On Λγ-sets in Fuzzy Bitopological Spaces

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Abstract: The aim of this paper is to introduce the concept of Λ operator of a fuzzy set in a fuzzy bitopological space. Then we study (i, j)-fuzzy Λγ-set and its properties. Moreover, we define (i, j)-fuzzy Λ-closed set, (i, j)-fuzzy Λγ-closed set and (i, j)-fuzzy generalized closed set in fuzzy bitopological space. The concepts of (i, j)-fuzzy Λ-closed set and (i, j)-fuzzy generalized closed set are independent to each other but jointly they give the τi-fuzzy closed set. To this end, as the application of (i, j)-fuzzy Λγ-closed set we shall study (i, j)-fuzzy Λγ-continuity and (i, j)-fuzzy Λγ-generalized continuity and their properties.

Key Words: (i, j)-fuzzy Λ-set, (i, j)-fuzzy Λγ-set, (i, j)-fuzzy Λγ-closed set and (i, j)-fuzzy Λγ-continuous function.

Contents

1 Introduction 285
2 (i, j)-Fuzzy Λγ-Sets 287
3 (i, j)-Fuzzy Λγ-Closed Sets 290
4 (i, j)-Fuzzy Λγ-Generalized Closed Sets 294
5 (i, j)-Fuzzy Λ-Continuity and (i, j)-Fuzzy Λγ-Continuity 296

1. Introduction

In 1986 Maki [18] introduced a very nice concept in topological space, so called Λ-set. After that, several authors have studied this notion in different directions in topological space such as Λα-sets (2000), pre Λ-sets (2002), Asp-sets (2004), Λn-sets (2005), Λδ-sets (2006), (Λ, α)-sets (2007), Λκ-sets (2009) and Λτ-sets (2011) respectively [5,12,14,11,7,6,8,15]. Then using the concept of Λ-set in 1997, Arenas et al. [2] introduced a generalized concept of the Makis idea and investigated the various characterizations in terms of different notions in topological space. In 2006, M. E. El-Shafei [17] has introduced the Λ-fuzzy set in fuzzy topological space in the sense of Maki. Fuzzy Λδ-sets are studied by G. Aslim and G. Gunel [1] in fuzzy topological space. Kandil [16], in 1989 introduced the concepts of fuzzy bitopological space. After that several authors were interested to do their work in that field. In 2013 B. C. Tripathy et al. [22] have introduced the concepts of (i, j)-fuzzy γ-open sets in fuzzy bitopological space.

The purpose of our paper is to continue the research work in the similar direction but in different approach. In literature we have seen that every closed set is a \( \Lambda \)-closed set but the converse may not be true. In this paper we shall try to investigate that relationship under which the converse is true. In this paper we shall introduce a generalized idea of (i, j)-fuzzy \( \gamma \)-open sets namely (i, j)-fuzzy \( \Lambda \gamma \)-sets in fuzzy bitopological space. We study the concept of (i, j)-fuzzy \( \Lambda \gamma \)-closed set in fuzzy bitopological space and try to establish one equivalent form of \( \tau \gamma \)-closed set in section 3.

Section 4 is devoted to study (i, j)-fuzzy \( \Lambda \gamma \)-generalized closed set. In [10], M. Caldas et al. have shown one equivalent condition using the locally closed set but we formulate the same equivalent condition with the help of a weaker form of fuzzy locally closed set.

In section 5 we study the application part of (i, j)-fuzzy \( \Lambda \gamma \)-closed set i.e (i, j)-fuzzy \( \Lambda \gamma \)-generalized continuity and discover some related results.

A system \((X, \tau_i, \tau_j)\) consisting of a set \(X\) with two fuzzy topologies \(\tau_i\) and \(\tau_j\) on \(X\) is called a fuzzy bitopological space [16]. For a fuzzy set \(\lambda\) of \(X\) the closure of \(\lambda\) and the interior of \(\lambda\) with respect to \(\tau_i\) are denoted by \(\tau_i\text{-cl}(\lambda)\) and \(\tau_i\text{-int}(\lambda)\) respectively, for \(i = 1, 2\).

First we recall some definitions from topology, fuzzy topology and fuzzy bitopological spaces.

**Definition 1.1.** A fuzzy subset \(\lambda\) of \(X\) is called (i, j)-fuzzy \(\gamma\)-open [22], if \(\lambda \land \mu\) is (i, j)-fuzzy pre-open for every (i, j)-fuzzy pre-open set \(\mu\) in \(X\).

**Definition 1.2.** [3] (i) A fuzzy subset \(\lambda\) of a fuzzy topological space \((X, \tau)\) is called a generalized closed (g-closed, for short) fuzzy set if \(\lambda \leq \eta\) and \(\eta \in \tau\) implies that \(cl(\lambda) \leq \eta\).

(ii) [3] A fuzzy topological space \((X, \tau)\) is called \(T_{1/2}\)-space iff every fuzzy generalized closed set is closed.

**Definition 1.3.** A fuzzy subset \(\lambda\) of a fuzzy topological space \((X, \tau)\) is called a \(\Lambda\)-fuzzy set [17] if \(\lambda = \lambda^\Lambda\), where \(\lambda^\Lambda = \land\{\eta; \lambda \leq \eta, \eta \in \tau\}\).

**Definition 1.4.** A subset \(B\) of a topological space \((X, \tau)\) is called a \(\lambda\)-closed [9] if \(B = C \land D\) where \(C\) is a \(\Lambda\)-set and \(D\) is a closed set.
Definition 1.5. A fuzzy topological space $(X,\tau)$ is called a fuzzy submaximal space \cite{19} if $\text{cl}(\lambda) = 1$ for any non-zero fuzzy set $\lambda$ in $(X,\tau)$, then $\lambda \in \tau$.

Definition 1.6. A fuzzy set $\lambda$ in a fuzzy topological space $(X,\tau)$ is called fuzzy dense \cite{21} if there exists no fuzzy closed set $\mu$ in $(X,\tau)$ such that $\lambda < \mu < 1$.

Definition 1.7. A subset $S$ of a topological space $(X,\tau)$ is said to be locally closed if \cite{13} $S = U \wedge F$, here $U$ is open and $F$ is closed.

Definition 1.8. A fuzzy set $\mu$ of a fuzzy topological space $(X,\tau)$ is called $\Lambda$-generalized fuzzy closed (briefly $\Lambda$-gf-closed) \cite{4} if $\text{cl}(\mu) \leq \beta$ whenever $\mu \leq \beta$ and $\beta$ is $\lambda f$-open.

Definition 1.9. Fuzzy pairwise continuous \cite{16} if the induced functions $f:(X,\tau_1) \rightarrow (Y,\sigma_1)$ and $f:(X,\tau_2) \rightarrow (Y,\sigma_2)$ are both fuzzy continuous.

Definition 1.10. \cite{3} A map $f:(X,\tau) \rightarrow (Y,\sigma)$ is called generalized fuzzy continuous (in short gf-continuous) if the inverse image of every fuzzy closed set in $Y$ is gf-closed in $X$.

Definition 1.11. \cite{6} A function $f$ from a topological space $X$ to another topological space $Y$ is called $(\lambda, \alpha)$-continuous if $f^{-1}(V)$ is a $(\lambda, \alpha)$-open subset of $X$ for every open subset $V$ of $Y$.

2. $(i, j)$-Fuzzy $\Lambda^\gamma$-Sets

As a particular case using $\tau_i$-fuzzy open set we can define $(i, j)$-fuzzy $\Lambda$-set and $(i, j)$-fuzzy $V$-set in fuzzy bitopological space as follows:

Definition 2.1. Let $\mu$ is any fuzzy subset in a fuzzy bitopological space $(X,\tau_i,\tau_j)$. Then

(i) Kernel of $\mu$ is denoted by $(i, j)-\Lambda(\mu)$ and is defined by
(i, j)-\Lambda(\mu)=\wedge\{\eta: \mu \leq \eta\}, where \eta is a \tau_{\tau_j} \text{-fuzzy open set in } (X, \tau_i, \tau_j)\}
and
(ii)(i, j)-V(\mu)\text{of a fuzzy set } \mu \text{ is defined by}
(i, j)-V(\mu)=\vee\{\eta: \eta \leq \mu\}, where \eta is a \tau_{\tau_j} \text{-fuzzy closed set in } (X, \tau_i, \tau_j)\}.

Based on this definition we define (i, j)-fuzzy \Lambda^\gamma\text{-set and (i, j)-fuzzy } V^\gamma\text{-set in a fuzzy bitopological space as follows:}

**Definition 2.2.** Let \lambda be any fuzzy subset in a fuzzy bitopological space \((X, \tau_i, \tau_j)\).
Then
(i) \gamma\text{-Kernel of } \lambda \text{ is denoted by (i, j)-}\Lambda^\gamma(\lambda)=\wedge\{\eta: \lambda \leq \eta\}, \text{where } \eta \text{ is a (i, j)-fuzzy}\\gamma\text{-open set in } (X, \tau_i, \tau_j)\}
and
(ii)(i, j)-V^\gamma(\lambda) \text{ of a fuzzy set } \lambda \text{ is defined by (i, j)-}\Lambda^\gamma(\lambda)=\vee\{\eta: \eta \leq \lambda\}, \text{where } \eta \text{ is a (i, j)-fuzzy }\gamma\text{-closed set in } (X, \tau_i, \tau_j)\}.

**Lemma 2.3.** Let \eta, \mu \text{ and } \mu_k \text{ be fuzzy subsets of a fuzzy bitopological space } (X, \tau_i, \tau_j) \text{ for every } k \in \Gamma \text{ (an index set)} \text{ and } x_p \text{ be any point of } X, \text{ then the following properties holds:}

(i) \mu \leq (i, j)-\Lambda^\lambda(\eta),
(ii) if \mu \leq \mu \text{ then, (i, j)-}\Lambda^\lambda(\mu) \leq (i, j)-\Lambda^\lambda(\lambda),
(iii) (i, j)-\Lambda^\lambda((i, j)-\Lambda^\lambda(\lambda))=(i, j)-\Lambda^\lambda(\eta),
(iv) if \eta \in (i, j) \text{\text{-O}(X) then } \eta = (i, j)-\Lambda^\lambda(\eta),
(v) (i, j)-\Lambda^\lambda(\vee\{\mu_k: k \in \Gamma\})=\vee\{\eta: (i, j)\text{-}\Lambda^\lambda(\mu_k: k \in \Gamma)\},
(vi) (i, j)-\Lambda^\lambda(\bigwedge\{\mu_k: k \in \Gamma\}) \leq \bigwedge\{\eta: (i, j)\text{-}\Lambda^\lambda(\mu_k: k \in \Gamma)\}
and
(vii)(i, j)-\Lambda^\lambda(I_X-\eta)=I_X-(i, j)-\Lambda^\lambda(\eta).

**Proof:** Here (i), (ii), (iii) and (iv) can be proved easily from the definition.
To prove (v)
Let \mu = \vee\{\mu_k: k \in \Gamma\} \geq \mu_k \text{ for all } k \in \Gamma.
Hence from (ii), (i, j)-\Lambda^\lambda(\mu_k) \leq (i, j)-\Lambda^\lambda(\mu) \text{ for all } k \in \Gamma \text{ i.e.}
\vee\{(i, j)-\Lambda^\lambda(\mu_k): k \in \Gamma\} \leq (i, j)-\Lambda^\lambda(\vee\mu_k: k \in \Gamma)\).
On the other hand let \(x_p \notin \vee\{(i, j)-\Lambda^\lambda(\vee\mu_k): k \in \Gamma\}\)
then \(x_p \notin \vee\{(i, j)-\Lambda^\lambda(\vee\mu_k): k \in \Gamma\}\)
so \(x_p \notin \mu_k\).
Now for each \(k \in \Gamma\) there exists \(\eta_k \in (i, j)\text{-O}(X)\) such that \(\mu_k \leq \eta_k\) and \(x_p \notin \eta_k\)
for all \(k \in \Gamma\). Then for \(\bigvee_{k \in \Gamma}\mu_k \leq \bigvee_{k \in \Gamma}\eta_k\) and \(\bigvee_{k \in \Gamma}\eta_k\) is an (i, j)-fuzzy \gamma\text{-open set not containing fuzzy point } x_p. Thus \(x_p \notin \vee\{(i, j)-\Lambda^\lambda(\vee\mu_k): k \in \Gamma\}\).
Thus we have \(\bigvee\{\eta_k: k \in \Gamma\} \subseteq \vee\{(i, j)-\Lambda^\lambda(\vee\mu_k): k \in \Gamma\}\).
Therefore \(\bigvee\{\eta_k: k \in \Gamma\} = (i, j)-\Lambda^\lambda(\vee\mu_k: k \in \Gamma)\).
To prove (vi)
Let \(\mu = \bigwedge\{\mu_k: k \in \Gamma\}\).
So \(\mu \leq \mu_k \text{ for all } k \in \Gamma\).
Then from (ii) \(\bigwedge\{\mu_k\} \leq \bigwedge\{\eta_k\}\).
Hence \(\bigwedge\{\eta_k\} \leq \bigwedge\{\mu_k: k \in \Gamma\}\).
To prove (vii)
On Λγ-sets in Fuzzy Bitopological Spaces

1_X-(i, j)-Vγ(η)=1_X - ∨\{β; β ≤ η, β ∈ (i, j)FγO(X}\}
= ∧\{μ; 1_X - η ≤ μ, μ is a (i, j)-fuzzy γ-open subset of X\}.
= (i, j)-Λγ(1_X - η)(here μ = 1_X - β).

By using the above lemma we can easily prove the following results.

Lemma 2.4. (i) (i, j)-Vγ(η) ≤ η,
(ii) if η ≤ μ then, (i, j)-Vγ(η) ≤ (i, j)-Vγ(μ),
(iii) if η ∈ (i, j)FγC(X) then η = (i, j)-Vγ(η),
(iv) (i, j)-Vγ((i, j)-Vγ(η)) = (i, j)-Vγ(η),
(v) (i, j)-Vγ(∧μk:k∈Γ) = ∧\{μ(i, k): k∈Γ\} and
(vi) (i, j)-Vγ(∨μk:k∈Γ) ≥ ∨\{μ(i, k): k∈Γ\}.

Definition 2.5. In a fuzzy bitopological space (X,τi,τj), a fuzzy subset µ is said to be (i, j)-fuzzy Λγ-set (resp. (i, j)-fuzzy Vγ-set) if µ = (i, j)-Λγ(µ) (resp. µ = (i, j)-Vγ(µ)). The collection of all (i, j)-fuzzy Λγ-set (resp. (i, j)-fuzzy Vγ-set) is denoted by (i, j)FΛγ(X) (resp. (i, j)FVγ(X)).

Theorem 2.6. In a fuzzy bitopological space (X,τi,τj) the following properties are satisfied
(i) 0_X and 1_X are (i, j)-fuzzy Λγ-set.
(ii) Arbitrary intersection of (i, j)-fuzzy Λγ-sets is a (i, j)-fuzzy Λγ-set.
(iii) Arbitrary union of (i, j)-fuzzy Λγ-sets is a (i, j)-fuzzy Λγ-set.

Proof: (i) It is obvious.
(ii) Let µ = ∧\{μk:k∈Γ\}, where µk is a (i, j)-fuzzy Λγ-set i.e. µk = (i, j)-Λγ(µk).
From lemma 2.3 (vi)
(i, j)-Λγ(∧μk:k∈Γ) ≤ ∧\{(i, j)-Λγ(µk): k∈Γ\} = ∧\{(i, j)-Λγ(µk): k∈Γ\}.
Hence (i, j)-Λγ(∧μk:k∈Γ) ≤ ∧\{(i, j)-Λγ(µk): k∈Γ\}.
But from 2.3(i) \{∧μk:k∈Γ\} ≤ ∧\{(i, j)-Λγ(µk): k∈Γ\} .
Hence ∧\{μk:k∈Γ\} = (i, j)-Λγ{∧μk:k∈Γ}\}.
Thus arbitrary intersection of (i, j)-fuzzy Λγ-sets is a (i, j)-fuzzy Λγ-set.
In a similar way using the lemma 2.3(v) we can show that arbitrary union of (i, j)-fuzzy Λγ-sets is a (i, j)-fuzzy Λγ-set.
Thus from the above proposition we can say that the collection of (i, j)-fuzzy Λγ-sets forms an Alexandroff space in a fuzzy bitopological space (X,τi,τj).

Remark 2.7. Since in a fuzzy bitopological space (X,τi,τj), the (i, j)-fuzzy γ-open set and τi,τj-fuzzy open sets are independent of each other, thus we can conclude that the concept of (i, j)-fuzzy Λγ-set and (i, j)-fuzzy Λ-set are also independent.
of each other.

**Definition 2.8.** A fuzzy subset $\mu$ in a fuzzy bitopological space $(X, \tau_i, \tau_j)$ is called $(i, j)$-fuzzy generalized closed set if $\tau_j \cdot \text{cl}(\mu) \leq \eta$ whenever $\mu \leq \eta$ and $\eta \in \tau_i \text{FO}(X)$.

**Proposition 2.9.** If $\mu$ is a $(i, j)$-fuzzy generalized closed set and $\mu \leq (i, j) \cdot \Lambda^\gamma(\mu)$ then $(i, j) \cdot \Lambda^\gamma(\mu)$ is $(i, j)$-fuzzy generalized closed set.

**Proof:** Since $\mu$ is a $(i, j)$-fuzzy generalized closed set and $\mu \leq (i, j) \cdot \Lambda^\gamma(\mu)$ then $(i, j) \cdot \Lambda^\gamma(\mu)$ implies that $\tau_j \cdot \text{cl}(\mu) \leq \tau_j \cdot \text{cl}(\tau_j \cdot \text{cl}(i, j) \cdot \Lambda^\gamma(\mu)) \leq \tau_j \cdot \text{cl}(\mu)$. Therefore, we get $\tau_j \cdot \text{cl}((i, j) \cdot \Lambda^\gamma(\mu)) = \tau_j \cdot \text{cl}(\mu)$. Hence $(i, j) \cdot \Lambda^\gamma(\mu)$ is $(i, j)$-fuzzy generalized closed set as $\mu$ is $(i, j)$-fuzzy generalized closed set. \[\Box\]

**Proposition 2.10.** If $\lambda$ is a $\tau_i$-fuzzy open set in a fuzzy bitopological space $X$ then there exist a $(i, j)$-fuzzy set $\mu$ and $(i, j)$-fuzzy regular open set $\eta$ such that $\lambda \leq \mu \land \eta$.

**Proof:** Since $\lambda \leq \tau_j \cdot \text{cl}(\lambda) \Rightarrow \tau_i \cdot \text{int}(\lambda) \land \mu \leq \tau_i \cdot \text{int}(\tau_j \cdot \text{cl}(\lambda)) \land \mu 
\Rightarrow \lambda \leq \eta \land \eta$, where $\eta$ is a $(i, j)$-fuzzy regular open set. \[\Box\]

### 3. $(i, j)$-Fuzzy $\Lambda^\gamma$-Closed Sets

In general $(i, j)$-fuzzy $\Lambda^\gamma$-closed set is not $\tau_i$-fuzzy closed set. The objective of this section to find the condition under which every $(i, j)$-fuzzy $\Lambda^\gamma$-closed set is $\tau_i$-fuzzy closed set.

**Definition 3.1.** A fuzzy subset $\lambda$ of a fuzzy bitopological space $(X, \tau_i, \tau_j)$ is called $(i, j)$-fuzzy $\Lambda$-closed set if $\lambda = \mu \land \delta$, where $\mu$ is $(i, j)$-fuzzy-$\Lambda$ set and $\delta$ is an $\tau_i$-fuzzy closed set. The family of all $(i, j)$ fuzzy $\Lambda$-closed set is denoted by $(i, j) \text{FAC}(X)$.

A fuzzy subset of a fuzzy bitopological space $(X, \tau_i, \tau_j)$ is called $(i, j)$-fuzzy $\Lambda$-open if its complement is $(i, j)$-fuzzy $\Lambda$-closed set.
Lemma 3.2. For a fuzzy subset $\lambda$ of a fuzzy bitopological space $(X, \tau_i, \tau_j)$ the following conditions are equivalent.

(i) $\lambda \in (i, j)\mathcal{FAC}(X)$ and

(ii) $\lambda = \mu \land \tau_i\text{-cl}(\lambda)$, for some $(i, j)$ fuzzy $\Lambda$-set $\mu$.

Proof: (i)$\Rightarrow$(ii) Let $\lambda$ be any $(i, j)$-fuzzy $\Lambda$-closed set in $X$. Then $\lambda = \mu \land \delta$, where $\mu$ is $(i, j)$-fuzzy $\Lambda$-set and $\delta$ is $\tau_i$-fuzzy-closed set in $X$. Now $\lambda \leq \delta$.

\[ \Rightarrow \tau_i\text{-cl}(\lambda) \leq \tau_i\text{-cl}(\delta) = \delta \]

\[ \Rightarrow \tau_i\text{-cl}(\lambda) \land \mu \leq \delta \land \mu = \lambda \]

Again $\tau_i\text{-cl}(\lambda) \geq \lambda$.

\[ \Rightarrow \tau_i\text{-cl}(\lambda) \land \mu \geq \lambda. \]

From the above two relation we get $\lambda = \mu \land \tau_i\text{-cl}(\lambda)$.

(ii)$\Rightarrow$(i) It is obvious.

Theorem 3.3. A $(i, j)$-fuzzy generalized closed set is $\tau_i$-fuzzy closed set iff it is $(i, j)$-fuzzy $\Lambda$-closed set.

Proof: Let $\lambda$ be $(i, j)$-fuzzy generalized closed set. Thus $\tau_i\text{-cl}(\lambda) \leq \mu$ whenever $\lambda \leq \mu$, where $\mu \in \tau_j \text{FO}(X)$. But since $\lambda$ is $(i, j)$-fuzzy $\Lambda$-closed set, then $\lambda = (i, j)\Lambda(\lambda) \land \tau_i\text{-cl}(\lambda) = \tau_i\text{-cl}(\lambda)$ [since $(i, j)\Lambda(\lambda) \geq \tau_i\text{-cl}(\lambda)$]. Therefore $\lambda$ is $\tau_i$-fuzzy closed set.

Converse part follows from the fact that every $\tau_i$-fuzzy closed set is $(i, j)$-fuzzy $\Lambda$-closed set.

Definition 3.4. A fuzzy set $\eta$ in fuzzy bitopological space $(X, \tau_i, \tau_j)$ is said to be $(i, j)$-fuzzy locally closed set if $\eta = \mu \land \beta$ where $\beta$ is a $\tau_i$-fuzzy closed set and $\mu$ is a $\tau_j$-fuzzy open set in $X$.

Remark 3.5. Every $(i, j)$-fuzzy locally closed set is $(i, j)$-fuzzy $\Lambda$-closed set which follows from the definition.

Definition 3.6. A fuzzy subset $\lambda$ of a fuzzy bitopological space $(X, \tau_i, \tau_j)$ is called $(i, j)$-fuzzy dense set if $\lambda$ is $\tau_i$-fuzzy dense set or $\tau_j$-fuzzy dense set.

Definition 3.7. A fuzzy bitopological space $(X, \tau_i, \tau_j)$ is said to be $(i, j)$-fuzzy sub-maximal space if each $(i, j)$-fuzzy dense set is a $\tau_i$-fuzzy open set.
Proposition 3.8. Every fuzzy subset of a \((i, j)\)-fuzzy submaximal space \((X, \tau_i, \tau_j)\) is \((i, j)\)-fuzzy \(\Lambda\)-closed set.

Proof: Let \((X, \tau_i, \tau_j)\) be a \((i, j)\)-fuzzy submaximal space and we know that every fuzzy subset \(\lambda\) of a \((i, j)\)-fuzzy submaximal space can be expressed as an intersection of \(\tau_j\)-fuzzy open set \(\mu\) and \(\tau_i\)-fuzzy closed set \(\delta\). It means that \(\mu\) is a \((i, j)\)-fuzzy \(\Lambda\)-set and therefore \(\lambda = \mu \land \delta\). Thus \(\lambda\) is \((i, j)\)-fuzzy \(\Lambda\)-closed set.

\(\square\)

Proposition 3.9. In a \((i, j)\)-fuzzy submaximal space \((X, \tau_i, \tau_j)\) every \((i, j)\)-fuzzy \(\Lambda\)-open set is \(\tau_i\)-fuzzy open if there does not exist \((i, j)\)-fuzzy regular closed set other than \(1_X\).

Proof: Let \(\lambda\) be any \((i, j)\)-fuzzy \(\Lambda\)-open set then \(\lambda\) can be expressed as \(\lambda = \mu \lor \tau_i\text{-cl(}\lambda\text{)}\), where \(\mu\) be any \((i,j)\)-fuzzy \(\Lambda\)-set and \(\delta\) be \(\tau_i\)-fuzzy open set .

Now \(\tau_i\text{-cl(}\lambda\text{)} = \tau_i\text{-cl(}\mu \lor \delta\text{)}\geq \tau_i\text{-cl(}\mu\text{)} \lor \tau_i\text{-cl(}\tau_i\text{-int(}\delta\text{)}\text{)}\geq \tau_i\text{-cl(}\mu\text{)} \lor 1_X\).

Therefore \(\lambda\) is \(\tau_i\)-fuzzy open since \((X, \tau_i, \tau_j)\) is \((i, j)\)-fuzzy submaximal space .

\(\square\)

Definition 3.10. A fuzzy subset \(\lambda\) of a fuzzy bitopological space \((X, \tau_i, \tau_j)\) is called \((i, j)\)-fuzzy \(\Lambda\)-closed set if \(\lambda = \mu \land \tau_i\text{-cl(}\lambda\text{)}\), where \(\mu\) is \((i, j)\)-fuzzy \(\Lambda\)-set and \(\delta\) is \(\tau_i\)-fuzzy closed set. The family of all \((i, j)\)-fuzzy \(\Lambda\)-closed sets is denoted by \((i, j)\text{FCL}(X)\).

A fuzzy subset \(\lambda\) of a fuzzy bitopological space \((X, \tau_i, \tau_j)\) is called \((i, j)\)-fuzzy \(\Lambda\)-open if its complement is \((i, j)\)-fuzzy \(\Lambda\)-closed set.

Lemma 3.11. For a fuzzy subset \(\lambda\) of a fuzzy bitopological space \((X, \tau_i, \tau_j)\) the following conditions are equivalent:

(i) \(\lambda\) is \((i, j)\)-fuzzy \(\Lambda\)-closed set.

(ii) \(\lambda = \mu \land \tau_i\text{-cl(}\lambda\text{)}\), where \(\mu\) is a \((i, j)\)-fuzzy \(\Lambda\)-set.

(iii) \(\lambda = (i, j)\text{-CL}(\lambda) \land \tau_i\text{-cl(}\lambda\text{)}\).

Proof: (i)\(\Rightarrow\) (ii) Let \(\lambda\) be a \((i, j)\)-fuzzy \(\Lambda\)-closed set. Therefore \(\lambda = \mu \land \delta\) where \(\mu\) is a \((i, j)\)-fuzzy \(\Lambda\)-set and \(\delta\) is \(\tau_i\)-fuzzy closed set. Since \(\lambda \leq \delta\) implies \(\tau_i\text{-cl(}\lambda\text{)} \leq \delta\) and \(\lambda = \mu \land \delta \geq \mu \land \tau_i\text{-cl(}\lambda\text{)} \geq \lambda\). Therefore we have \(\lambda = \mu \land \tau_i\text{-cl(}\lambda\text{)}\).

(ii)\(\Rightarrow\)(iii) Let \(\lambda = \mu \land \tau_i\text{-cl(}\lambda\text{)}\), where \(\mu\) is a \((i, j)\)-fuzzy \(\Lambda\)-set. Since \(\lambda \leq \mu\) implies \(\lambda \leq (i, j)\text{-CL}(\lambda)\) and \(\lambda = \mu \land \delta \geq (i, j)\text{-CL}(\lambda) \land \tau_i\text{-cl(}\lambda\text{)} \geq \lambda\).

Therefore we have \(\lambda = (i, j)\text{-CL}(\lambda) \land \tau_i\text{-cl(}\lambda\text{)}\).

(iii)\(\Rightarrow\)(i) It is obvious.

\(\square\)

Remark 3.12. Every \((i, j)\)-fuzzy \(\Lambda\)-closed set and \((i, j)\)-fuzzy \(\Lambda\)-open set are independent of each other since every \((i, j)\)-fuzzy \(\Lambda\)-closed set and \((i, j)\)-fuzzy \(\Lambda\)-set are independent.
Proposition 3.13. If \( \lambda \) be any \((i, j)\)-fuzzy dense set and \((i, j)\)-fuzzy \( \Lambda^\gamma \)-closed set then \( \lambda \) is \((i,j)\)-fuzzy \( \Lambda^\gamma \)-set.

Proof: Let \( \lambda \) be any \((i, j)\)-fuzzy dense set and \((i, j)\)-fuzzy \( \Lambda^\gamma \)-closed set then 
\[
\lambda = (i, j)\Lambda^\gamma(\lambda) \land \tau_i \text{-cl}(\lambda) = (i,j)\Lambda^\gamma(\lambda) \land 1_X = (i, j)\Lambda^\gamma(\lambda).
\]
Thus \( \lambda \) is a \((i,j)\)-fuzzy \( \Lambda^\gamma \)-set. \( \square \)

Definition 3.14. A fuzzy bitopological space \((X, \tau_i, \tau_j)\) is said to be fuzzy \((i, j)\)-fuzzy \( T_{1/2} \) space if every \((i, j)\)-fuzzy generalized closed set is a \( \tau_i \)-fuzzy closed set.

Proposition 3.15. In a fuzzy bitopological space \((X, \tau_i, \tau_j)\) the following conditions are equivalent:

(i) \((X, \tau_i, \tau_j)\) is \((i, j)\)-fuzzy \( T_{1/2} \) space.

(ii) Every \((i, j)\)-fuzzy generalized closed set is a \((i, j)\)-fuzzy \( \Lambda^\gamma \)-closed set.

Proof: (i)\(\Rightarrow\)(ii) Given \((X, \tau_i, \tau_j)\) be a \((i, j)\)-fuzzy \( T_{1/2} \) space. Hence by the definition of \((i, j)\)-fuzzy \( T_{1/2} \) space every \((i, j)\)-fuzzy generalized closed set is a \( \tau_i \)-fuzzy closed set which is a \((i, j)\)-fuzzy \( \Lambda^\gamma \)-closed set.

(ii)\(\Rightarrow\)(i) Using theorem 3.3, it can be proved easily. \( \square \)

Definition 3.16. A fuzzy subset \( \lambda \) in a fuzzy bitopological space \((X, \tau_i, \tau_j)\) is said to be \((i, j)\)-fuzzy semi-\( \gamma \)-closed set if 
\[
\tau_i \text{-int}_\gamma (\tau_j \text{-cl}(\lambda)) \leq \lambda.
\]

Proposition 3.17. A \((i, j)\)-fuzzy \( \Lambda^\gamma \)-closed set is \( \tau_i \)-fuzzy closed set if it is \((i, j)\)-fuzzy semi-\( \gamma \)-closed set.

Proof: Let \( \lambda \) be any \((i, j)\)-fuzzy \( \Lambda^\gamma \)-closed set, so 
\[
\lambda = (i, j)\Lambda^\gamma \land \tau_i \text{-cl}(\lambda).
\]
Given \( \lambda \) is a \((i, j)\)-fuzzy semi-\( \gamma \)-closed set. Thus 
\[
\tau_i \text{-int}_\gamma (\tau_j \text{-cl}(\lambda)) \leq \lambda.
\]
Let 
\[
\tau_i \text{-int}_\gamma (\tau_j \text{-cl}(\lambda)) \leq \lambda \leq \delta_k, k \in I,
\]
where \( \delta_k \) are \((i, j)\)-fuzzy \( \gamma \)-open set. Therefore 
\[
\tau_i \text{-cl}(\lambda) \leq \Lambda(\lambda).
\]
This implies that 
\[
\tau_i \text{-cl}(\lambda) \leq (i, j)\Lambda^\gamma(\lambda).
\]
Therefore \( \lambda = \tau_i \text{-cl}(\lambda) \), hence \( \lambda \) is a \( \tau_i \)-fuzzy closed set. \( \square \)

Remark 3.18. A \((i, j)\)-fuzzy \( \Lambda^\gamma \)-closed set is \( \tau_i \)-fuzzy closed set if it is \((i, j)\)-fuzzy semi-\( \gamma \)-closed set.

Definition 3.19. A fuzzy point \( x_p \) is called a \((i, j)\)-fuzzy \( \Lambda^\gamma \)-cluster point of \( \lambda \) if for every \((i, j)\)-fuzzy \( \Lambda^\gamma \)-open set \( \delta \) containing \( x_p \) such that 
\[
\lambda \land \delta \neq 0_X.
\]

Definition 3.20. We define \((i, j)\)-\( \Lambda^\gamma(\tau_i \text{-cl}) \) for any fuzzy set \( \lambda \) in a fuzzy bitopological space \((X, \tau_i, \tau_j)\) as follows: 
\[
(i, j)\Lambda^\gamma(\tau_i \text{-cl}(\lambda)) = \lambda \{ \mu : \lambda \leq \mu \} \text{ and } \mu \text{ is } (i, j)\text{-fuzzy } \Lambda^\gamma\text{-closed set}.
\]
Proposition 3.21. If $\lambda_i$ are $(i, j)$-fuzzy $\Lambda^\gamma$-closed set for each $i \in I$, then $\wedge \lambda_i$ is $(i, j)$-fuzzy $\Lambda^\gamma$-closed set.

Proof: Suppose $\lambda = \wedge \lambda_i$ and $x_p \in (i, j)\Lambda^\gamma(\tau_i\text{-cl}(\lambda))$. Then $x_p$ is a $(i, j)$-fuzzy $\Lambda^\gamma$-cluster point of $\lambda$. Thus there exist a $(i, j)$-fuzzy $\Lambda^\gamma$-open set $\delta$ containing $x_p$ such that $\lambda \wedge \delta \neq 0_X$ and $\lambda \wedge \delta \neq 0_X$. This implies that $(\wedge \lambda \wedge \delta \neq 0_X)$. Thus $\lambda_i \wedge \delta \neq 0_X$ for each $i \in I$. If $x_p \notin \lambda$ for each $i \in I$ then $x_p \notin \lambda_i$. Since $\lambda_i$ is $(i, j)$-fuzzy $\Lambda^\gamma$-closed, $\lambda_i = (i, j)\Lambda^\gamma(\tau_i\text{-cl}(\lambda_i))$ and hence $x_p \in (i, j)\Lambda^\gamma(\tau_i\text{-cl}(\lambda_i))$. Therefore $x_p$ is not a $(i, j)$-fuzzy $\Lambda^\gamma$-cluster point of $\lambda_i$. So there exist a $(i, j)$-fuzzy $\Lambda^\gamma$-open set $\mu$ containing $x_p$ such that $\lambda_i \wedge \mu = 0_X$. Hence by the contradiction $x_p \notin \lambda$. Therefore $(i, j)-\Lambda^\gamma(\tau_i\text{-cl}(\lambda_i)) \subseteq \lambda$ and hence $\lambda = (i, j)\Lambda^\gamma(\tau_i\text{-cl}(\lambda_i))$. Therefore $\wedge \lambda_i$ is $(i, j)$-fuzzy $\Lambda^\gamma$-closed set. \qed

4. $(i, j)$-Fuzzy $\Lambda^\gamma$-Generalized Closed Sets

Definition 4.1. A fuzzy set $\lambda$ of fuzzy bitopological space $(X, \tau_i, \tau_j)$ is called $(i, j)$-fuzzy $\Lambda^\gamma$-generalized closed set if $\tau_j\text{-cl}(\lambda) \subseteq \mu$ whenever $\lambda \subseteq \mu$ and $\mu$ is $(i, j)$-fuzzy $\Lambda^\gamma$-open set, where $i \neq j$ and $i, j = 1, 2$.

Definition 4.2. A fuzzy set $\lambda$ of fuzzy bitopological space $(X, \tau_i, \tau_j)$ is called $(i, j)$-fuzzy $\Lambda^\gamma$-generalized closed set if $\tau_j\text{-cl}(\lambda) \subseteq \mu$ whenever $\lambda \subseteq \mu$ and $\mu$ is a $(i, j)$-fuzzy $\Lambda^\gamma$-open set, where $i \neq j$ and $i, j = 1, 2$.

Proposition 4.3. (i) Every $\tau_j$-fuzzy closed set is $(i, j)$-fuzzy $\Lambda^\gamma$-generalized closed set.
(ii) Every $(i, j)$-fuzzy $\Lambda^\gamma$-generalized closed set $(i, j)$-fuzzy $\Lambda^\gamma$-generalized closed set.

Proof: (i) Let $\lambda$ be any $\tau_i$-fuzzy closed set and $\mu$ be any $(i, j)$-fuzzy $\Lambda^\gamma$-open set such that $\lambda \subseteq \mu$, $\tau_i\text{-cl}(\lambda) = \lambda$. Thus $\lambda$ is a $(i, j)$-fuzzy $\Lambda^\gamma$-generalized closed set. (ii) Since every $\tau_i$-fuzzy open set is $(i, j)$-fuzzy $\Lambda^\gamma$-open set, so every $(i, j)$-fuzzy $\Lambda^\gamma$-generalized closed set is $(i, j)$-fuzzy generalized closed set follows from the definition. \qed

Remark 4.4. Every $(i, j)$-fuzzy $\Lambda^\gamma$-generalized closed set need not be $\tau_i$-fuzzy closed set as seen in the following example.

Example 4.5. Let $X = \{x, y\}$, $\tau_i = \{(x, 0.1), (y, 0.1), 0_X, 1_X\}$ and $\tau_j = \{(x, 0.2), (y, 0.2), 0_X, 1_X\}$. Here $\{i, j\} F \gamma O(X) = \{(x, p), (y, q), 0_X, 1_X\}$ where $0 \leq p \leq 0.2, 0 \leq q \leq 0.2$ and $p > 0.8, q > 0.8$. Let us suppose $\lambda = \{(x, 0.3), (y, 0.3)\}$, here $(i, j)\Lambda^\gamma(\lambda) = \{(x, 0.8), (y, 0.8)\}$ which also contains the $\tau_j\text{-cl}(\lambda)$. Thus $\lambda$ is a $(i, j)$-fuzzy $\Lambda^\gamma$-generalized closed set, but not a $\tau_i$-fuzzy closed set.

Remark 4.6. Every $(i, j)$-fuzzy generalized closed set need not be $(i, j)$-fuzzy $\Lambda^\gamma$-generalized closed set.
Example 4.7. In the above example-4.5, if we consider $\lambda = \{(x, 0.2), (y, 0.2)\}$, then $\lambda$ is a (i, j)-fuzzy generalized closed set but it is not a (i, j)-fuzzy $\Lambda^\gamma$-generalized closed set.

Remark 4.8. Every (i, j)-fuzzy $\Lambda^\gamma$-closed set need not be (i, j)-fuzzy $\Lambda^\gamma$-generalized closed set as shown in the following example.

Example 4.9. Let $X = \{x, y\}$, $\tau_i = \{\{(x, 0.1), (y, 0.1)\}, 0_X, 1_X\}$ and $\tau_j = \{\{(x, 0.2), (y, 0.2)\}, 0_X, 1_X\}$. Here (i, j) $F\gamma O(X) = \{\{(x, p), (y, q)\}, 0_X, 1_X\}$ where $0 \leq p \leq 0.2, 0 \leq q \leq 0.2$ and $p > 0.8, q > 0.8$. Let us suppose that $\lambda = \{(x, 0.85), (y, 0.85)\}$, which implies that $\lambda$ is (i, j)-fuzzy $\Lambda^\gamma$-closed set. Again $\lambda \leq \lambda$ and $\tau_j-cl(\lambda) = 1_X$. Therefore, $\lambda$ is not (i, j)-fuzzy $\Lambda^\gamma$-generalized closed set.

Remark 4.10. Every (i, j)-fuzzy $\Lambda^\gamma$-generalized closed set need not be a (i, j)-fuzzy $\Lambda^\gamma$-closed.

Example 4.11. From the above example 4.5, we see that $\lambda$ is a (i, j)-fuzzy $\Lambda^\gamma$-generalized closed set. But here (i, j) $\Lambda^\gamma(\lambda) \cap \tau_i-cl(\lambda) \neq \lambda$. It implies that $\lambda$ is not (i, j)-fuzzy $\Lambda^\gamma$-closed.

Theorem 4.12. [10] Let $A$ be a locally closed subset of a topological space $(X, \tau)$. For the set $A$, the following properties are equivalent:

(i) $A$ is closed.
(ii) $A$ is $\Lambda^\gamma$-closed.
(iii) $A$ is generalized-closed.

From the above equivalent conditions we can easily say that the collection of all generalized-open sets forms a topology in $(X, \tau)$.

In this paper we originate the above equivalent condition in fuzzy bitopological space $(X, \tau_i, \tau_j)$ with the help of (i, j)-fuzzy $\Lambda$-closed which is a weaker form of (i, j)-fuzzy locally closed set.

Theorem 4.13. Let $\lambda$ be any (i, j)-fuzzy $\Lambda$-closed set in a fuzzy bitopological space $(X, \tau_i, \tau_j)$. For the fuzzy set $\lambda$ the following properties are equivalent:

(i) $\lambda$ is $\tau_i$-fuzzy closed set.
(ii) $\lambda$ is (i, j)-fuzzy $\Lambda$-generalized closed.
(iii) $\lambda$ is (i, j)-fuzzy generalized closed set.

Proof: (i)$\Rightarrow$ (ii) Using proposition 4.3(i), the proof can be done easily.
(ii)$\Rightarrow$ (iii) Using proposition 4.3(ii), it can be proved easily.
(iii)$\Rightarrow$ By using the proposition 3.3, one can easily establish the relation.
From the above result we can conclude that the family of all (i, j)-fuzzy generalized closed sets forms a fuzzy topology in the light of (i, j)-fuzzy $\Lambda$-closed set, though it is not true in general.

$\square$
5. (i, j)-Fuzzy $\Lambda$-Continuity and (i, j)-Fuzzy $\Lambda^\gamma$-Continuity

In this section, we define the idea of continuous function using (i, j)-fuzzy $\Lambda$-set and (i, j)-fuzzy $\Lambda^\gamma$-set. As an application we have shown that $\tau_i$-fuzzy continuous and (i, j)-fuzzy $\Lambda^\gamma$-continuous functions are equivalent up to certain extent.

**Definition 5.1.** Let $f: (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$ be a function from a fuzzy bitopological space $(X, \tau_i, \tau_j)$ into another fuzzy bitopological space $(Y, \sigma_i, \sigma_j)$. Then $f$ is called (i, j)-fuzzy $\Lambda$-continuous (resp., (i, j)-fuzzy $\Lambda^\gamma$-continuous, (i, j)-fuzzy generalized continuous, (i, j)-fuzzy $\Lambda^\gamma$-generalized continuous) if $f^{-1}(\mu)$ is (i, j)-fuzzy $\Lambda$-closed (resp., (i, j)-fuzzy $\Lambda^\gamma$-closed, (i, j)-fuzzy generalized closed, (i, j)-fuzzy $\Lambda^\gamma$-generalized closed set) in $X$ for each $\sigma_i$-fuzzy closed set $\mu$ in $Y$.

**Proposition 5.2.** Every $\tau_i$-fuzzy continuous function is (i, j)-fuzzy $\Lambda$-continuous function.

**Proof:** It is straightforward from the definition. \qed

**Remark 5.3.** Converse of the above proposition may not be true as seen in the following example.

**Example 5.4.** Let $X = \{x, y\}$, $\tau_i = \{(x, 0.3), (y, 0.3)\}, 0_X, 1_X$ and $\tau_j = \{(x, 0.6), (y, 0.7)\}, 0_X, 1_X$, $\sigma_i = \{(x, 0.6), (y, 0.7)\}, 0_Y, 1_Y$ and $\sigma_j = \{(x, 0.2), (y, 0.5)\}, 0_Y, 1_Y$.

Now we consider a function $f: (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$ such that $f(x) = x$ and $f(y) = y$. Thus $f$ is a (i, j)-fuzzy $\Lambda$-continuous function but not a $\tau_i$-fuzzy continuous. Since the inverse image of $\sigma_i$-fuzzy closed set in $Y$ is (i, j)-fuzzy $\Lambda^\gamma$-closed set in $X$ which is not a $\tau_i$-fuzzy closed set.

In our next theorem we have shown that the converse part is true under some particular circumstances.

**Theorem 5.5.** Let $f: (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$ be a function, where $X$ is a (i, j)-fuzzy submaximal space and the only (i, j)-fuzzy regular open set in $X$ is $0_X$ then the following conditions are equivalent:

(i) $f$ is $\tau_i$-fuzzy-continuous.

(ii) $f$ is (i, j)-fuzzy $\Lambda$-continuous.

**Proof:** (i) $\Rightarrow$ (ii) Let $\mu$ be any $\sigma_i$-fuzzy closed set in $Y$. Since $f$ is $\tau_i$-fuzzy-continuous, so $f^{-1}(\mu)$ is $\tau_i$-fuzzy closed set in $X$. Thus $f^{-1}(\mu)$ is (i, j)-fuzzy $\Lambda$-closed set in $X$ since every $\tau_i$-fuzzy closed set is (i, j)-fuzzy $\Lambda$-closed set. Therefore $f$ is (i, j)-fuzzy $\Lambda$-continuous.

(ii) $\Rightarrow$ (i) Let $\mu$ be any $\sigma_i$-fuzzy closed set in $Y$. Thus for any $\sigma_i$-fuzzy closed set we have $f^{-1}(\mu)$ is (i, j)-fuzzy $\Lambda$-closed set. Hence $f^{-1}(\mu)$ is a $\tau_i$-fuzzy closed set in $X$, since in a (i, j)-fuzzy submaximal space every (i, j)-fuzzy $\Lambda$-closed set is a $\tau_i$-fuzzy closed set. Therefore $f$ is a $\tau_i$-fuzzy continuous. \qed
Theorem 5.6. For any function \( f:(X,\tau_1,\tau_j)\to (Y,\sigma_i,\sigma_j) \) following conditions are equivalent:

(i) \( f \) is \((i, j)\)-fuzzy \( \Lambda^\gamma \)-continuous.

(ii) \( f^{-1}(\sigma_i,\text{int}(\lambda)) \leq (i, j)\)-\( \Lambda^\gamma \)(\( \tau_1,\text{int}(f^{-1}(\lambda)) \)), for any fuzzy subset \( \lambda \) of \( Y \).

(iii) \((i, j)\)-\( \Lambda^\gamma \)(\( \tau_1,\text{cl}(f^{-1}(\lambda)) \))\leq f^{-1}(\sigma_i,\text{cl}(\lambda)), for any fuzzy subset \( \lambda \) of \( Y \).

Proof: (i)\(\Rightarrow\)(ii) Let \( f \) be \((i, j)\)-fuzzy \( \Lambda^\gamma \)-continuous and \( \lambda \) be any subset of \( Y \).

Since \( \sigma_i,\text{int}(\lambda) \leq \lambda \Rightarrow f^{-1}(\sigma_i,\text{int}(\lambda)) \leq f^{-1}(\lambda) \). Now since \( f^{-1}(\sigma_i,\text{int}(\lambda)) \) is \((i, j)\)-fuzzy \( \Lambda^\gamma \)-open in \( X \). Therefore \( f^{-1}(\sigma_i,\text{int}(\lambda)) \leq (i, j)\)-\( \Lambda^\gamma \)(\( \tau_1,\text{int}(f^{-1}(\lambda)) \)).

(ii)\(\Rightarrow\)(iii) Let \( \lambda \) be any subset of \( Y \), then

\[ jf^{-1}(\sigma_i,\text{int}(1_{Y} - \lambda)) \leq (i, j)\)-\( \Lambda^\gamma \)(\( \tau_1,\text{int}(f^{-1}(\sigma_i,\text{int}(1_{Y} - \lambda)) \)) \Rightarrow 1_X - f^{-1}(\sigma_i,\text{cl}(\lambda)) \leq (i, j)\)-\( \Lambda^\gamma \)-\( \tau_1,\text{cl}(f^{-1}(\lambda)) \) \Rightarrow (i, j)\)-\( \Lambda^\gamma \)(\( \tau_1,\text{cl}(f^{-1}(\lambda)) \)) \leq f^{-1}(\sigma_i,\text{cl}(\lambda)) \).

(iii)\(\Rightarrow\)(i) Let \( \lambda \) be any \( \sigma_i \)-fuzzy closed set in \( Y \).

Now \( f^{-1}(\lambda) = f^{-1}(\sigma_i,\text{cl}(\lambda)) \) is \((i, j)\)-fuzzy \( \Lambda^\gamma \)-closed.

Thus \( f^{-1}(\lambda) \) is \((i, j)\)-fuzzy \( \Lambda^\gamma \)-closed in \( X \).

\[ \square \]

Remark 5.7. Every \((i, j)\)-fuzzy \( \Lambda^\gamma \)-continuous function and \((i, j)\)-fuzzy \( \Lambda^\gamma \)-continuous function are independent of each other.

Theorem 5.8. A \((i, j)\)-fuzzy \( \Lambda^\gamma \)-continuous function from a fuzzy bitopological space \( X \) to another fuzzy bitopological space \( Y \) is \( \tau_1 \)-fuzzy continuous if every \((i, j)\)-fuzzy \( \Lambda^\gamma \)-closed set is \((i, j)\)-fuzzy semi \( \gamma \)-closed set in \( X \).

Proof: Let \( f \) be \((i, j)\)-fuzzy \( \Lambda^\gamma \)-continuous function from \( X \) to \( Y \) and let \( \mu \) be any \( \sigma_i \)-fuzzy closed set in \( Y \). Therefore \( f^{-1}(\mu) \) is \((i, j)\)-fuzzy \( \Lambda^\gamma \)-closed set in \( X \).

Since every \((i, j)\)-fuzzy \( \Lambda^\gamma \)-closed set is \((i, j)\)-fuzzy semi \( \gamma \)-closed set in \( X \) and hence by proposition 3.17, \( f^{-1}(\mu) \) is \( \tau_1 \)-fuzzy closed set in \( X \). Therefore \( f \) is \( \tau_1 \)-fuzzy continuous.

\[ \square \]

Theorem 5.9. Let \( f:(X,\tau_1,\tau_j)\to (Y,\sigma_i,\sigma_j) \) be a function. Then the followings are equivalent.

(i) \( f \) is \( \tau_1 \)-fuzzy continuous function.

(ii) \( f \) is \((i, j)\)-fuzzy generalized continuous and \((i, j)\)-fuzzy \( \Lambda^\gamma \)-continuous functions.

Proof: (i)\(\Rightarrow\)(ii) Proof is straightforward.

(ii)\(\Rightarrow\)(i) Let \( \mu \) be any \( \sigma_i \)-fuzzy closed set in \( Y \). Thus \( f^{-1}(\mu) \) is \((i, j)\)-fuzzy generalized closed and \((i, j)\)-fuzzy \( \Lambda^\gamma \)-closed set by the given condition. Hence \( f^{-1}(\mu) \) is \( \tau_1 \)-fuzzy closed set using theorem 4.3. Therefore \( f \) is \( \tau_1 \)-fuzzy continuous.

The notion of \((i, j)\)-fuzzy \( \Lambda \)-continuous and \((i, j)\)-fuzzy \( \Lambda^\gamma \)-continuous functions are independent of each other. But in the light of Theorem 5.5 and 5.9 one can verify that they are sameness as an application.  

\[ \square \]
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