



Item response theory in the production of indicators of socioeconomic metropolitan region of Maringá, Paraná State, Brazil

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ABSTRACT. This study aimed to identify and produce through models of Item Response Theory (IRT) a socio-economic indicator based in the items observed in 2000 Census, following the methodology by Soares (2005). By the IRT Methodology, this indicator, as a latent variable, is obtained through the construction of specific models and scales, making it possible to measure this variable, which according to Andrade et al. (2000), IRT analyzes each item which compose the measuring instrument. This case consists of binary or dichotomous items, which assess the possession of certain assets of domestic comfort. The characteristics of each item were analyzed, as the ability to discrimination and income necessary for the possession of certain property. It was concluded that with 13 items, a trustworthy questionnaire can be done for the construction of a socioeconomic index of Maringá's metropolitan region.

Keywords: latent variable, microdata 2000 Census, trustworthy questionnaire.

Teoria de resposta ao item na produção de indicadores sócio-econômicos da região metropolitana de Maringá, Estado do Paraná, Brasil

RESUMO. Este estudo teve como proposta identificar e produzir por meio de modelos da Teoria de Resposta ao Item (TRI) um indicador sócio-econômico baseado nos itens observados no Censo Demográfico 2000, seguindo a metodologia de Soares (2005). Pela metodologia TRI este indicador, por ser uma variável latente, é obtido por meio da construção de modelos específicos e de escalas, tornando possível medir esta variável, segundo Andrade et al. (2000), a TRI analisa cada item que constitui o instrumento de medida. Neste estudo os itens são binários (dicotômicos) e avaliam a posse de determinados bens de conforto doméstico. Características de cada item foram analisadas como a capacidade de discriminação e renda necessária para a posse do determinado bem. Foi possível concluir que com 13 itens têm-se um questionário fidedigno para a construção do índice sócio-econômico na Região Metropolitana de Maringá.

Palavras-chave: variável latente, microdados Censo 2000, questionário fidedigno.

Introduction

There's an existent need in evaluate latent variables, in other words, a variable which is not directly measurable, as socio-occupational condition, satisfaction, learning, happiness, et cetera. These variables, which can not be directly measured, are evaluated by a scale of values based in instruments like tests or questionnaires. These instruments are constituted by items (specific questions, according to the applying model) which are associated to the variable of interest.

Thus, the questions presented in the instrument are indicators to measure the constructs.

There are different techniques to evaluate an instrument e consequently obtain the measure, which can be divided into classic techniques, in which the model for the construct is directly based

on the result given by the instrument as a whole, and techniques in which specific models are made for each item of the instrument and the scale is built considering these individual models. The techniques based on the item response theory (IRT) can be classified in this last case.

According to Valle (2000), the Item Response Theory manage the problem of estimating ability and knowledge of an individual in an essentially different way: the analysis focus does not attaches itself on the proofs (Classical Test Theory) but on the items; if in the Classical Test Theory the items' statistics depend on the population of respondents and on the proof whose items belongs, the Item Response Theory assumes that the concepts of the items' parameters are made, obtained on the statistical procedure of calibration of the difficulty parameters, discrimination and

items' random hits are specific characteristics of them. The characteristic of the Items' measure (represented by their parameters) is often considered to be undeviating along time with a few exceptions. Considered the previous ponderations, a primal characteristic for the feasibility of knowledge and ability comparison of respondents who were submitted to different proofs is that the Item Response Theory models the probability of an item to be right, also known as Item Characteristic Curve, by means of non linear function of knowledge. This property of Item Response Theory modeling has great importance, since this manner allows the comparison of respondents whose knowledge were measured by different proofs, only requiring that those proofs measure the same characteristics.

This work was developed according to Soares (2005) article, entitled "Utilização da Resposta ao Item na Produção de Indicador Sócio-Econômico" (Application of Item Response Theory in the Production of Socio-Economic Status), in which were compared three types of indicators, produced by the models of Item Response Theory, for the socio-occupational condition of students who participated in educational assessment programs in the state of Minas Gerais. The goal of the study accomplished by Soares (2005) was to guide the production of that type of indicator in the sphere of the assessment program and finding the minimum variables to be introduced in the questionnaire. Under these issues, the work demonstrated that, in a total of 20 variables originally available, only 13 or 14 of them could be used to produce an index with quality.

The Metropolis' Observatory (Observatório das Metrópoles), Regional Center of Maringá

(Maringá's metropolitan region), is an institutional program which, initially, classifies people from Maringá's metropolitan region by socio-occupational categories based in the variables income, education, field of activity, occupation and position in this occupation, these variables belong to the micro-data of IBGE Census 2000. By the Item Response Theory methodology, it was possible to characterize the socio-occupational condition based on a series of questions which evaluate the possession of certain assets of domestic comfort, such as radios, refrigerators, air conditioners, vehicles and others. The characteristics of each item were analyzed as the capability to discriminate and the necessary income to possess such asset.

Material and methods

The items analyzed in this study were obtained from Micro-data of Census 2000, presented on Table 1, these items had two classes, therefore, they were dichotomized according to Soares (2005) items, being 1 for possession of the asset and 0 for the opposite, the population was 16,270 residences.

The logistic model used was the two-parameter logistic, which measure the difficulty and discrimination for dichotomous items.

The analyses were realized using the software BILOG-MG for Windows, Version: 3.0.2327.2, January 2003 (SCIENTIFIC SOFTWARE INTERNATIONAL, 2003).

Logistic models

According to Andrade et al. (2000), the logistic models for dichotomous items are the most used item response models, whereas there basically three types, who differ themselves by the number of parameters they use to describe the item.

Table 1. Indicating issues of socioeconomic conditions (2000 Census).

| Item | Description |
|---------|---|
| Item 1 | Rooms with dormitories, number (0 for 0 and 1; 1 for 2 or more) |
| Item 2 | Water supply, source (0 for other; 1 for general network) |
| Item 3 | Residence, occupational condition (0 for other; 1 for ownership) |
| Item 4 | Bathrooms, number (0 for 0 e 1; 1 for 2 or more) |
| Item 5 | Waste, destination (0 for other; 1 for collected by waste management service) |
| Item 6 | Electrical lighting, existence (1 for YES; 0 for NO) |
| Item 7 | Radio, existence (1 for YES; 0 for NO) |
| Item 8 | Refrigerator or freezer, existence (1 for YES; 0 for NO) |
| Item 9 | Videocassette recorders (VCR), existence (1 for YES; 0 for NO) |
| Item 10 | Washing machine, existence (1 for YES; 0 for NO) |
| Item 11 | Microwave oven, existence (1 for YES; 0 for NO) |
| Item 12 | Installed telephone line, existence (1 for YES; 0 for NO) |
| Item 13 | Microcomputer, existence (1 for YES; 0 for NO) |
| Item 14 | Television, existence (1 for YES; 0 for NO) |
| Item 15 | Vehicles for private use, number (0 for 0 ; 1 for 1 or more) |
| Item 16 | Air conditioner, number (0 for 0 ; 1 for 1 or more) |

These models are known as logistic models of 1, 2 and 3 parameters, which consider, respectively:

- I) only the difficulty of the item;
- II) the difficulty and the discrimination of the item;
- III) the difficulty, the discrimination and the probability of a correct response, given by an individual of low ability.

The logistic model of 2 parameters, this model is given by:

$$P(U_{ij}=1|\theta_j) = \frac{1}{1 + e^{-Da_i(\theta_j - b_i)}},$$

with item $i = 1, 2, \dots, I$ and individual $j = 1, 2, \dots, n$, wherein:

U_{ij} is a dichotomous variable that assumes the values 1, when the individual j possesses the asset i , or 0 when it does not possess.

θ_j represents the ability (latent trace) of the j -th individual.

$P(U_{ij}=1|\theta_j)$ is the probability of an individual j with the ability θ_j possesses the asset i and it is called Item Response Function (IRF).

b_i Is the difficulty parameter of the item i , measured in the same scale as ability.

a_i is the discrimination parameter of item i , with a proportional value to Item Characteristic Curve's inclination at the point b_i , which has an example showed in Figure 1.

D is a scale factor, constant and equals 1. It can be equal to 1,7 when wishes that the logistic function gives results similar to those from the normal ogive function.

The higher the a_i parameter's value, the more sensitive becomes the model to ability variations around its difficulty point. Therefore, it is known as item's discrimination parameter. Figure 1 presents the so-called Item Characteristic Curve, in other words, the representation of the values under the form of a model's graph as a function of θ .

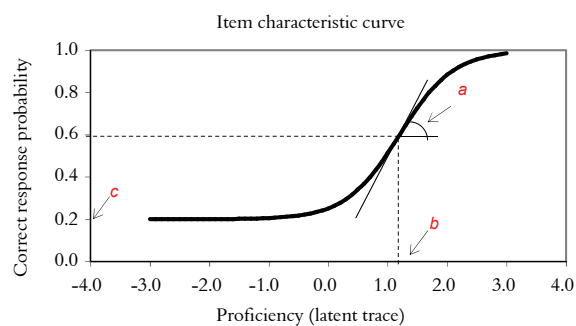


Figure 1. Example of an Item Characteristic Curve.

It is observed that the proposed model, which is based in the fact that the individuals with higher ability have higher probability of possessing the asset and that this ratio is non linear.

The scale of possession is an arbitrary scale, wherein the existent order relations are important and not necessary their magnitude. The parameter b is measured in the same scale as possession.

The parameter b represents the necessary income for an asset's possession probability. Therefore, the higher the b value, the more difficult is the probability of possess that asset.

The parameter a determines the discrimination power of the items.

Biserial correlation

According to Soares (2005), the point-biserial correlation is a statistical mensuration that measures the item's discrimination capability, as well as parameter a , against the test's result. To do so, admit that S represents the raw score obtained in the test. Admit that Y represents the result of a response assigned to an item, a dichotomous variable (in the case of an item that evaluate the socio-occupational condition, $Y = 1$ represents the possession of an asset, for an example). This correlation is known as Pearson's and it is defined as:

$$\rho_{SY} = \frac{E(SY) - E(S)E(Y)}{\sigma_Y \sigma_S}$$

wherein a natural estimation obtained over the test's result is given by:

$$\bar{\rho}_{pb} = \frac{\bar{S}_p - \bar{S}}{\bar{\sigma}_S} \sqrt{\frac{\bar{p}}{\bar{q}}}$$

with \bar{S}_p being the test's mean score for those which got the item right, \bar{S} being the test's mean score for all, \bar{p} being the mean proportion of those which got the item right, $\bar{\sigma}_S$ being the standard deviation of the scores obtained in the tests by the respondents and the estimation $\bar{\rho}_{pb}$ for Pearson's correlation, which is often named in the literature as point-biserial correlation.

Calibration

The calibration provides to each item, parameters that characterize its technical qualities, independent from the population under investigation, in other words, is the phase in which parameters are estimated. According to Andrade et al. (2000), the two-parameter logistic model is

calculated to determinate the item discrimination index and expresses the relation between the latent variable θ and the response given to the item. Admit that Y is a dichotomous random variable assuming both 0 and 1 values. In the case of socio-occupational questionnaires, the 0 value is associated to negative response in relation to comfort assets and 1 the affirmative response to that asset, as described below:

$$P_i(Y=1; \theta, a_i, b_i) = \frac{e^{Da_i(\theta-b_i)}}{1 + e^{Da_i(\theta-b_i)}},$$

wherein the i index represents the item; a_i is denominated item discrimination parameter; b_i is denominated item difficulty parameter, D is a scale factor, a constant and usually equals 1.7, used to approximate the logistic curve to the normal distribution, the higher a_i parameter's value, the more sensitive the model becomes against the variations around its difficulty point, therefore it is known as item discrimination parameter.

Results and discussion

To determinate if one variable presents significant correlation with the raw score produced by the variable series, initially was observed the item's biserial correlation. This step is crucial to choose which items have, indeed, internal consistence and associate themselves well to the score that will be produced. According to Soares (2005), normally, to this correlation, values above 0.03 are accepted. Table 2 shows the correlations for each item. By Table 2, is possible to observe that none of the variables have correlations below 0.03, therefore, all of them correlates well with the raw score. Although, observing the values of the possessions proportions, on Table 2, it can be seen that most of the variables presents a high proportion, which can be reflected into an index with low information, due to the absence of items with difficulty to access the higher asset. However, the item 6, which evaluates the existence, or not, of electrical energy, presents a higher proportion, 0.994, such that it probably will not bring information to the index that will be made. Thus, it can be eliminated of the questionnaire.

In the second phase, which consists in the index analysis, the parameters of the model are observed, being a what determines the item's discrimination and b what represents the necessary income to possess the particular asset.

By means of the model's parameters, can be observed that the item with highest discrimination value, $a = 3.968$, coincides with item 6 (existence of electrical energy), already commented as the item with the highest proportion of residences which possess that asset.

Table 2. Biserial correlation values and items' parameters estimates.

| Item | Possessions' Proportions | Biserial Correlation | Discrimination Parameter a (E.P.) | Difficulty Parameter b (E.P.) |
|------|--------------------------|----------------------|-------------------------------------|---------------------------------|
| 1 | 0.743 | 0.301 | 0.656 (0.11) | -1.769 (0.07) |
| 2 | 0.529 | 0.238 | 0.404 (0.18) | -0.300 (0.04) |
| 3 | 0.914 | 0.123 | 0.090 (0.01) | -26.237 (0.25) |
| 4 | 0.259 | 0.752 | 2.252 (0.25) | 0.810 (0.23) |
| 5 | 0.882 | 0.015 | 0.042 (0.002) | -47.962 (6.35) |
| 6 | 0.994 | 0.523 | 3.968 (0.35) | -3.109 (0.55) |
| 7 | 0.924 | 0.294 | 0.823 (0.08) | -3.364 (0.56) |
| 8 | 0.963 | 0.525 | 2.620 (0.20) | -2.169 (0.45) |
| 9 | 0.381 | 0.715 | 2.225 (1.35) | 0.373 (0.07) |
| 10 | 0.472 | 0.590 | 1.560 (0.37) | 0.096 (0.003) |
| 11 | 0.280 | 0.760 | 2.629 (0.65) | 0.690 (0.45) |
| 12 | 0.523 | 0.700 | 2.310 (0.85) | -0.082 (0.05) |
| 13 | 0.130 | 0.741 | 2.554 (0.21) | 1.375 (0.21) |
| 14 | 0.339 | 0.721 | 2.053 (0.41) | 0.533 (0.17) |
| 15 | 0.488 | 0.634 | 1.851 (0.25) | 0.033 (0.001) |
| 16 | 0.620 | 0.683 | 2.244 (0.55) | 1.987 (0.08) |

Figure 2 presents the Item Characteristic Curve for this index.

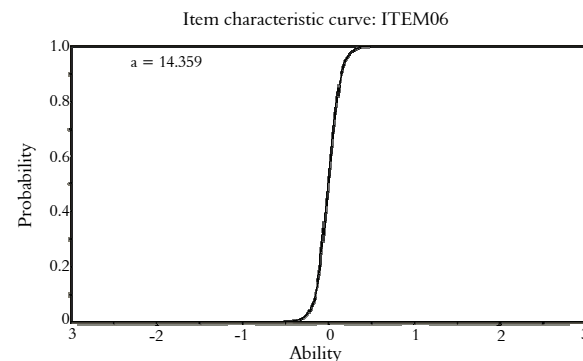


Figure 2. Characteristic curve of item 6 (existence of electrical energy).

Item 6 is quite discriminatory, which corresponds to an asset (electrical energy) that most of the individuals possess in their residences, therefore, this item do not helps in the construction of the index. Item 5 is the one with minor discrimination power, which evaluates the existence of waste collection service in the residence, which is slightly discriminatory and also do not satisfies the construction of the index.

Figure 3 presents the Item Characteristic Curve for Item 5.

By observing Figures 2 and 3, it is possible to see a clear difference between the curves' inclinations. This occurs by the fact that low values of a indicate that the item has low discrimination power (individuals with

quite different purchasing power have the same probability to possess the asset) and very high values with quite steep characteristic curves discriminate the individuals in basically two groups: those who have abilities below parameter b value and those who have abilities above parameter b value.

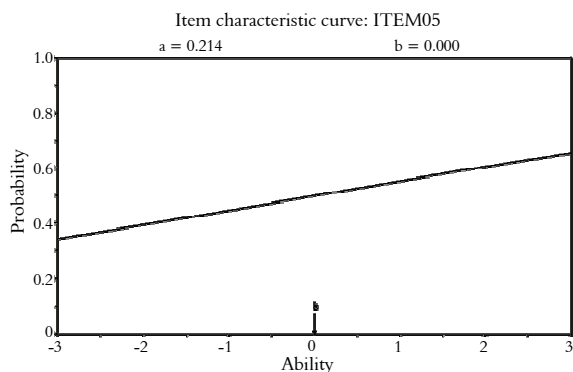


Figure 3. Characteristic Curve of Item 5 (waste management service).

Concerning the difficulty parameter, b , it can be verified that the highest value is found in the item 16, which evaluates the possession of air conditioning machines, in other words, it is an item with a very high difficulty, therefore, it is the asset in which the individual needs higher income to possess it and also has to be eliminated from the questionnaire.

The item that presented lowest difficulty was item 5 (existence of waste management service), this item also presented the minor discrimination value, therefore, the income can be low for the individual to have this service. So, as this item presented low discrimination power, it does not help in the income measure, as mentioned before.

After the analyses of all 16 models' parameters, this is, the model of each item and also the analyses of correlations and proportions of assets of domestic comfort possession, it was possible to construct a trustworthy index by IRT, for Maringá's Metropolitan Region with only 13 items. As presented on Table 3.

Table 3. Indicative issues of socio-occupational condition, trustworthy according to IRT.

| Item | Description |
|---------|--|
| Item 1 | Rooms with dormitories, number (0 for 0 and 1; 1 for 2 or more) |
| Item 2 | Water supply, source (0 for other; 1 for general network) |
| Item 3 | Residence, occupational condition (0 for other; 1 for ownership) |
| Item 4 | Bathrooms, number (0 for 0 e 1; 1 for 2 or more) |
| Item 5 | Radio, existence (1 for YES; 0 for NO) |
| Item 6 | Refrigerator or freezer, existence (1 for YES; 0 for NO) |
| Item 7 | Videocassette recorders (VCR), existence (1 for YES; 0 for NO) |
| Item 8 | Washing machine, existence (1 for YES; 0 for NO) |
| Item 9 | Microwave oven, existence (1 for YES; 0 for NO) |
| Item 10 | Installed telephone line, existence (1 for YES; 0 for NO) |
| Item 11 | Microcomputer, existence (1 for YES; 0 for NO) |
| Item 12 | Television, existence (1 for YES; 0 for NO) |
| Item 13 | Vehicles for private use, number (0 for 0; 1 for 1 or more) |

Conclusion

With the purpose of finding a trustworthy indicator for the socio-occupational condition, the research pointed that, for an amount of 16 variables originally available, only three presented non satisfactory values. Item 6, existence of electrical lighting, must be eliminated by the fact that its possession's proportion and discrimination value are quite high, meaning that the majority of the individuals possess that service in their residences. Item 5, which evaluates the existence of waste management service in the residence, must be eliminated of the questionnaire for presenting the lowest discrimination value and low difficulty, which means that this service appears in all types of income, since lower to higher. Item 16, existence of air conditioning machine, must be eliminated from the questionnaire by the fact of having a too high difficulty, meaning that only those with higher income could possess that asset. Therefore, the analyses resulted that 13 items made a trustworthy questionnaire for the construction of a socio-occupational condition index for Maringá's Metropolitan Region, corroborating with those found by Soares (2005), in which only 13 or 14 items made an index of quality for Minas Gerais State.

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