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Approximate orbits of comets

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ABSTRACT. This paper presents a geometrical method to calculate orbits of comets, especially Halley's and Hale Bopp's besides, it discusses the important notion of odograph of cometary movement.

Key words: Kepler's law, Halley's and Hale-Bopp's comet, physics and astronomy teaching.

RESUMO. Órbitas aproximadas de cometas. Este artigo apresenta um método geométrico para calcular órbitas de cometas, especialmente aquelas dos cometas Halley e Hale-Bopp, discutindo ainda a importante noção de odógrafa de um movimento cometário.

Palavras-chave: leis de Kepler, cometas Halley e Hale-Bopp, ensino de física e astronomia.

In the present article we will develop a geometrical technique based on the second law of Kepler (Law of Areas). Basically it consists of a reproduction of triangles of the same area on an ellipsis from an original triangle.

Method of triangles in ellipsis

The second law of Kepler states that in a given amount of time a line joining any planet to the sun sweeps out the same amount of area no matter where the planet is on its elliptical orbit.

Figure 1 exemplifies the geometrical method of reproduction of triangles on a curve that allows the possibility of triangles of the same area. The method consists of the following stages: from the principal focus (occupied by the sun) we construct the first triangle Δ_1. When the height h_1 of the triangle is found, we will extend it to keep a right angle, perpendicular to cathetus of the first triangle. With two setsquares we will find a point in the ellipsis which will give us the same height h_1. When this point is found, we will pass a straight line through it till we find the sun, the principal focus of the ellipsis. Thus, we have a second triangle Δ_2 with the same area as the first Δ_1 using as a common base one side (cathetus) of the previous triangle. Figure 2 shows this method (called Method of Triangles in Ellipsis, or MTE) applied in its totality on an ellipsis with a great eccentricity.

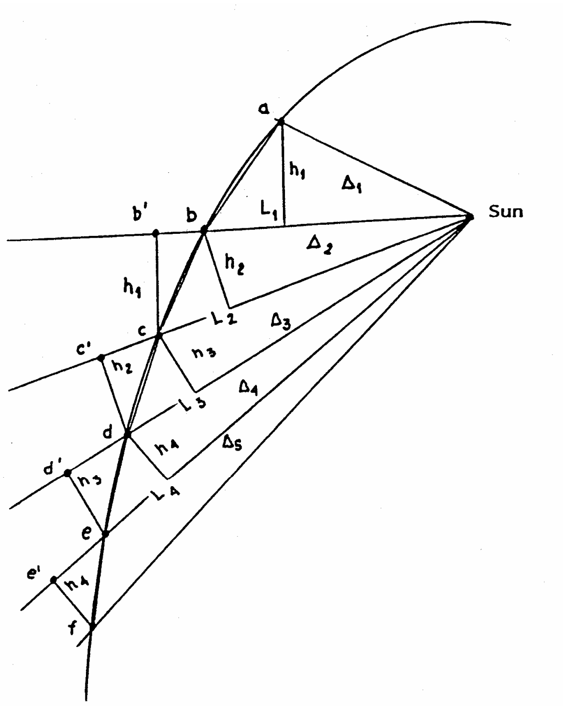

Figure 1. Method of triangle in ellipsis (MTE)

Let us apply this method to the orbit of a well-known comet: Halley's comet. First, we have to know some of its orbital characteristics:

- Period of orbital revolution: 76.008 years;
- Minimum distance of comet from sun (perihelion): a' = 0.587 A.U.;

- Inclination of orbit's plane in relation to ecliptic: i = 162.24°;
- Major semi-axis: a = 17.94 A.U.;
- Longitude of ascendent node: Ω = 58.15°;
- Argument of perihelion: ϖ = ω - Ω = 111.80°;
- Minimum distance of comet from Earth (perigee): 0.42 A.U. (in April 11, 1986);
- Time of perihelion passage: February 9, 1986;
- Eccentricity: ε = 0.97;
- Direction of orbital movement: retrograde.

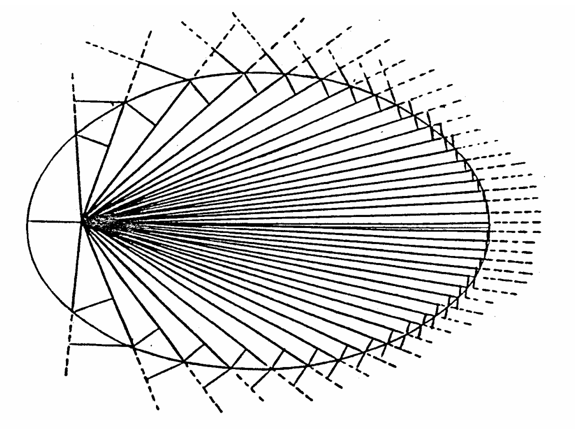

Figure 2. Method of triangles in ellipsis (MTE) applied in its totality

Figure 3 shows the orbital elements of Halley's comet. To apply MTE we should, first construct the characteristic ellipsis of a comet orbit. Figure 4 demonstrates the elements of the ellipsis. Below, formula (1) agrees with the equation of ellipsis in its polar form:

$$r = [a\,(1 - \varepsilon^2)] / (1 - \varepsilon \cos\theta) \qquad (1)$$

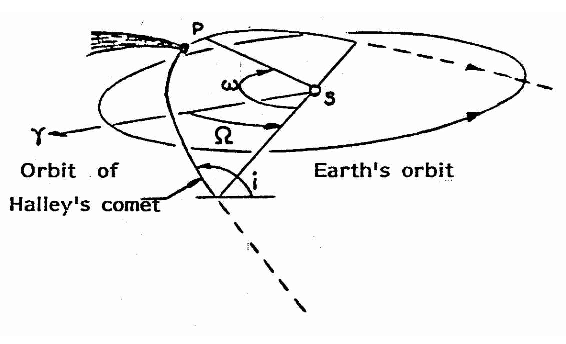

Figure 3. Orbital elements of Halley's comet

We see that the only variable present in the equation (1) is the angle θ. Thus, if we alter the latter (on the condition that we have previously chosen a scale for the astronomical unit, A.U.) we get the angular variation of *r*, giving us the possibility of constructing the ellipsis.

For the construction of the first triangle we have a fundamental datum: the date of the perihelion passage of the comet (the passage of time referring to previous appearances in 1759, 1835-36 and 1909-11). If we want to monitor the comet's orbit every ten days, for instance, we have to know the area travelled by the comet during this interval of time.

We know that the area of an ellipsis is given as $S_T = \pi a\, b$[1]. So, finding the partial area S_P in an interval of ten days is an easy matter, using only the simple rule of proportion.

Having the value of the perihelion distance a' (Figure 4) and the partial area SP, we can find the height of triangle Δ1 in an approximative method:

$$h_1 = (2\, S_P) / a' \qquad (2)$$

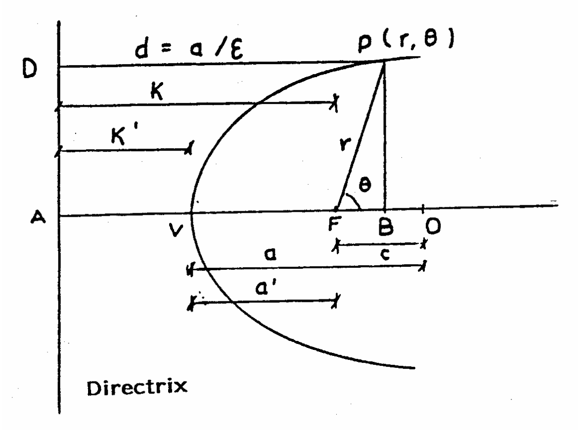

Figure 4. Elements of an ellipsis

When the first triangle is found, we will reproduce it along the ellipsis (due to its extreme eccentricity of the comet's orbit it will not be reproduced along all the ellipsis) using MTE.

Figure 5 shows the orbit of the comet deduced by MTE and table 1 exhibits the values of ecliptical longitude λ for the points present in the Figure and obtained by the method of decentered circumferences (Neves and Argüello, 1986). Table 2 compares some important events between the values calculated and the values referred to in the bibliography (Anuário Astronômico, 1986).

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[1] $a^2 = b^2 + c^2$ and $c = \varepsilon\, a$ are proper to ellipsis. Thus, $b = a\,(1 - \varepsilon^2)^{1/2}$

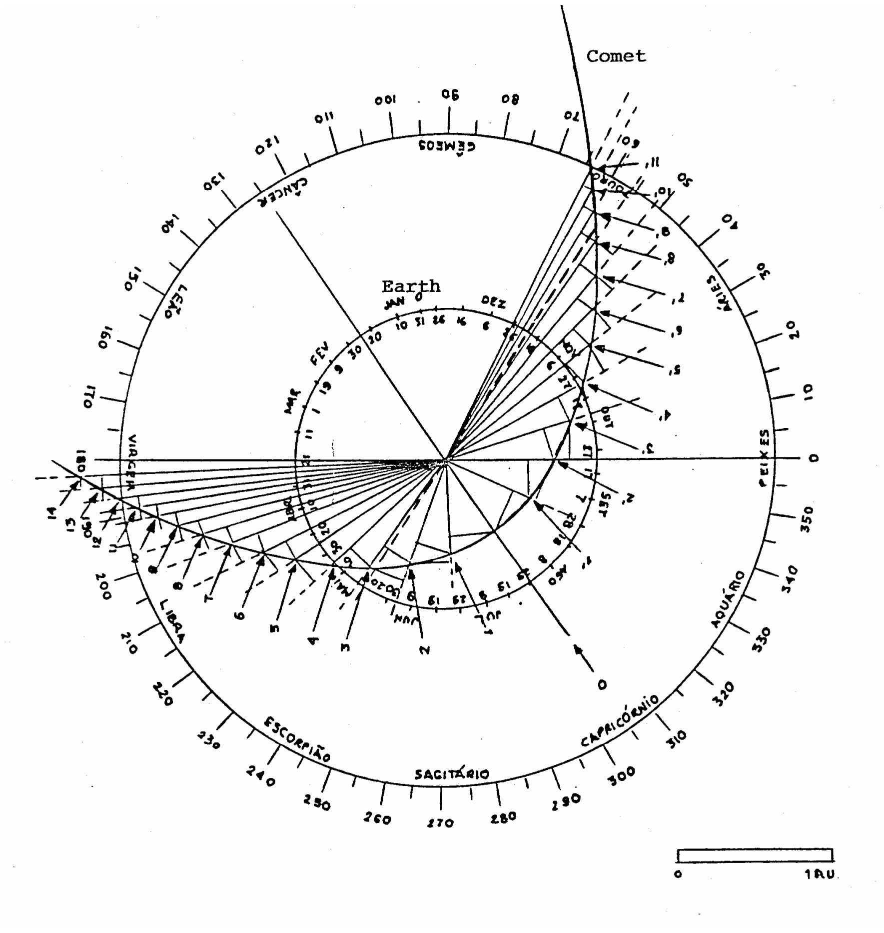


Figure 5. Orbit of Halley's comet deduced by MTE

To delineate the apparent movement of the comet, we must know the various ecliptic latitudes β. To calculate this in a simple way (this method was presented for appreciation to high school students) we take Figure 5 and draw it once more. However, this time it will be inverted. From this figure we can still cut the part that corresponds to Halley's orbit in the strict sense and invert it too. We can thus construct a tridimensional model (Figure 6) which allows us to find latitude β with certain ease.

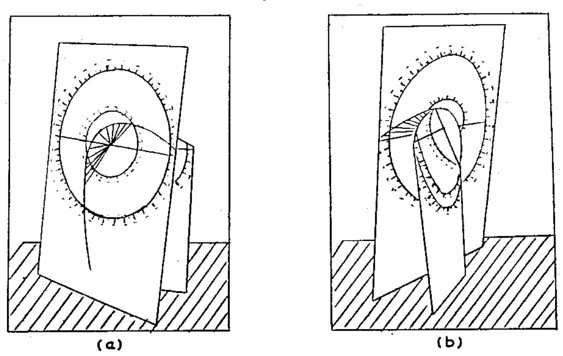


Figure 6. Tridimensional model

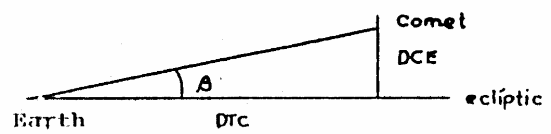
We deduce an elementary trigonometric relation from the tridimensional model so that we may find the ecliptic geocentric latitude (as seen from the Earth – see Figure 7). From this last figure we have:

$$\beta = \text{arc}\,[\tan(DCE/DTC)] \qquad (3)$$

where,

DCE is the distance of the comet to the plane of the ecliptic.

DTC is the distance of the Earth to the comet.



β = Ecliptic geocentic latitude

Figure 7. Trigonometric relation from the tridimensional model

Table 3 presents values of some ecliptic latitudes found.

Figure 8 shows the apparent trajectory calculated by MTE and Figure 9, the same trajectory, but given in the bibliography (Anuário Astronômico, 1986).

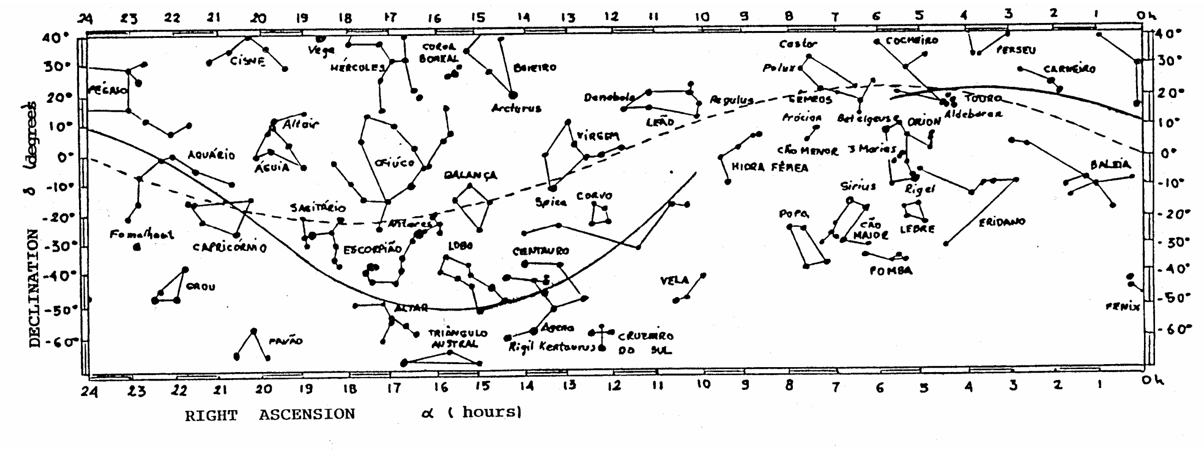


Figure 8. Apparent trajectory of Halley's comet by MTE

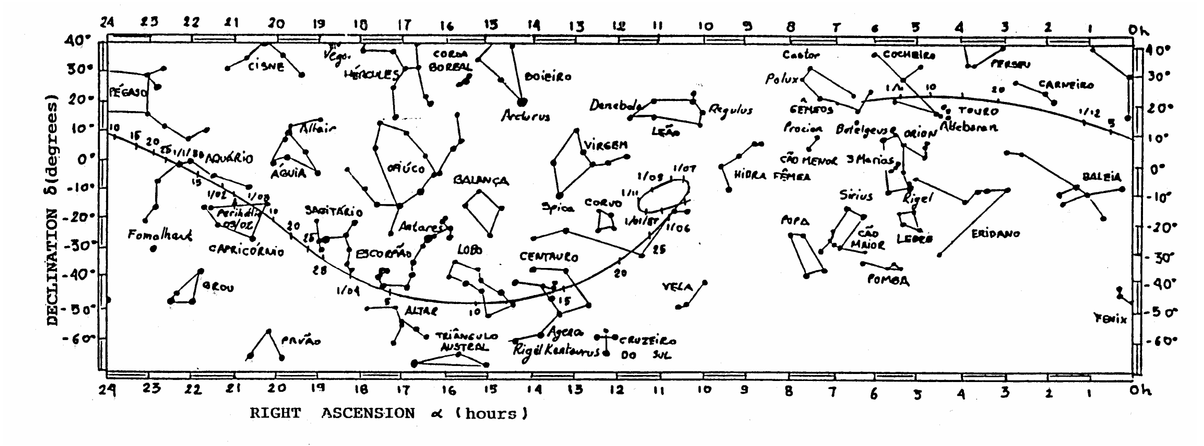


Figure 9. Apparent trajectory given in the bibliography

Table 1. Ecliptic longitudes

Points in the orbit of Halley's comet	Corresponding dates	Ecliptic longitude (degrees)	Ecliptic longitude (hours)
13'	10.01.1985	93	6.2
12'	10.11	91	6.0
11'	10.21	87	5.8
10'	10.31	84	5.6
9'	11.10	72	4.8
8'	11.20	52	3.5
7'	12.01	25	1.7
6'	12.11	0	0.0
5'	12.21	344	22.9
4'	12.31	335	22.3
3'	01.10.1986	328	21.9
2'	01.20	325	21.7
1'	01.30	321	21.4
0	02.09 (perihelion)	315	21.0
1	02.19	309	20.6
2	03.01	305	20.3
3	03.11	299	19.9
4	03.21	291	19.4
5	03.31	273	18.2
6	04.10	232	15.5
7	04.20	187	12.5
8	04.30	172	11.5
9	05.10	164	10.9
10	05.20	160	10.7
11	05.30	159	10.6
12	06.09	159	10.6
13	06.19	160	10.7

Table 2. Comparative values

Event	Calculated date	Date given in bibliography (Anuário Astronômico, 1986)
1st Opposition	Between 20 and 11.21.85	11.18.1985
Conjuction	Between 04 and 02.05.86	02.06.1986
2nd Opposition	Between 16 and 04.17.86	04.17.1986
Perigee	Between 11 and 04.12.86	04.11.1986

Table 3. Ecliptic latitudes and longitudes (the sign (+) indicates superior position to the ecliptic, and sign (-), position inferior to it)

Days	Point in the comet's orbit	Latitude (degrees)	Longitude (hours)
11.01.85	≅ 10'	- 2.2	≅ 5.6
11.27.85	Between 8' and 7'	+ 3.8	≅ 2.6
01.01.86	≅ 4'	+ 8.7	≅ 22.3
02.01.86	≅ 1'	+ 9.1	≅ 21.4
02.09.86	≅ 0	+ 9.1	≅ 21.0
03.01.86	≅ 2	+ 4.2	≅ 20.3
04.01.86	≅ 5	- 21.8	≅ 18.2
04.11.86	≅ 6	- 32.9	≅ 15.5
05.01.86	≅ 8	- 28.5	≅ 11.5
06.01.86	≅ 11	- 20.4	≅ 10.6
07.04.86	Between 11 and 12	- 15.6	≅ 10.4

Hale-Bopp Comet

Using the same method for a comet that was recently discovered in our skies, we can also calculate the orbit of the Hale-Bopp's comet. The Figure 10 shows the comet as photographed in the skies of Maringá (city in the south of Brazil, latitude 23.5°S and longitude 52°), toward west direction, below the Constellation of Orion, on the 04.29.1997, at 19h00min (local time).

Figure 10. Hale-Bopp's comet in the skies of Maringá

This comet was recently discovered (on July 23, 1995) by the astronomers Alan Hale and Thomas Bopp. Its initial periodicity is estimated in a value already 4,200 years, but due to the intense force gravitational of Jupiter, your orbital period is estimated in about 2,380 years.

The orbital elements of that comet are:

- Age of comet: ≅ 4.5 billion years.
- Nucleus' diameter: 40 km.
- Dimension: 137,000,000 km.
- Minimun distance from Earth: 195,250,000 km.
- Date of perigee: March 22, 1997.
- Period: 2,380 years.
- Minimun distance from the sun (perihelion): 137,000,000 km.
- Argument of perihelion: ω = 130.59°.
- New date of perihelion: year 4,377 a.C.
- New aphelion: year 3,187 a.C.
- Last date of aphelion: 103 b.C.
- Orbital velocity in the perihelion: v = 44 km/sec.
- Eccentricity: ε = 0.9951.
- Inclination with respect to ecliptic: $i \cong 90°$.
- Longitude of ascendent node: Ω = 282.47°.

The Figure 11 shows the trajectory of Hale-Bopp's comet using MTE and the Figure 12 shows the apparent trajectory (right ascension and declination): the full line is that one given in the bibliography (Anuário Astronômico, 1997) and the dotted line those calculated by the method. The differences in the values of the right ascensions are verified because they were not made the three-dimensional model to obtain the values of those variables. However, the results obtained by the method it is possible to locate, approximately, the position of the comet.

Odograph of a Comet's Movement

If we wish to verify the precision of the method developed here (and to confirm forecasts of other comets, for instance, Hale-Bopp's comet) we may use the notion of odograph[2].

The odograph of a movement corresponds to a polar diagram of a vectorial velocity (Boczko, 1984), that is, to the curve obtained by the union of the extremities of vectorial velocities, drawn from a point considered as the odographic pole (pole **O** – see Figures 13).

The odograph is a way of verifying the conservation of the angular momentum and the law of the areas for a cometary (or planetary) movement, based on figures obtained in the previous section and, in a special manner, in Figure 5.

For the conservation of angular momentum, we have,

$$\vec{r} \times \vec{v} = constant \quad (4)$$

In polar coordinates we would have,

$$r^2\, d\theta / dt = A = constant \quad (5)$$

where r and θ are the polar coordinates of the ellipsis and A is a constant that has a dimension of area, or better, of areolar velocity which, according to Kepler's law, is always constant.

We can deduce the equations of velocities in the diagram v_x *versus* v_y:

$$v_x = -(A / R) \sin \theta \quad (6)$$

$$v_y - (A\, \varepsilon) / R = (A / R) \cos \theta \quad (7)^{*}$$

Thus,

$$v_x^2 + \{v_y - [(A\, \varepsilon) / R]\}^2 = (A^2 / R^2) \quad (8)$$

[2] The idea of a odograph originated with A. F. Möebius (1843) and, in an independent way, with Sir William Rowan Hamilton (1846) to whom we owe the name and certain original developments of the theory (Encyclopaedia Britannica, 1971).

[*] The equation to v_y obtained in the bibliography (Toth and Bardócz, 1983) is incorrect. The mistake occured when the authors differentiated y with respect to time t. This fact affected the final interpretation of the diagrams obtained by the dynamic demonstration model of planetary movements.

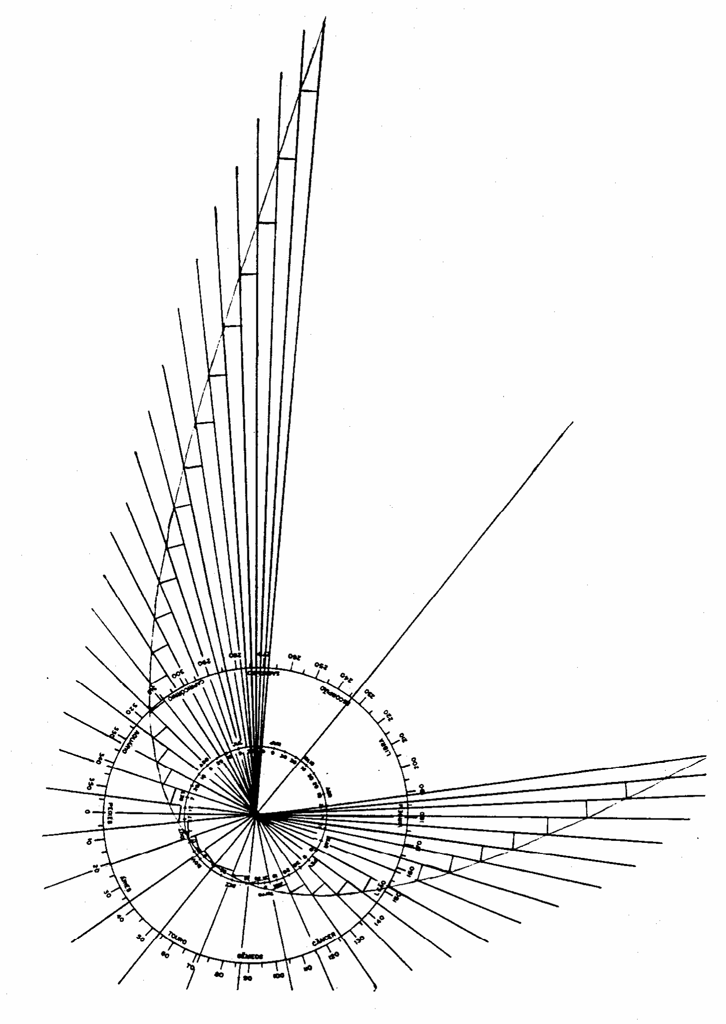


Figure 11. Hale-Bopp's comet deduced by MTE

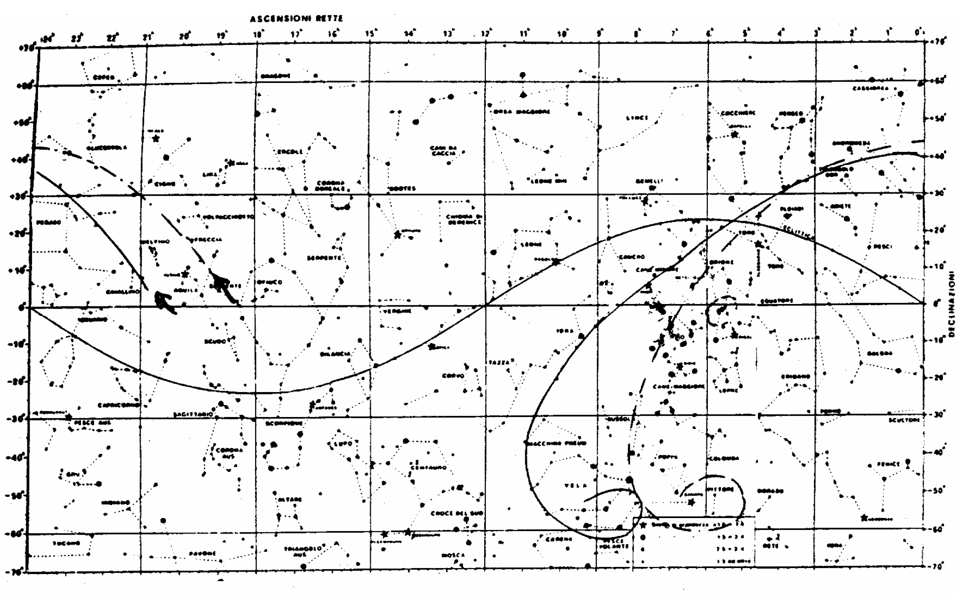


Figure 12. Comparative apparent trajectory of Hale-Bopp's comet

which corresponds to the equation of a circumference of radius *(A / R)* and of focus *F* displaced *[(A ε) / R]* from the center *O* of the circumference thus obtained [*R* is a semi-latus rectum and equal to $\rightarrow a\,(1 - \varepsilon^2)$].

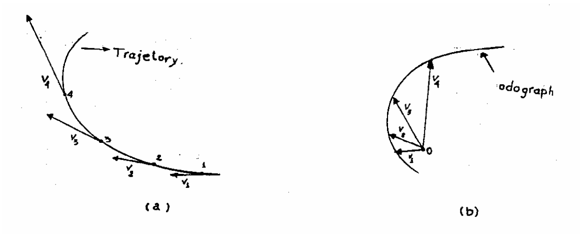


Figure 13. Diagrams of vectorial velocities

By equation (8) we perceive that the odograph of an elliptic movement is a circumference. This may be seen in Figures 14 and 15 which correspond to the odograph of Halley's elliptic movement, obtained from Figure 5 of the previous section, with the pole O of the former on the principal focus of the comet's orbit, that is, on the sun.

We also perceive on Figure 14 how the linear velocity of the comet varies with respect to its distance from the sun.

We may finish this article since we have discussed the odograph. Using a relatively simple geometrical technique, it is possible to teach Kepler's laws (especially the first and second) ones in an alternative form. It is practical and accessible to students of college level. Depending on the presentation of the theme by the teacher, it is also accessible to upper secondary school students as it was proved in previous given courses to the writing of this paper.

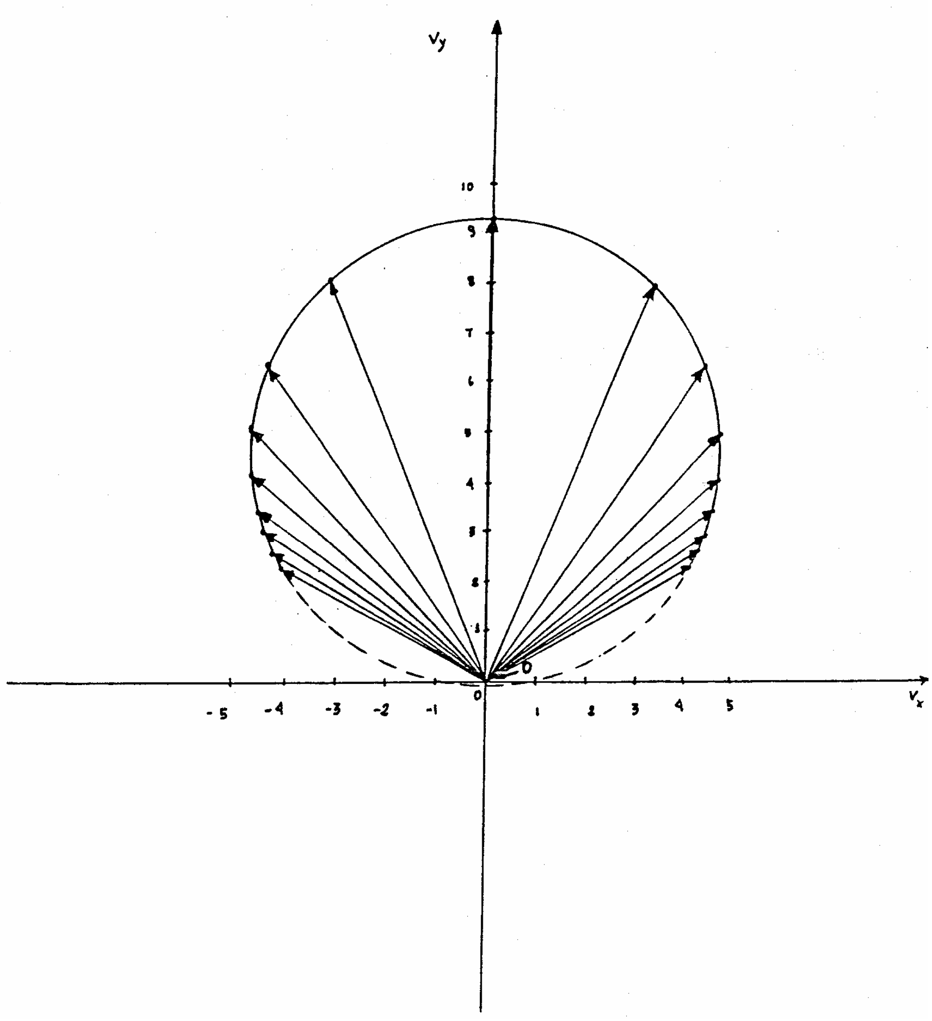


Figure 14. Odograph of Halley's comet

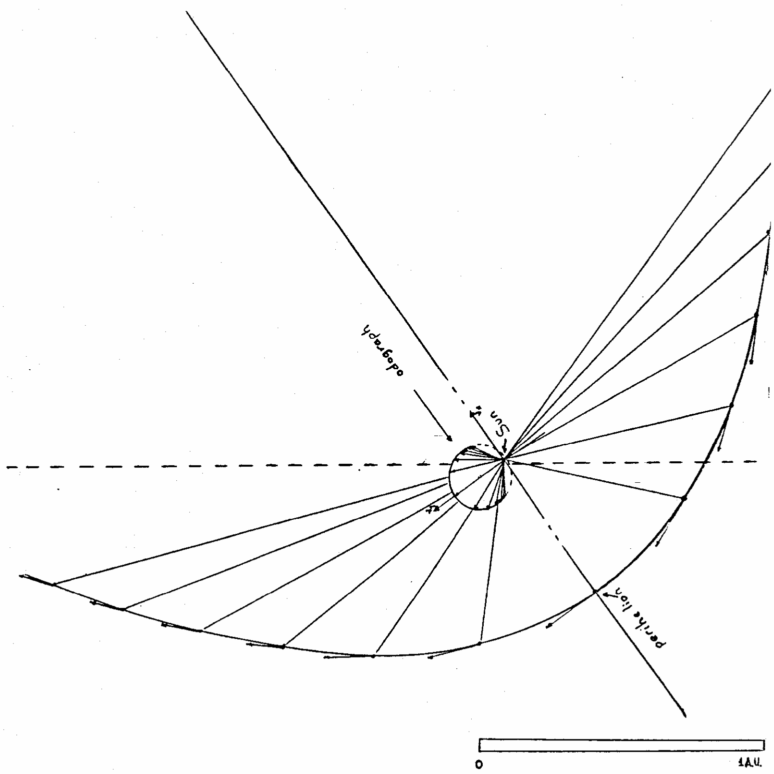


Figure 15. Odograph of Halley's elliptic movement

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References

Anuário Astronômico. São Paulo: Instituto Astronômico e Geofísico da Universidade de São Paulo, 1986.

Anuário Astronômico. São Paulo: Instituto Astronômico e Geofísico da Universidade de São Paulo, 1997.

Boczko R. *Conceitos de astronomia.* São Paulo: Edgard Blücher, 1984.

Encyclopaedia Britannica, William Benton Publisher, 1971. v.11.

Neves, M.C.D.; Argüello, C.A. *Astronomia de régua e compasso:* de Kepler a Ptolomeu. Campinas: Papirus, 1986.

Tóth, L.S.; Bardócz, A. A dynamical model for the demonstration of planetary motions or 'celestial mechanics on the desk'. *Europ. J. Phys.*, *4*:24-33, 1983.

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