



Bending stiffness evaluation of Teca and Guajará lumber through tests of transverse and longitudinal vibration

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ABSTRACT. The grading of structural lumber besides contributing for increasing the structure's safety, due to the reduction of the material variability, also allows its rational use. Due to the good correlation between strength and bending stiffness, the latter has been used in estimating the mechanical strength of lumber pieces since the 60's. For industrial application, there are equipment and techniques to evaluate the bending stiffness of lumber, through dynamic tests such as the longitudinal vibration technique, also known as stress wave, and the transverse vibration technique. This study investigated the application of these two techniques in the assessment of the modulus of elasticity in bending of Teca beams (*Tectona grandis*), from reforestation, and of the tropical species Guajará (*Micropholis venulosa*). The modulus of elasticity estimated by dynamic tests showed good correlation with the modulus measured in the static bending test. Meantime, we observed that the accuracy of the longitudinal vibration technique was significantly reduced in the evaluation of the bending stiffness of Teca pieces due to the knots existing in this species.

Keywords: lumber, non-destructive evaluation, dynamic tests.

Avaliação da rigidez à flexão de peças estruturais de Teca e Guajará pelos ensaios de vibração transversal e vibração longitudinal

RESUMO. A classificação de peças estruturais de madeira, além de contribuir para aumentar a segurança das estruturas, em função da redução da variabilidade do material, permite também a utilização racional desse material. Pela boa correlação entre resistência mecânica e rigidez à flexão, essa última tem sido empregada na estimativa da resistência mecânica de peças de madeira serrada desde a década de 60. Para a aplicação industrial, existem equipamentos e técnicas que permitem avaliar a rigidez à flexão da madeira serrada, por meio de ensaios dinâmicos como, por exemplo, a técnica de vibração longitudinal, também conhecida como ondas de tensão e a técnica de vibração transversal. O objetivo deste trabalho foi investigar a aplicação dessas duas técnicas na avaliação do módulo de elasticidade à flexão de vigas da espécie Teca (*Tectona grandis*), proveniente de reflorestamento, e da espécie tropical Guajará (*Micropholis venulosa*). O módulo de elasticidade estimado com os ensaios dinâmicos mostrou boa correlação com o módulo de elasticidade medido no ensaio estático de flexão. Entretanto, observou-se que a exatidão da técnica de vibração longitudinal foi reduzida significativamente na avaliação da rigidez à flexão das peças Teca em função dos nós existentes nessa espécie.

Palavras-chave: madeira serrada, avaliação não-destrutiva, ensaios dinâmicos.

Introduction

The structural grading of lumber consists of grouping into classes the pieces with strength and stiffness of the same order of magnitude. This procedure provides the rational use of the lumber, since it becomes possible to employ the best quality lumber to the higher stressed parts, and lower quality lumber in less loaded parts of a structure.

Due to the lower variability of mechanical properties within each class, it has also an increase in

the design strength because the coefficient k_{mod3} changes from 0.8 to 1.0, which results in less wood consumption without compromising the structural security.

The method traditionally used to measure the strength of wooden structural members is to conduct destructive tests such as bending or compression parallel to the grain. However, after knowing its strength, the piece becomes unusable. Thus, these tests can not be developed, for

example, in a Glued Laminated Timber industry where all wood boards laminating need to be graded. Fortunately, in the 60's, researchers in North America had found that there is a strong correlation between the modulus of elasticity and modulus of rupture in bending. Since then, the bending stiffness has been used for Non-Destructive Evaluation (NDE) of strength of structural wooden members.

The modulus of elasticity of structural lumber can be determined from the static bending test, as established by ASTM D198 (ASTM, 2008). Nevertheless, in an industry the classification speed is essential, and thus devices have been developed to estimate the bending stiffness of structural lumber in a few seconds. More recently, with the advances in computing and mathematical tools, dynamic tests have been employed to evaluate the bending stiffness of structural lumber. The most common dynamic tests are: transverse vibration, longitudinal vibration, also known as stress waves, and the ultrasound technique.

The present study aimed at examining the application of the transverse vibration and longitudinal vibration techniques in the evaluation of the bending modulus of elasticity of wood beams of the species Teca (*Tectona grandis*) from reforestation, and of the tropical species Guajará (*Micropholis venulosa*). The Teca and Guajará beams were obtained from the cities of Alta Floresta and Cuiabá, respectively, in the Mato Grosso State, Brazil.

Theoretical bases

The estimate of the longitudinal modulus of elasticity of lumber is based on the theory of propagation of longitudinal waves in a bar. The differential equation that governs the motion of an infinitesimal element of a bar under a longitudinal vibration is shown in Equation 1 (CLOUGH; PENZIEN, 1995).

$$EA \frac{\partial^2 u(x,t)}{\partial x^2} - m' \frac{\partial^2 u(x,t)}{\partial t^2} = 0 \quad (1)$$

where:

- E: modulus of elasticity;
- A: cross sectional area;
- m': mass per unit length;
- u: longitudinal displacement;
- t: time;
- x: longitudinal coordinate.

The natural frequencies of a bar in free-free suspension subjected to longitudinal vibration can be calculated by the Equation 2.

$$\omega_n = \frac{n\pi}{L} \sqrt{\frac{EA}{m'}} \quad (2)$$

where:

- n: mode of interest;
- L: bar length (m);
- E: modulus of elasticity (N m⁻²);
- A: cross sectional area (m²);
- m': mass per unit length (m' = ρA).

By isolating the modulus of elasticity of the Equation 2 and expressing the first mode natural frequency in Hz, we get the expression that provides the elastic modulus of a bar under a longitudinal vibration (Equation 3).

$$E = 4\rho(Lf)^2 \quad (3)$$

where:

- E: modulus of elasticity (N m⁻²);
- ρ: apparent density (kg m⁻³);
- L: bar length (m);
- f: first mode natural frequency (Hz).

The equation of motion of a straight beam in free transverse vibration is shown in the Equation 4 (TIMOSHENKO, 1938);

$$m'(y) \frac{\partial^2 v(y,t)}{\partial t^2} + \frac{\partial^2}{\partial y^2} \left[EI \frac{\partial^2 v(y,t)}{\partial y^2} \right] = 0 \quad (4)$$

where:

- E: modulus of elasticity;
- I: moment of inertia of the cross section;
- m': mass per unit length;
- v: transverse displacement;
- t: time;
- y: coordinate in the transverse direction.

Equation 5 provides the natural frequency of the first bending mode of a beam in free-free suspension.

$$f = \frac{3.560}{L^2} \sqrt{\frac{EI}{\rho A}} \quad (5)$$

By isolating the modulus of elasticity (E) in Equation 5:

$$E = \frac{f^2 L^4 \rho A}{12.6791} \quad (6)$$

where:

- E: modulus of elasticity (N m⁻²);
- f: first mode natural frequency (Hz);
- L: bar length (m);

ρ : apparent density (kg m^{-3});
 A : cross sectional area (m^2);
 I : moment of inertia of the cross section (m^4).

Material and methods

In order to develop this work, 24 wood beams were used: 12 of Guajará (*Micropholis venulosa*) with nominal dimensions (5 x 11 x 300 cm) and 12 of Teca (*Tectona grandis*) with nominal dimensions (4 x 10 x 200 cm).

These species were selected to verify the application of NDE techniques in two different situations: tropical lumber of relatively high density and reforestation wood with low density.

Importantly to note that while the Guajará beams do not contain any knot, Teca beams showed significant amount; and were visually graded as class 2 (S2), according to the project of the new version of the NBR 7190 standard (ABNT, 1997).

Measurement of dimensions and mass

The dimensions of the beams were measured employing a steel measuring tape with 1 mm resolution. Afterwards, the mass of beams were weighed on a Toledo digital scale model 2098, with 50 g resolution.

With the mass and volume of each beam, the apparent density was calculated in the moisture content of the test.

Determining moisture content

The moisture content of the beams was determined with an electric moisture-meter model Digisystem DL 2000. The moisture of each beam was taken as the arithmetic mean of three measurements along the length.

Bending static test

The static bending test was developed in accordance with ASTM D198 (ASTM, 2008) with the configuration of simply supported beam with concentrated force in half lengthwise. The beams were bent in relation to the axis with the lowest inertia. Figure 1 schematically illustrates the static bending test of three points.

The beams were supported on metal roller and initially, at half the length of the beams it was applied a concentrated force of 50 N (F_1), and the resulting vertical displacement (Δ_1) at half the length was then recorded. The concentrated force was increased to 100 N (F_2) and the new displacement was recorded (Δ_2). For the measurement of the

vertical displacements it was used a Mitutoyo dial indicator with 0.01 mm resolution and full scale of 50 mm.

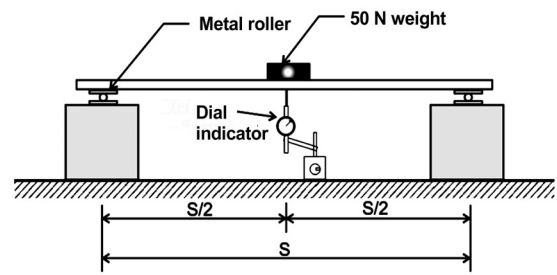


Figure 1. Scheme of static bending test.

As the ratio $L h^{-1}$ (length/height) of all beams was higher than 20, the static modulus of elasticity was calculated without considering the shear effect, using the Equation 7, ASTM D198 (ASTM, 2008)

$$E_{\text{est}} = \frac{FS^3}{4\Delta bh^3}, \quad \text{with } F = F_2 - F_1 \quad \text{and} \quad \Delta = \Delta_2 - \Delta_1 \quad (7)$$

where:

- E_{est} : static bending modulus of elasticity (Pa);
- F : force increase (N);
- S : span (m);
- Δ : vertical displacement increase (m);
- b : width of cross sectional (m);
- h : height of cross sectional (m).

Transverse vibration test

In the transverse vibration test, it was adopted the free-free boundary condition, due to the ease of get it effectively. To this end, the beams were suspended by two nylon ropes (3 mm diameter) attached to two low-stiffness springs, in turn, attached to a metal frame, as shown in Figure 2.



Figure 2. Transverse vibration test.

Nylon ropes were placed on the nodal points of the first bending mode of a beam in free-free suspension, to minimize the influence of the suspension system on the natural frequencies of the first mode.

At one end of the beams, it was attached a MEMS (Micro Electro-Mechanical Systems) accelerometer LIS3L02AS4 manufactured by the ST Microelectronics. The beams were excited by the impact applied to the opposite end of the accelerometer. For this, we used the impulse hammer Endevco model 2302. The signs of the hammer and accelerometer were applied to a signal conditioner developed by the first author and the conditioned signal was then applied to the data acquisition card USB 6009 from National Instruments.

The signal analysis was performed using a program developed in LabVIEW, from which one can get the Frequency Response Function (FRF) of the beams (Figure 3). From the FRF, the natural frequencies were obtained using the Least Squares Complex Exponential (LSCE) algorithm.

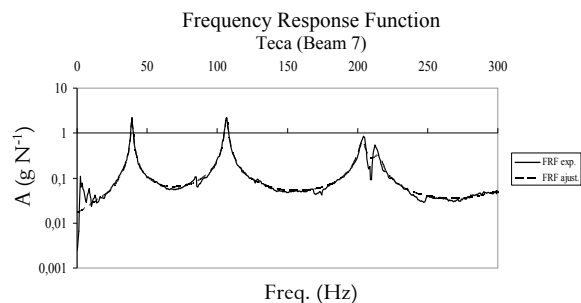


Figure 3. Example of the FRF obtained during the transverse vibration test.

With the natural frequencies of the first bending mode, dimensions and mass of the beams, the dynamic modulus of elasticity estimated by the

transverse vibration test, here called of E_{VT} , was calculated using the Equation 6.

Longitudinal vibration test

The longitudinal vibration test was conducted with the same equipment and with the same boundary condition of the transverse vibration test, but with the accelerometer attached in the longitudinal direction (Figure 4) and the impact applied in this same direction.



Figure 4. Position of the accelerometer in the longitudinal vibration test.

It was not possible to get the FRF in the longitudinal direction due to the high intensity of noise observed below 300 Hz (Figure 5). Thus, the natural frequencies of longitudinal vibration were obtained by means of the Fast Fourier Transform (FFT) of the accelerometer signal.

The modulus of elasticity, estimated by the longitudinal vibration test, here called the E_{VL} was calculated according to the Equation 3.

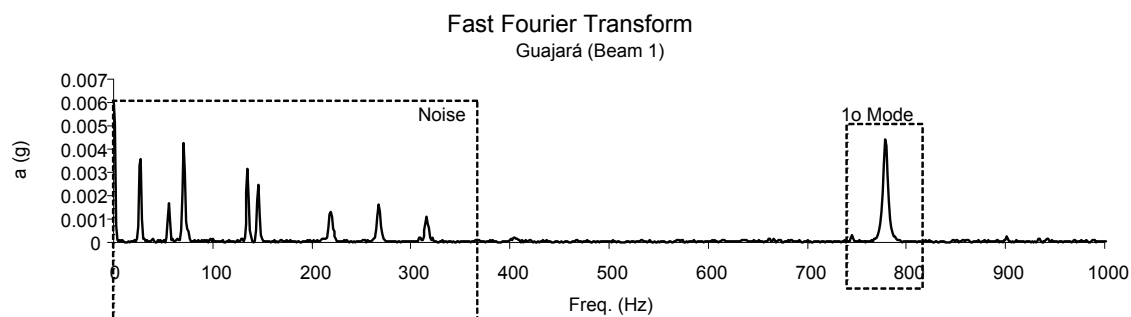


Figure 5. Example of the FFT obtained in the longitudinal vibration tests.

Results and discussion

Apparent density and moisture content

Table 1 lists the density and moisture content for Teca and Guajará species.

Table 1. Apparent density and moisture content of the beams.

Beam	Teca		Guajará	
	ρ kg m ⁻³	U (%)	ρ kg m ⁻³	U (%)
1	553.1	16.2	803.6	41.9
2	569.3	11.0	966.9	40.3
3	594.9	14.3	916.3	37.1
4	503.2	15.4	877.0	42.0
5	599.9	13.4	824.0	31.1
6	508.7	13.9	847.1	40.5
7	514.1	11.5	772.9	27.2
8	667.8	13.2	982.3	43.2
9	571.0	7.0	826.9	47.2
10	519.0	14.2	817.4	22.0
11	539.8	13.8	746.2	26.0
12	535.2	11.4	910.5	21.5
Mean	556.3	12.9	857.6	35.0

The average moisture content of Teca beams was next to 12%, while the Guajará beams had moisture content of around 35%, i.e. above the fiber saturation point.

Teca showed relatively low density, lower than that found for *Pinus* sp. (~ 600 kg m⁻³). The mean

density of Guajará corrected for 12% moisture content by the Kollman and Côte Jr. (1968) graph is 810 kg m⁻³.

Mechanical tests

Table 2 presents the results of the mechanical tests with the Guajará beams.

Table 3 shows the results of mechanical tests on Teca beams.

Correlation between the modulus of elasticity obtained in static and dynamic tests

In order to check the accuracy of the estimates of E_{est} , from the dynamic tests, linear regression models were fitted between the modulus of elasticity obtained in the dynamic tests and static modulus of elasticity. Figures 6 and 7 show the graphs obtained for the species Teca and Guajará, respectively.

Both graphs have pointed out a good correlation between the static modulus of elasticity and the modulus of elasticity obtained in dynamic tests.

However, there is a greater dispersion of points on the diagram of correlation between E_{es} and E_{VL} for Teca beams, resulting in a lower value for R^2 .

Table 2. Results of dynamic and static bending tests in the Guajará beams.

Beam	L (m)	S (m)	b (m)	h (m)	Mass kg	Displac. 50 N	(0.01 mm) 100 N	F _{Transv.} (Hz)	F _{Long.} (Hz)	E _{est} (GPa)	E _{VT} (GPa)	E _{VL} (GPa)
1	3.03	2.82	0.12	0.05	14.00	107	214	26.60	778.72	18.14	18.39	17.89
2	3.06	2.86	0.11	0.05	15.60	146	293	21.74	689.47	16.72	17.04	17.22
3	3.06	2.86	0.11	0.05	16.65	84	167	27.78	781.31	20.60	20.79	20.95
4	3.09	2.90	0.11	0.05	15.50	120	241	24.06	692.59	16.28	16.42	16.07
5	3.10	2.90	0.11	0.05	14.05	123	250	24.61	768.83	17.46	17.69	18.72
6	3.65	3.43	0.11	0.05	16.85	249	506	15.87	579.95	14.34	14.22	15.18
7	3.05	2.86	0.11	0.06	15.25	114	229	24.89	618.08	11.78	12.02	10.99
8	3.05	2.86	0.11	0.05	15.10	166	336	20.83	680.35	16.77	17.47	16.92
9	3.51	3.31	0.11	0.06	17.40	192	386	19.23	585.62	12.85	14.73	13.98
10	3.08	2.88	0.11	0.05	14.40	109	222	26.38	772.85	17.18	18.17	18.53
11	3.48	3.28	0.11	0.05	13.40	213	433	19.35	707.60	16.91	17.08	18.10
12	3.54	3.34	0.11	0.06	19.50	128	260	20.80	662.75	19.20	19.62	20.05
Mean										16.52	16.97	17.05
Stand. Dev.										2.50	2.40	2.73
Coeff. Var										15%	14%	16%

Table 3. Results of dynamic and static bending tests with the Teca beams.

Beam	L (m)	S (m)	b (m)	h (m)	Mass kg	Displac. 50 N	(0.01 mm) 100 N	F _{Transv.} (Hz)	F _{Long.} (Hz)	E _{est} (GPa)	E _{VT} (GPa)	E _{VL} (GPa)
1	2.15	2.00	0.099	0.0395	4.65	160	324	36.33	1004.03	10.02	9.59	10.31
2	2.15	2.00	0.098	0.0395	4.75	133	268	39.06	1033.93	12.23	11.42	11.25
3	2.15	2.00	0.096	0.0395	4.85	152	306	36.54	974.12	11.01	10.44	10.44
4	2.15	2.00	0.097	0.0398	4.15	139	281	40.48	1097.41	11.58	10.70	11.21
5	2.15	2.00	0.099	0.0398	5.05	155	311	35.40	941.77	10.39	9.76	9.84
6	2.15	2.00	0.099	0.0393	4.25	183	370	35.13	982.06	8.93	8.35	9.07
7	2.15	2.00	0.099	0.0398	4.35	144	290	39.16	1039.43	11.02	10.23	10.27
8	2.15	2.00	0.099	0.0398	5.65	125	252	37.61	1000.37	12.71	12.26	12.36
9	2.15	2.00	0.097	0.0393	4.65	166	332	35.35	962.30	10.29	9.50	9.78
10	2.15	2.00	0.099	0.0398	4.40	146	293	38.44	1028.44	10.88	9.95	10.15
11	2.15	2.00	0.096	0.0393	4.35	159	319	37.57	1013.79	10.82	10.14	10.26
12	2.15	2.00	0.099	0.0395	4.50	134	270	39.37	1069.33	12.01	10.90	11.32
Mean										10.99	10.27	10.52
Stand. Dev.										1.04	1.00	0.88
Coeff. Var.										9%	10%	8%

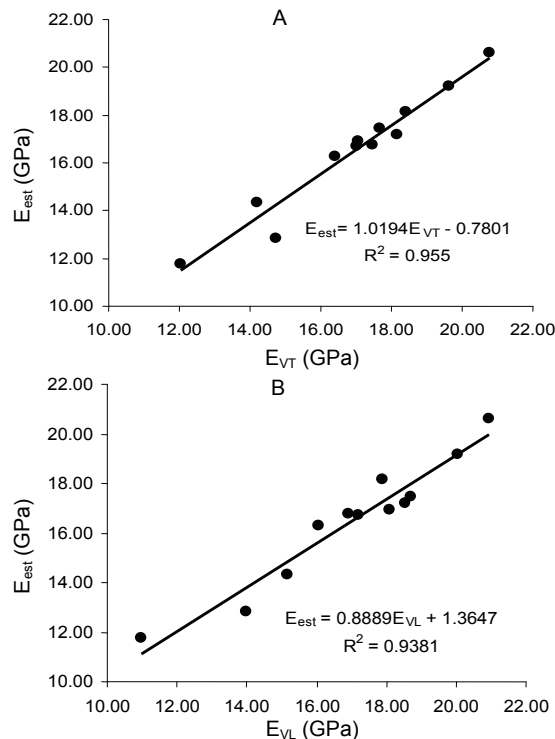


Figure 6. Correlations between the modules of elasticity of Guajará: A) $E_{VT} \times E_{est}$; B) $E_{VL} \times E_{est}$.

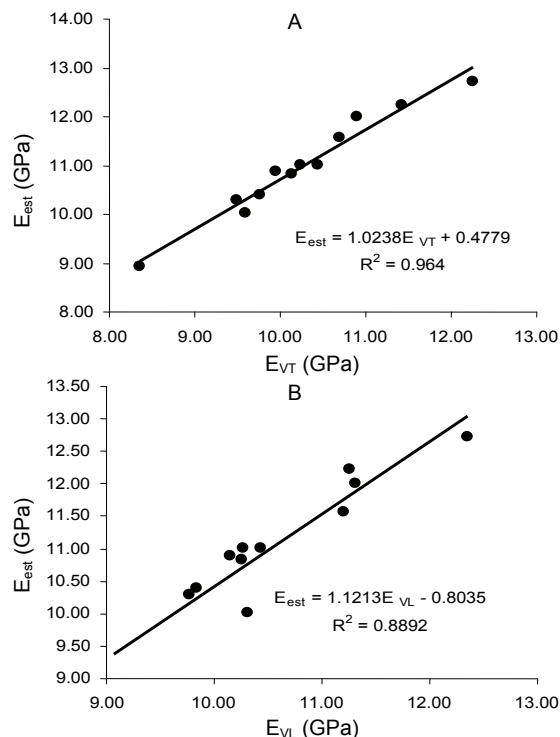


Figure 7. Correlations between the modulus of elasticity of Teca beams: A) $E_{VT} \times E_{est}$; B) $E_{VL} \times E_{est}$.

The coefficients of determination (R^2) shown in Figures 6 and 7, respectively for the Teca and Guajará species, were higher than those found by Candian and Sales (2009), for example, that found

correlation coefficients equal to 0.76, and 0.80, respectively for the correlation between the static bending modulus of elasticity and the modulus of elasticity obtained with the ultrasound and transverse vibration techniques.

The average values for the modulus of elasticity of Teca obtained from transverse vibration and longitudinal vibration techniques were lower than the average elastic modulus measured with the static bending test, at 7 and 4%, respectively. These results were compatible with those found by Carreira and Candian (2008).

Conclusion

It was observed that the transverse vibration technique had better correlation than the longitudinal vibration test.

The accuracy of longitudinal vibration technique was significantly reduced in the evaluation of bending stiffness of the Teca beams. Probably this was due to the deviations of the shock wave caused by the presence of knots. Thus, we believe it is necessary to develop further research with longitudinal vibration technique to calibrate the mathematical model of the problem in order to obtain more accurate estimates of the static modulus of elasticity of beams with knots.

References

- ASTM-American Society of Testing and Material. **ASTM D 198** - Standard test methods of static tests of lumber in structural sizes. Philadelphia, 2008.
- ABNT-Associação Brasileira de Normas Técnicas. **NBR 7190** - Projeto de estruturas de madeira. Rio de Janeiro, 1997.
- CANDIAN, M.; SALES, A. Aplicação das técnicas não-destrutivas de ultra-som, vibração transversal e ondas de tensão para avaliação de madeira. **Ambiente Construído**, v. 9, n. 4, p. 83-98, 2009.
- CARREIRA, M. R.; CANDIAN, M. Teste de um equipamento para classificação de peças estruturais de madeira pela técnica da vibração transversal. **Semina: Ciências Exatas e da Terra**, v. 29, n. 1, p. 3-14, 2008.
- CLOUGH, R. W.; PENZIEN, J. **Dynamics of Structures**. 3rd ed. Berkeley: Computers and Structures Inc., 1995.
- KOLLMAN, F. F. P.; CÔTE JR, W. A. **Principles of wood science and technology**: solid wood. New York: Springer Vellag, 1968.
- TIMOSHENKO, S. **Vibration problems in engineering**. 2nd ed. New York: John Wiley and Sons, 1938.

Received on July 23, 2010.

Accepted on January 31, 2011.

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