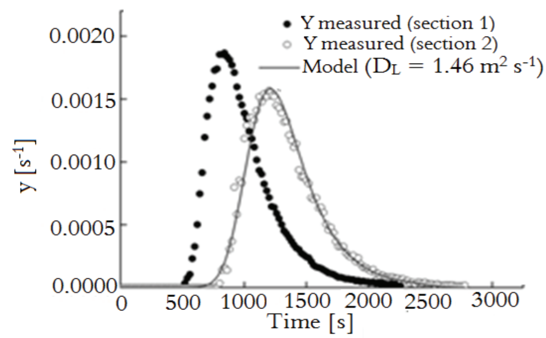

Figure 2. Concentration response curves from upstream (1) and downstream (2) sections, normalzed to Jaú Stream (test 3).

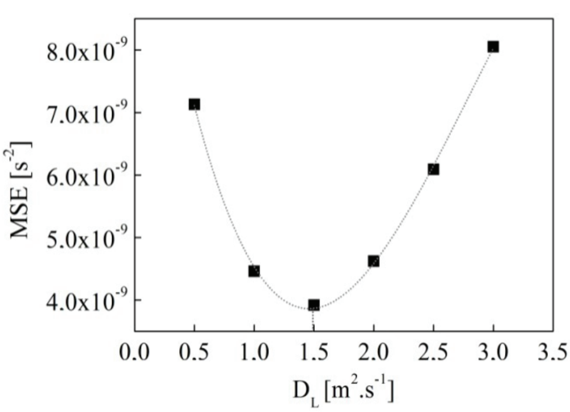

Figure 3. Mean square error in function of D_L (test 3).

Considering the 15 field tests performed, Table 1 presents the calculated values of D_L as well as the geometric and hydraulic characteristics of the watercourses.

Development of a New Equation

Studies related to the process of mixing of pollutants in rivers have shown that the longitudinal dispersion coefficient is influenced by the properties of the fluid, defined by the density (ρ), dynamic viscosity (μ), and by hydraulic and geometric characteristics of the channel, such as: velocity (U), width (B), depth (H) and shear velocity (U_*) (Seo & Cheong, 1998; Deng et al. 2001; Kashefipour & Falconer, 2002; Devens et al., 2010; Ribeiro et al., 2010; Soares, Pinheiro, & Zucco, 2013). So, D_L can be written (Equation 15):

$$D_L = f(\rho, \mu, U, U_*, B, H) \quad (15)$$

Applying the Buckingham π theorem with M, L, and T as fundamental dimensions and ρ, U_* and H as repeating variables, four dimensionless groups are produced (Equation 16):

$$\frac{D_L}{U_* H} = f\left(\frac{B}{H}, \frac{U_*}{U}, Re_*\right) \quad (16)$$

A similar analysis was performed by Seo and Cheong (1998). However, they considered that the roughness Reynolds number (Re_*) could be neglected for turbulent flow in rough channels, such as the natural channels.

For the development of the new prediction equation of D_L, a multiple linear regression analysis was applied to the set of data generated by field experiments, summarized in Table 1.

Table 1. Calculated values of D_L, and geometric and hydraulic characteristics of the watercourses.

Test	Watercourse	Q [$m^3 s^{-1}$]	D_L [$m^2 s^{-1}$]	B [m]	H [m]	U [$m\ s^{-1}$]	S [-]	U_*[$m\ s^{-1}$]	L_x [m]
1	Jaú	0.272	3.39	3.1	0.3	0.31	0.00812	0.15	8.94
2	Jaú	0.324	0.70	3.5	0.4	0.16	0.00812	0.18	4.26
3	Jaú	0.445	1.46	3.6	0.5	0.27	0.00812	0.27	3.81
4	Jaú	1.113	1.06	4.9	0.7	0.32	0.00812	0.21	4.28
5	Jaú	1.106	5.37	4.9	0.7	0.56	0.00812	0.21	8.08
6	Jaú	2.839	1.56	3.9	1.4	0.45	0.00812	0.26	2.54
7	Jaú	2.519	1.50	3.9	1.4	0.42	0.00812	0.26	2.41
8	Lageado	0.037	0.47	2.4	0.2	0.05	0.00956	0.11	1.61
9	Lageado	1.234	9.89	3.8	0.5	0.71	0.00956	0.17	26.19
10	Lageado	2.092	1.39	4.6	0.4	0.49	0.00956	0.20	15.48
11	Lageado	1.947	3.46	4.6	0.4	0.65	0.00956	0.20	20.61
12	Lageado	2.222	8.77	4.6	0.4	0.58	0.00956	0.20	18.45
13	Lageado	1.564	7.37	4.6	0.4	0.82	0.00956	0.20	25.89
14	Lageado	1.546	10.44	4.6	0.4	0.94	0.00956	0.20	29.72
15	Lageado	1.710	3.87	4.6	0.4	0.66	0.00956	0.20	21.07

Considering the dimensionless ratio defined by Equation 16, in which B/H, U_*/U and Re_* are independent variables and $D_L/U_*H/U$ is the dependent variable, and adopting the power function model to describe the relationship of dependency between them, the result is:

$$\frac{D_L}{U_*H}=A\left(\frac{B}{H}\right)^b\left(\frac{U_*}{U}\right)^c Re_*^d \quad (17)$$

Linearizing the Equation 17, the result is:

$$\log\left(\frac{D_L}{U_*H}\right)=\log A+b\log\left(\frac{B}{H}\right)+c\log\left(\frac{U_*}{U}\right)+d\log Re_* \quad (18)$$

From the Equation 18 and based in Table 1, the multiple linear regression analysis was applied resulting in the Equation 19.

$$\frac{D_L}{U_*H}=4.18x10^9\left(\frac{B}{H}\right)^{-0.66}\left(\frac{U_*}{U}\right)^{-1.59} Re_*^{-1.63} \quad (19)$$

As $Re^\star = (U_*H)\ v^{-1}$ and kinematic viscosity of water (v) is equal to $10^{-6}\ m^2\ s^{-1}$ for 20°C, Equation 19 is rearranged, becoming:

$$D_L=0,744\frac{H^{0.036}U^{1.59}}{U_*^{2.22}B^{0.66}} \quad (20)$$

To check the quality of adjustment carried out by multiple linear regression, the correlation coefficient (r^2) was analyzed and the statistical F-test was applied. The r^2 was equal to 0.87, which means that 87.0% of the variation of the dependent variable (D_L/U_*H) is being explained by the equation deduced from the regression (Equation 19), suggesting that this equation is appropriate. Considering the F-test with a significance level $\alpha = 0.1\%$, the value of 'F' tabled to 3.11 degrees of freedom (F = 11.56) was lower than the calculated value (F = 32.53), which implies rejection of the null hypothesis of the parameters and acceptance of the regression with 99.9% confidence. Figure 4 presents the comparison between the measured values of D_L by the routing procedure (x-axis) and the values estimated by Equation 20 (y-axis). It is notable that the points are fairly well distributed in the surrounding area of the values corresponding to the ratio $D_{L(}estimated)\ D_{L(}measured)^{-1} = 1$.

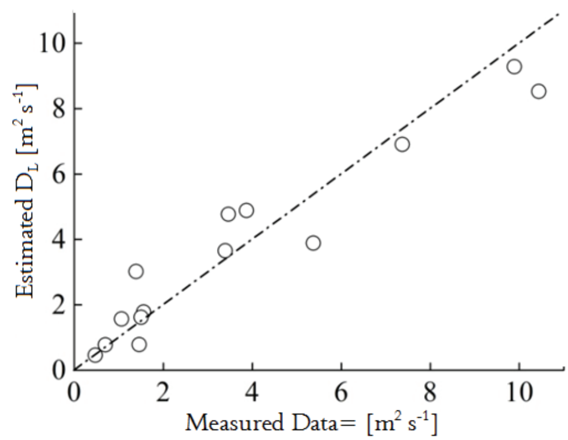

Figure 4. Comparison between the measured values of D_L by the routing procedure and the values estimated by equation 20.

Validation of the Equation

To validate the produced model, 31 sets of measured data (Table 2) were selected from studies that related the longitudinal dispersion coefficient with geometric and hydraulic characteristics of the channel (Barbosa Jr. et al, 2005; Ribeiro et al., 2010; Soares et al., 2013). These studies were carried out in rivers with flow rates ranging from 0.025 to 42.6 $m^3\ s^{-1}$. The comparison between the estimated and measured values was performed using error estimation, namely standard error (SE), normal mean error (NME) and mean multiplicative error (MME). The SE and the NME were determined using Equations 21 and 22:

$$SE=\left(\sum_{i=1}^{n}\frac{(D_{Le}-D_{Lm})_i^2}{n}\right)^{1/2} \quad (21)$$

$$NME=\frac{100\%}{n}\sum_{i=1}^{n}\left(\frac{D_{Le}-D_{Lm}}{D_{Lm}}\right)_i \quad (22)$$

where:

n is the number of measures,

D_{Le} e D_{Lm} are respectively the estimated and measured values of the longitudinal dispersion coefficient.

The MME was used by Moog and Jirka (1998) for evaluation of the errors produced by estimative equations of surface reaeration coefficient. These authors showed that this method is less sensitive to extreme errors and, therefore, can be a good alternative for quality evaluation of produced equations.

Table 2. Parameters of the watercourses used to validate the model.

Source	D_L [m² s⁻¹]	B [m]	H [m]	U [s⁻¹]	U_* [m s⁻¹]
Barbosa Jr. et al. (2005)	1.30	4.00	0.61	0.28	0.13
	2.42	4.00	0.62	0.28	0.13
	1.10	4.00	0.51	0.26	0.12
	2.01	4.50	0.81	0.33	0.15
	5.05	10.0	0.52	0.50	0.14
	4.22	10.0	0.54	0.52	0.14
	6.09	11.0	0.65	0.60	0.16
Ribeiro et al. (2010)	35.0	26.0	1.79	0.92	0.09
	10.0	25.5	1.17	0.66	0.08
	8.50	21.0	1.36	0.57	0.08
	12.0	23.0	1.31	0.77	0.08
	15.0	28.0	1.43	0.83	0.08
	16.0	44	1.34	0.65	0.14
	17.8	81	2.42	0.50	0.20
	19.7	40	1.37	0.65	0.15
Soares et al. (2013)	0.74	1.80	0.20	0.30	0.25
	0.86	1.05	0.20	0.33	0.24
	0.36	3.60	0.39	0.20	0.12
	0.60	2.15	0.26	0.53	0.10
	1.16	1.18	0.17	0.12	0.24
	0.10	1.33	0.32	0.09	0.30
	1.21	1.33	0.16	0.15	0.23
	0.13	1.58	0.32	0.10	0.31
	0.51	1.36	0.19	0.25	0.05
	0.23	2.85	0.27	0.25	0.07
	0.62	2.15	0.21	0.26	0.06
	6.57	13.8	0.35	0.42	0.11
	10.5	14.6	0.34	0.40	0.08
	17.0	6.95	0.37	0.41	0.08
	13.9	8.35	0.32	0.41	0.08
	4.64	5.25	0.29	0.31	0.07

The MME is defined as shown in Equation 23:

$$MME = \exp\left[\frac{\sum_{i=1}^{N}\left|\ln(D_{Le}/D_{Lm})_i\right|}{n}\right] \quad (23)$$

Figure 5 presents values of SE, NME and MME for six equations of D_L prediction, including the equation developed in this study.

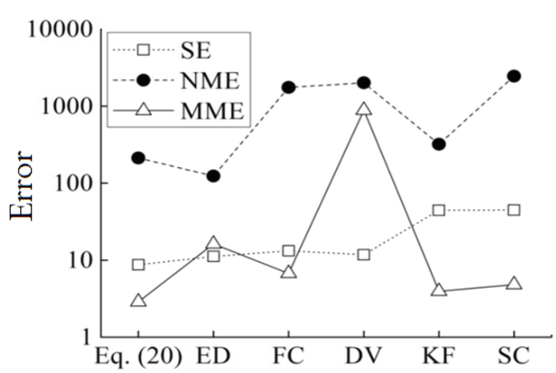

Figure 5. Standard error, normal mean error, and mean multiplicative error for different prediction equations of longitudinal dispersion coefficient: ED, Elder (1959); FC, Fischer (1975); DV, Devens et al. (2006); KF, Kashefipour and Falconer (2002); and SC, Seo and Cheong (1998).

The SE presented lower values than the NME and remains practically constant for the first fourequations in the graph, getting higher for Kashefipour and Falconer (2002) and Seo and Cheong (1998) equations. The NME values are high for all the equations, which show that, according to this error estimator, none of them would be suitable to estimate the dispersion coefficient. However, since SE and NME are differential errors, the results are considered biased, especially in cases in which underestimated values occur (Moog & Jirka, 1998). The MME, considered to have a better accuracy for error estimation, varies for the considered equations, having the smallest value (2.02) for the equation developed in this study.

In addition to the three methods of evaluation of errors (SE, NME and MME), the variances between the measured and estimated values of the longitudinal dispersion coefficient were analyzed using the discrepancy ratio (Equation 24) defined by White, Milli, & Crabbe (1973) and subsequently applied by Seo and Cheong (1998):

$$R_d = \log\frac{D_{Le}}{D_{Lm}} \quad (24)$$

where R_d is the discrepancy ratio.

Using this analysis it is possible to check whether D_L is underestimated ($R_d < 0$) or overestimated ($R_d > 0$), as well as to quantify the accuracy of each equation, which is defined as the percentage of the results of R_d that is within the range -0.3 to 0.3 (Seo & Cheong, 1998).

The R_d values depending on the aspect ratio (B/H) for each equation are presented in Figure 6. This figure shows that Elder's (1959) equation, Fischer's (1975) equation, and Devens' et al. (2006) equation underestimate D_L in most cases, resulting in low accuracy: 3, 29% and zero, respectively. Seo and Baek (2004) warn that the Elder's equation tends to underestimate the values of D_L, since the transversal variation of the velocity profile is not considered in its formulation. The inaccuracy produced by Devens' equation can be explained by the fact that this equation has been formulated using field tests in streams with hydraulic and geometric characteristics that go beyond the values of Table 2, used for the validation of the equations.

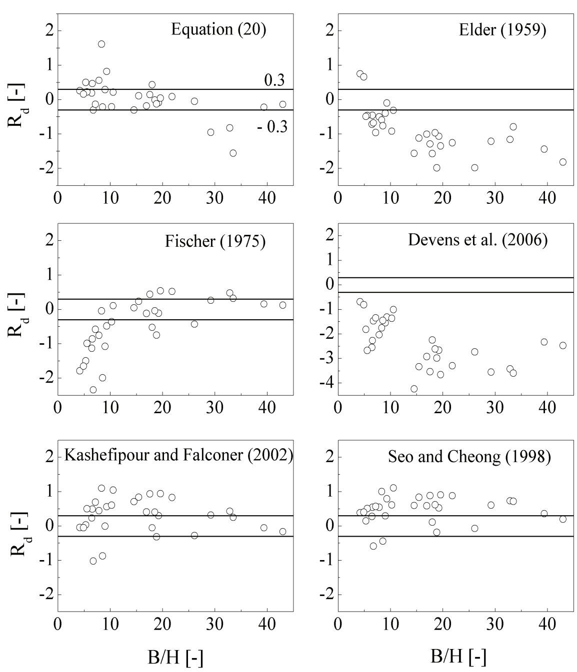

Figure 6. Comparison between estimated and measured D_L.

In contrast, Kashefipour and Falconer's (2002) and Seo and Cheong's (1998) equations in most cases overestimate D_L, and, similarly to the other equations, feature low accuracy, 39 and 23%, respectively. Differently, the equation developed in this study, although there is some points out of the range -0.3 to 0.3, has accuracy of 71%; the minimum value of R_d obtained was -1.5 and the maximum was 1.6. These results demonstrate that the proposed equation for prediction of longitudinal dispersion coefficient is superior to the existing equations analyzed in this study.

Conclusion

This study resulted in a semi-empirical equation for predicting the longitudinal - dispersion coefficient using dimensional analysis and regression analysis of experimental data measured in two streams of the Grande River watershed (State of Minas Gerais, Brazil). The equation presented good results for predicting D_L when compared with other existing equations in literature. However, the analysis of other studies about this subject in the literature shows that the possibility of existence of a single empirical or semi-empirical equation that is able to estimate this coefficient for different hydraulic and geometric conditions as those found at a variety of existing watercourses is becoming increasingly distant.

The used tracer, sodium chloride, proved to be a good alternative for the estimation of D_L in low discharge watercourses because it is cheap and easily measurable. Despite the fact that this tracer is not conservative, a correction technique of tracer loss can be used, such as the one used by Devens et al. (2006).

The proposed semi-empirical equation can be useful when the laborious field tests for quantifying the longitudinal dispersion coefficient are not possible. Also, it can be applied in other watercourses since their hydraulic and climatic conditions are similar to those for which the equation was obtained.

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