pre-g-bi-irresolute and pre-g-stable in ditopological texture spaces

Hariwan Zikri Ibrahim

ABSTRACT: We introduce and study new notions of continuity, compactness and stability in ditopological texture spaces based on the notions of pre-g-open and pre-g-closed sets and some of their characterizations are obtained.

Key Words: Texture, difunction, pre-g-bi-irresolute, pre-g-stability.

Contents

1 Introduction 247
2 Preliminaries 247
3 pre-g-bicontinuous, pre-g-bi-irresolute, pre-g-compact and pre-g-stable 250

1. Introduction

Textures and ditopological texture spaces were first introduced by L. M. Brown as a point-based setting for the study of fuzzy topology. The study of compactness and stability in ditopological texture spaces was started to begin in [6]. In this paper, we introduce and study the concepts of pre-g-bicontinuity, pre-g-bi-irresolute, pre-g-compactness and pre-g-stability in ditopological textures spaces.

2. Preliminaries

The following are some basic definitions of textures we will need later on.

Texture space: [6] Let $S$ be a set. Then $\varphi \subseteq P(S)$ is called a texturing of $S$, and $S$ is said to be textured by $\varphi$ if

1. $(\varphi, \subseteq)$ is a complete lattice containing $S$ and $\phi$ and for any index set $I$ and $A_i \in \varphi$, $i \in I$, the meet $\bigwedge_{i \in I} A_i$ and the join $\bigvee_{i \in I} A_i$ in $\varphi$ are related with the intersection and union in $P(S)$ by the equalities

$$\bigwedge_{i \in I} A_i = \bigcap_{i \in I} A_i$$

for all $I$, while

$$\bigvee_{i \in I} A_i = \bigcup_{i \in I} A_i$$

for all finite $I$.

2. $\varphi$ is completely distributive.

2000 Mathematics Subject Classification: 35B40, 35L70
3. \( \varphi \) separates the points of \( S \). That is, given \( s_1 \neq s_2 \) in \( S \) we have \( L \in \varphi \) with \( s_1 \in L, s_2 \notin L \), or \( L \in \varphi \) with \( s_2 \in L, s_1 \notin L \).

If \( S \) is textured by \( \varphi \) then \( (S, \varphi) \) is called a texture space, or simply a texture.

**Complementation:** [6] A mapping \( \sigma : \varphi \to \varphi \) satisfying \( \sigma(\sigma(A)) = A \), for all \( A \in \varphi \) and \( A \subseteq B \) implies that \( \sigma(B) \subseteq \sigma(A) \), for all \( A, B \in \varphi \) is called a complementation on \( (S, \varphi) \) and \( (S, \varphi, \sigma) \) is then said to be a complemented texture.

For a texture \( (S, \varphi) \), most properties are conveniently defined in terms of the \( p \)-sets

\[
P_s = \bigcap \{ A \in \varphi : s \in A \}
\]

and the \( q \)-sets,

\[
Q_s = \bigvee \{ A \in \varphi : s \notin A \}.
\]

**Ditopology:** [6] A dichotomous topology on a texture \( (S, \varphi) \), or ditopology for short, is a pair \( (\tau, k) \) of subsets of \( \varphi \), where the set of open sets \( \tau \) satisfies

1. \( S, \emptyset \in \tau \),
2. \( G_1, G_2 \in \tau \) implies that \( G_1 \cap G_2 \in \tau \), and
3. \( G_i \in \tau, i \in I \) implies that \( \bigvee_i G_i \in \tau \),

and the set of closed sets \( k \) satisfies

1. \( S, \emptyset \in k \),
2. \( K_1, K_2 \in k \) implies that \( K_1 \cup K_2 \in k \), and
3. \( K_i \in k, i \in I \) implies that \( \bigwedge_i K_i \in k \).

Hence a ditopology is essentially a "topology" for which there is no a priori relation between the open and closed sets.

For \( A \in \varphi \) we define the closure \( [A] \) and the interior \( ]A[ \) of \( A \) under \( (\tau, k) \) by the equalities

\[
[A] = \bigcap \{ K \in k : A \subseteq K \} \quad \text{and} \quad ]A[ = \bigvee \{ G \in \tau : G \subseteq A \}
\]

We refer to \( \tau \) as the topology and \( k \) as the cotopology of \( (\tau, k) \).

If \( (\tau, k) \) is a ditopology on a complemented texture \( (S, \varphi, \sigma) \), then we say that \( (\tau, k) \) is complemented if the equality \( k = \sigma(\tau) \) is satisfied. In this study, a complemented ditopological texture space is denoted by \( (S, \varphi, \tau, k, \sigma) \).

In this case we have \( \sigma([A]) = ]\sigma(A)[ \) and \( \sigma([A]) = [\sigma(A)] \).

We denote by \( O(S, \varphi, \tau, k) \), or when there can be no confusion by \( O(S) \), the set of open sets in \( \varphi \). Likewise, \( C(S, \varphi, \tau, k) \), \( C(S) \) will denote the set of closed sets.

Let \( (S_1, \varphi_1) \) and \( (S_2, \varphi_2) \) be textures. In the following definition we consider the product texture \( [3] P(S_1) \otimes \varphi_2 \), and denote by \( \overline{P}_{(s,t)} \), \( \overline{Q}_{(s,t)} \), respectively the \( p \)-sets and \( q \)-sets for the product texture \( (S_1 \times S_2, P(S_1) \otimes \varphi_2) \).

**Direlation:** [5] Let \( (S_1, \varphi_1) \) and \( (S_2, \varphi_2) \) be textures. Then
One of the most useful notions of (ditopological) texture spaces is that of difunctions: A difunction is a special type of direlation. Let \( \phi : (S_1, \varphi_1) \to (S_2, \varphi_2) \) be a difunction. Then \( (f, F) : (S_1, \varphi_1) \to (S_2, \varphi_2) \) is called a direlation from \( (S_1, \varphi_1) \) to \( (S_2, \varphi_2) \).

Image and Inverse Image: \([5]\) Let \((f, F) : (S_1, \varphi_1) \to (S_2, \varphi_2) \) be a difunction.

1. For \( A \in \varphi_1 \), the image \( f^\rightarrow A \) and the co-image \( F^\rightarrow A \) are defined by

\[
\begin{align*}
    f^\rightarrow A &= \bigcap \{ Q_t : \text{for all } s, f \not\in \overline{Q}_s(t) \text{ implies that } A \subseteq Q_s \}, \\
    F^\rightarrow A &= \bigvee \{ P_t : \text{for all } s, \overline{P}_s(t) \not\subseteq F \text{ implies that } P_s \subseteq A \}.
\end{align*}
\]

2. For \( B \in \varphi_2 \), the inverse image \( f^\leftarrow B \) and the inverse co-image \( F^\leftarrow B \) are defined by

\[
\begin{align*}
    f^\leftarrow B &= \bigvee \{ P_t : \text{for all } s, f \not\in \overline{Q}_s(t) \text{ implies that } P_t \subseteq B \}, \\
    F^\leftarrow B &= \bigcap \{ Q_s : \text{for all } t, \overline{Q}_s(t) \not\subseteq F \text{ implies that } B \subseteq Q_t \}.
\end{align*}
\]

For a difunction, the inverse image and the inverse co-image are equal, but the image and co-image are usually not.

Bicontinuity: \([4]\) The difunction \((f, F) : (S_1, \varphi_1), \tau_1, k_1 \to (S_2, \varphi_2), \tau_2, k_2 \) is called continuous if \( B \in \tau_2 \) implies that \( F^\rightarrow B \in \tau_1 \), and bicontinuous if it is both continuous and cocontinuous.

Surjective difunction: \([5]\) Let \((f, F) : (S_1, \varphi_1) \to (S_2, \varphi_2) \) be a difunction. Then \((f, F) \) is called surjective if it satisfies the condition.

\[
\text{SUR. For } t, t' \in S_2, P_t \not\subseteq Q_{t'} \text{ implies that there exists } s \in S_1 \text{ with } f \not\in \overline{Q}_s(t, t') \text{ and } \\
\overline{P}_s(t) \not\subseteq F.
\]

If \((f, F)\) is surjective then \( F^\rightarrow (f^\leftarrow B) = B = f^\rightarrow (F^\leftarrow B) \) for all \( B \in \varphi_2 \) \([5], \text{ Corollary 2.33}\).
Definition 2.1. [5] Let \((f, F)\) be a difunction between the complemented textures \((S_1, \varphi_1, \sigma_1)\) and \((S_2, \varphi_2, \sigma_2)\). The complement \((f, F)' = (F', f')\) of the difunction \((f, F)\) is a difunction, where \(f' = \bigcap_{(s, t)} \exists u, v \text{ with } f \notin Q_{u,v}, \sigma_1(Q_s) \notin Q_u \text{ and } P_v \notin \sigma_2(P_t)\) and \(F' = \bigvee_{(s, t)} \exists u, v \text{ with } F_{u,v} \notin F, P_u \notin \sigma_1(P_s) \text{ and } \sigma_2(Q_s) \notin Q_v\).

If \((f, F) = (f, F)'\) then the difunction \((f, F)\) is called complemented.

Definition 2.2. [7] Let \((S, \varphi, \tau, k)\) be a ditopological texture space. A set \(A \in \varphi\) is called pre-open (pre-closed) if \(A \subseteq [A] [\cup [A] \subseteq A]\).

We denote by \(PO(S, \varphi, \tau, k)\), or when there can be no confusion by \(PO(S)\), the set of pre-open sets in \(\varphi\). Likewise, \(PC(S, \varphi, \tau, k)\), or \(PC(S)\) will denote the set of pre-closed sets.

Definition 2.3. [2] Let \((S, \varphi, \tau, k)\) be a ditopological texture space. A subset \(A \in \varphi\) is said to be generalized closed (g-closed for short) if \(A \subseteq G \in \tau\) then \([A] \subseteq G\).

Definition 2.4. [2] Let \((S, \varphi, \tau, k, \sigma)\) be a complemented ditopological texture space. A subset \(A \in \varphi\) is said to be generalized open (g-open for short) if \(\sigma(A)\) is g-closed.

We denote by \(gc(S, \varphi, \tau, k)\), or when there can be no confusion by \(gc(S)\), the set of g-closed sets in \(\varphi\). Likewise, \(go(S, \varphi, \tau, k, \sigma)\), or \(go(S)\) will denote the set of g-open sets.

Definition 2.5. [1] Let \((S, \varphi, \tau, k)\) be a ditopological texture space. A subset \(A \in \varphi\) is said to be pre-g-closed if \(A \subseteq G \in PO(S)\) then \([A] \subseteq G\).

We denote by \(pregc(S, \varphi, \tau, k)\), or when there can be no confusion by \(pregc(S)\), the set of pre-g-closed sets in \(\varphi\).

Definition 2.6. [1] Let \((S, \varphi, \tau, k, \sigma)\) be a complemented ditopological texture space. A subset \(A \in \varphi\) is called pre-g-open if \(\sigma(A)\) is pre-g-closed.

We denote by \(preg(S, \varphi, \tau, k, \sigma)\), or when there can be no confusion by \(preg(S)\), the set of pre-g-open sets in \(\varphi\).

Definition 2.7. [1] Let \((S, \varphi, \tau, k, \sigma)\) be a complemented ditopological texture space. For \(A \in \varphi\), we define the pre-g-closure \([A]_{pre-g}\) and the pre-g-interior \([A]_{pre-g}^\ast\) of \(A\) under \((\tau, k)\) by the equalities

\[ [A]_{pre-g} = \bigcap \{K \in preg(S) : A \subseteq K\} \text{ and } [A]_{pre-g}^\ast = \bigcup \{G \in preg(S) : G \subseteq A\}. \]

3. pre-g-bicontinuous, pre-g-bi-irresolute, pre-g-compact and pre-g-stable

Definition 3.1. The difunction \((f, F) : (S_1, \varphi_1, \tau_1, k_1, \sigma_1) \rightarrow (S_2, \varphi_2, \tau_2, k_2, \sigma_2)\) is called:
1. pre-g-continuous (pre-g-irresolute), if \( F^{-} (G) \in \text{prego}(S_1) \), for every \( G \in O(S_2) \) \( (G \in \text{prego}(S_2)) \).

2. pre-g-cocontinuous (pre-g-co-irresolute), if \( f^{-} (G) \in \text{pregc}(S_1) \), for every \( G \in k_2 \) \( (G \in \text{pregc}(S_2)) \).

3. pre-g-bicontinuous, if it is pre-g-continuous and pre-g-cocontinuous.

4. pre-g-bi-irresolute, if it is pre-g-irresolute and pre-g-co-irresolute.

**Corollary 3.2.** Let \((f, F) : (S_1, \varphi_1, \tau_1, k_1, \sigma_1) \rightarrow (S_2, \varphi_2, \tau_2, k_2, \sigma_2)\) be a difunction. Then:

1. Every continuous is pre-g-continuous.
2. Every cocontinuous is pre-g-cocontinuous.
3. Every pre-g-irresolute is pre-g-continuous.
4. Every pre-g-co-irresolute is pre-g-cocontinuous.

**Proof.** Clear.

**Theorem 3.3.** Let \((f, F) : (S_1, \varphi_1, \tau_1, k_1, \sigma_1) \rightarrow (S_2, \varphi_2, \tau_2, k_2, \sigma_2)\) be a difunction. Then:

1. The following are equivalent:
   (a) \((f, F)\) is pre-g-continuous.
   (b) \(F^{-} A^{S_2} \subseteq F^{-} A^{\text{pre}-g}_{S_2}\), for all \( A \in \varphi_1\).
   (c) \(f^{-} B^{S_2} \subseteq f^{-} B^{\text{pre}-g}_{S_1}\), for all \( B \in \varphi_2\).

2. The following are equivalent:
   (a) \((f, F)\) is pre-g-cocontinuous.
   (b) \(f^{-} [A]^{S_1}_{\text{pre}-g} \subseteq [f^{-} A]^{S_2}\), for all \( A \in \varphi_1\).
   (c) \([F^{-} B]^{S_1}_{\text{pre}-g} \subseteq [F^{-} B]^{S_2}\), for all \( B \in \varphi_2\).

**Proof.** We prove (1), leaving the dual proof of (2) to the interested reader. 

(a) \(\Rightarrow\) (b). Let \( A \in \varphi_1\). From \([5]\), Theorem 2.24 (2a) and the definition of interior,

\[
f^{-} F^{-} (A)^{S_2} \subseteq f^{-} (F^{-} (A)) \subseteq A.
\]

Since inverse image and co-image under a difunction are equal, \(f^{-} F^{-} (A)^{S_2} = F^{-} F^{-} (A)^{S_2}\). Thus, \(f^{-} F^{-} (A)^{S_2} \in \text{prego}(S_1)\), by pre-g-continuity. Hence

\[
f^{-} F^{-} (A)^{S_2} \subseteq [A]^{S_2}_{\text{pre}-g}.
\]
and applying [[5], Theorem 2.24 (2b)] gives

\[ |F^{\rightarrow}(A)|^{\mathbb{S}_2} \subseteq F^{\rightarrow}(f^{\rightarrow}(|F^{\rightarrow}(A)|^{\mathbb{S}_2}) \subseteq F^{\rightarrow}A|^{\mathbb{S}_1}_{pre-g}, \]

which is the required inclusion.

(b) \Rightarrow (c). Take \( B \in \varphi_2 \). Applying inclusion (b) to \( A = f^{\rightarrow}(B) \) and using [[5], Theorem 2.24 (2b)] gives

\[ |B|^{\mathbb{S}_2} \subseteq |f^{\rightarrow}(B)|^{\mathbb{S}_2} \subseteq |f^{\rightarrow}(B)|^{\mathbb{S}_1}_{pre-g}. \]

Hence, we have \( f^{\rightarrow}|B|^{\mathbb{S}_2} \subseteq f^{\rightarrow}|f^{\rightarrow}(B)|^{\mathbb{S}_2}_{pre-g} \subseteq |f^{\rightarrow}(B)|^{\mathbb{S}_1}_{pre-g} \) by [[5], Theorem 2.24 (2a)].

(c) \Rightarrow (a). Applying (c) for \( B \in O(S_2) \) gives

\[ f^{\rightarrow}(B) = f^{\rightarrow}|B|^{\mathbb{S}_2} \subseteq |f^{\rightarrow}(B)|^{\mathbb{S}_1}_{pre-g}, \]

so \( f^{\rightarrow}(B) = f^{\rightarrow}(B)|^{\mathbb{S}_1}_{pre-g} \in prego(S_1) \). Hence, \((f, F)\) is pre-g-continuous.

**Theorem 3.4.** Let \((f, F) : (S_1, \varphi_1, \tau_1, k_1, \sigma_1) \rightarrow (S_2, \varphi_2, \tau_2, k_2, \sigma_2)\) be a difunction. Then:

1. The following are equivalent:
   
   (a) \((f, F)\) is pre-g-irresolute.
   
   (b) \(|F^{\rightarrow}A|^{\mathbb{S}_2}_{pre-g} \subseteq F^{\rightarrow}|A|^{\mathbb{S}_1}_{pre-g}\), for all \( A \in \varphi_1 \).

   (c) \(|f^{\rightarrow}|B|^{\mathbb{S}_2}_{pre-g} \subseteq |f^{\rightarrow}|B|^{\mathbb{S}_1}_{pre-g}\), for all \( B \in \varphi_2 \).

2. The following are equivalent:

   (a) \((f, F)\) is pre-g-co-irresolute.

   (b) \(|F^{\rightarrow}|A|^{\mathbb{S}_1}_{pre-g} \subseteq |F^{\rightarrow}|A|^{\mathbb{S}_2}_{pre-g}\), for all \( A \in \varphi_1 \).

   (c) \(|F^{\rightarrow}|B|^{\mathbb{S}_1}_{pre-g} \subseteq |F^{\rightarrow}|B|^{\mathbb{S}_2}_{pre-g}\), for all \( B \in \varphi_2 \).

**Proof.** We prove (1), leaving the dual proof of (2) to the interested reader.

(a) \Rightarrow (b). Take \( A \in \varphi_1 \). Then

\[ f^{\rightarrow}|F^{\rightarrow}|A|^{\mathbb{S}_2}_{pre-g} \subseteq f^{\rightarrow}(F^{\rightarrow}A) \subseteq A \]

by [[5], Theorem 2.24 (2a)]. Now \( f^{\rightarrow}|F^{\rightarrow}|A|^{\mathbb{S}_2}_{pre-g} = F^{\rightarrow}|F^{\rightarrow}|A|^{\mathbb{S}_2}_{pre-g} \in prego(S_1) \) by pre-g-irresolute, so \( f^{\rightarrow}|F^{\rightarrow}|A|^{\mathbb{S}_2}_{pre-g} \subseteq |A|^{\mathbb{S}_1}_{pre-g} \) and applying [[5], Theorem 2.24 (2b)] gives

\[ |F^{\rightarrow}A|^{\mathbb{S}_2}_{pre-g} \subseteq F^{\rightarrow}(f^{\rightarrow}|F^{\rightarrow}|A|^{\mathbb{S}_2}_{pre-g} \subseteq F^{\rightarrow}|A|^{\mathbb{S}_1}_{pre-g}. \]
which is the required inclusion.

(b) $\Rightarrow$ (c). Take $B \in \varphi_2$. Applying inclusion (b) to $A = f^r B$ and using [[5], Theorem 2.24 (2b)] gives

$$[B_{pre-g}^S \subseteq F
\Rightarrow (f^r B)_{pre-g}^S \subseteq F \Rightarrow f^r B]_{pre-g}^S.$$  

Hence, $f^r B_{pre-g}^S \subseteq f^r F \Rightarrow f^r B_{pre-g}^S \subseteq f^r B_{pre-g}^S$ by [[5], Theorem 2.24 (2a)],

(c) $\Rightarrow$ (a). Applying (c) for $B \in \text{prego}(S_2)$ gives

$$f^r B = f^r f^r B_{pre-g}^S \subseteq f^r B_{pre-g}^S,$$

so $F^r B = f^r B = f^r B_{pre-g} \in \text{prego}(S_1)$. Hence, $(f, F)$ is pre-g-irresolute.

**Theorem 3.5.** Let $(S_j, \varphi_j, \tau_j, k_j, \sigma_j)$, $j = 1, 2$, complemented ditopology and $(f, F) : (S_1, \varphi_1) \rightarrow (S_2, \varphi_2)$ be complemented difunction. If $(f, F)$ is pre-g-continuous then $(f, F)$ is pre-g-cocontinuous.

**Proof.** Since $(f, F)$ is complemented, $(F', f') = (f, F)$. From [[5], Lemma 2.20], $\sigma_1(f'(f'(B))) = f'^r(\sigma_2(B))$ and $\sigma_1(f'(f'(B))) = F'^r(\sigma_2(B))$ for all $B \in \varphi_2$. The proof is clear from these equalities.

**Corollary 3.6.** Let $(S_j, \varphi_j, \tau_j, k_j, \sigma_j)$, $j = 1, 2$, complemented ditopology and $(f, F) : (S_1, \varphi_1) \rightarrow (S_2, \varphi_2)$ be complemented difunction. If $(f, F)$ is pre-g-irresolute then $(f, F)$ is pre-g-co-irresolute.

**Proof.** Clear.

**Definition 3.7.** A complemented ditopological texture space $(S, \varphi, \tau, k, \sigma)$ is called pre-g-compact if every cover of $S$ by pre-g-open sets has a finite subcover. Here we recall that $C = \{A_j : j \in J\}$, $A_j \in \varphi$ is a cover of $S$ if $\bigcup C = S$.

**Corollary 3.8.** Let $(S, \varphi, \tau, k, \sigma)$ be a complemented ditopological texture space. Then:

1. Every pre-g-compact is compact.
2. Every g-compact is pre-g-compact.

**Proof.** Clear.

**Theorem 3.9.** If $(S, \varphi, \tau, k, \sigma)$ is pre-g-compact and $L = \{F_j : j \in J\}$ is a family of pre-g-closed sets with $\cap L = \phi$, then $\cap \{F_j : j \in J\} = \phi$ for $J' \subseteq J$ finite.

**Proof.** Suppose that $(S, \varphi, \tau, k, \sigma)$ is pre-g-compact and let $L = \{F_j : j \in J\}$ be a family of pre-g-closed sets with $\cap L = \phi$. Clearly $C = \{\sigma(F_j) : j \in J\}$ is a family of pre-g-open sets. Moreover,

$$\bigvee C = \bigvee \{\sigma(F_j) : j \in J\} = \sigma(\cap \{F_j : j \in J\}) = \sigma(\phi) = S,$$

and so we have $J' \subseteq J$ finite with $\bigvee \{\sigma(F_j) : j \in J'\} = S$. Hence $\cap \{F_j : j \in J'\} = \phi$.  


Theorem 3.10. Let \((f, F): (S_1, \varphi_1, \tau_1, k_1, \sigma_1) \to (S_2, \varphi_2, \tau_2, k_2, \sigma_2)\) be an pre-g-irresolute difunction. If \(A \in \varphi_1\) is pre-g-compact then \(f^{-1}A \in \varphi_2\) is pre-g-compact.

**Proof.** Take \(f^{-1}A \subseteq \bigvee_{j \in J} G_j\), where \(G_j \in \text{prego}(S_2), j \in J\). Now by [[5], Theorem 2.24 (2a) and Corollary 2.12 (2)] we have

\[
A \subseteq F^c(f^{-1}A) \subseteq F^c(\bigvee_{j \in J} G_j) = \bigvee_{j \in J} F^c G_j.
\]

Also, \(F^cG_j \in \text{prego}(S_1)\) because \((f, F)\) is pre-g-irresolute. So by the pre-g-compactness of \(A\) there exists \(J' \subseteq J\) finite such that \(A \subseteq \bigcup_{j \in J'} F^c G_j\). Hence

\[
f^{-1}A \subseteq f^{-1}(\bigcup_{j \in J'} F^c G_j) = \bigcup_{j \in J'} f^{-1}(F^c G_j) \subseteq \bigcup_{j \in J'} G_j
\]

by [[5], Corollary 2.12 (2) and Theorem 2.24 (2b)]. This establishes that \(f^{-1}A\) is pre-g-compact.

Corollary 3.11. Let \((f, F): (S_1, \varphi_1, \tau_1, k_1, \sigma_1) \to (S_2, \varphi_2, \tau_2, k_2, \sigma_2)\) be a surjective pre-g-irresolute difunction. Then, if \((S_1, \varphi_1, \tau_1, k_1, \sigma_1)\) is pre-g-compact so is \((S_2, \varphi_2, \tau_2, k_2, \sigma_2)\).

**Proof.** This follows by taking \(A = S_1\) in Theorem 3.10 and noting that \(f^{-1}S_1 = f^{-1}(F^c S_2) = S_2\) by [[5], Proposition 2.28 (1c) and Corollary 2.33 (1)].

Definition 3.12. A complemented ditopological texture space \((S, \varphi, \tau, k, \sigma)\) is called pre-g-stable if every pre-g-closed set \(F \in \varphi \setminus \{S\}\) is pre-g-compact in \(S\).

Corollary 3.13. Let \((S, \varphi, \tau, k, \sigma)\) be a complemented ditopological texture space. Then:

1. Every pre-g-stable is stable.
2. Every g-stable is pre-g-stable.

**Proof.** Clear.

Theorem 3.14. Let \((S, \varphi, \tau, k, \sigma)\) be pre-g-stable. If \(G\) is an pre-g-open set with \(G \neq \phi\) and \(D = \{F_j : j \in J\}\) is a family of pre-g-closed sets with \(\bigcap_{j \in J} F_j \subseteq G\) then \(\bigcap_{j \in J} F_j \subseteq G\) for a finite subsets \(J'\) of \(J\).

**Proof.** Let \((S, \varphi, \tau, k, \sigma)\) be pre-g-stable, let \(G\) be an pre-g-open set with \(G \neq \phi\) and \(D = \{F_j : j \in J\}\) be a family of pre-g-closed sets with \(\bigcap_{j \in J} F_j \subseteq G\). Set \(K = \sigma(G)\). Then \(K\) is pre-g-closed and satisfies \(K \neq S\). Hence \(K\) is pre-g-compact. Let \(C = \{\sigma(F)|F \in D\}\). Since \(\cap D \subseteq G\) we have \(K \subseteq \bigcup C\), that is \(C\) is an pre-g-open cover of \(K\). Hence there exists \(F_1, F_2, ..., F_n \in D\) so that

\[
K \subseteq \sigma(F_1) \cup \sigma(F_2) \cup ... \cup \sigma(F_n) = \sigma(F_1 \cap F_2 \cap ... \cap F_n).
\]

This gives \(F_1 \cap F_2 \cap ... \cap F_n \subseteq \sigma(K) = G\), so \(\bigcap_{j \in J'} F_j \subseteq G\) for a finite subsets \(J' = \{1, 2, ..., n\}\) of \(J\).
Theorem 3.15. Let \((S_1, \varphi_1, \tau_1, k_1, \sigma_1)\), \((S_2, \varphi_2, \tau_2, k_2, \sigma_2)\) be two complemented ditopological texture spaces with \((S_1, \varphi_1, \tau_1, k_1, \sigma_1)\) is pre-g-stable, and \((f, F) : (S_1, \varphi_1, \tau_1, k_1, \sigma_1) \rightarrow (S_2, \varphi_2, \tau_2, k_2, \sigma_2)\) be an pre-g-bi-irresolute surjective difunction. Then \((S_2, \varphi_2, \tau_2, k_2, \sigma_2)\) is pre-g-stable.

Proof. Take \(K \in \text{pregc}(S_2)\) with \(K \neq S_2\). Since \((f, F)\) is pre-g-co-irresolute, so \(f^{-1}K \in \text{pregc}(S_1)\). Let us prove that \(f^{-1}K \neq S_1\). Assume the contrary. Since \(f^{-1}S_2 = S_1\), by [5], Lemma 2.28 (1c) we have \(f^{-1}S_2 \subseteq f^{-1}K\), whence \(S_2 \subseteq K\) by [5], Corollary 2.33 (1 ii) as \((f, F)\) is surjective. This is a contradiction, so \(f^{-1}K \neq S_1\). Hence \(f^{-1}(K)\) is pre-g-compact in \((S_1, \varphi_1, \tau_1, k_1, \sigma_1)\) by pre-g-stability. As \((f, F)\) is pre-g-irresolute, \(f^{-1}(f^{-1}K)\) is pre-g-compact for the ditopology \((\tau_2, k_2)\) by Theorem 3.10, and by [5], Corollary 2.33 (1) this set is equal to \(K\). This establishes that \((S_2, \varphi_2, \tau_2, k_2, \sigma_2)\) is pre-g-stable.

References


Hariwan Zikri Ibrahim
Department of Mathematics,
Faculty of Science,
University of Zakho,
Kurdistan-Region, Iraq
E-mail address: hariwan_math@yahoo.com