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Dirac-hyperbolic scarf problem including a coulomb-like tensor potential

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ABSTRACT. The Dirac equation have been solved for the q-deformed hyperbolic Scarf potential coupled to a Coulomb-like tensor potential under the spin symmetry. The parametric generalization of the Nikiforov-Uvarov method is used to obtain the energy eigenvalues equation and the normalized wave functions.

Keywords: Dirac equation, q-deformed hyperbolic Scarf, spin symmetry, Coulomb-like tensor.

Problema de Dirac-hiperbólico scarf incluindo um tensor de Coulomb como potenciais

RESUMO. A equação de Dirac foi resolvida para o potencial hiperbólico q-deformado de Scarf acopladas a um potencial de tensão tipo Coulomb sob a simetria de rotação. A generalização paramétrica do método Nikiforov-Uvarov foi usado para obter a equação dos valores energéticos e as funções normalizadas de ondas.

Palavras-chave: equação de Dirac, potencial hiperbólico q-deformado de Scarf, a simetria de rotação, tensor tipo Coulomb.

Introduction

One of the important tasks of quantum mechanics is to find exact analytic solutions of the wave equations are only possible for certain potentials of physical interest under consideration since they contain all the necessary information regarding the quantum system. It is well known that the exact solutions of these wave equations are only possible in a few simple cases such as the harmonic oscillator, the coulomb, pseudo harmonic potentials and others (IKHDAIR; SEVER, 2006, 2008; LANDAU; LIFSHITZ, 1977; NEITO, 1979; SCHIFF, 1968).

The exact solution of the Dirac equation with any potential is an important subject in relativistic quantum physics. These solutions are valuable tools in checking and improving models. Many authors have studied the Dirac under the spin and/or pseudo-spin symmetry for various potentials. For exemple, see (ARDA; SEVER, 2009; ESHGHI; MEHRABAN, 2011a and b; MOVAHEDI; HAMZAVI, 2011; PANAHI; BAKHSHI, 2011).

Tensor potentials were introduced into the Dirac equation with the substitution $\vec{p} \to \vec{p} - im\omega\beta \cdot \hat{r}U(r)$. In this way, a spin-orbit coupling term is added to the Dirac Hamiltonian. In this regard, see (AKCAY, 2009; AKCAY; TEZCAN, 2009; AYDOGDU; SEVER, 2010; HAMZAVI et al., 2010a and b; HAMZAVI et al., 2011; ZARRINKAMAR et al., 2010). Tensor coupling and exactly solvable tensor potential have been used to investigate nuclear properties (ALBERTO et al., 2005; FURNSTAHL et al., 1998; MAO, 2003) and have also some physical applications (MOSHINSKY; SIMIRNOV, 1996; PACHECO et al., 2003).

On the other hand, the spin symmetry appears when the magnitude of the scalar and vector potentials are nearly equal, i.e., $V_v(r) \cong V_s(r)$, in the nuclei (*i.e.*, when the difference potential $\Delta(r) = V_v(r) - V_s(r) = C_s = const.$) . However, the pseudo-spin symmetry occurs when $V_v(r) \cong -V_s(r)$ (*i.e.,* when the sum potential $\sum(r) = V_v(r) + V_s(r) = C_{ps} = const.$)

(GINOCCHIO, 1997). The bound states of nucleons seem to be sensitive to some mixtures of these potentials. The $\Delta(r) = 0$ and $\sum(r) = 0$ correspond to $SU(2)$ symmetries of the Dirac Hamiltonian (GINOCCHIO, 2005). The spin symmetry is relevant for mesons (PAGE et al., 2001) and the pseudo-spin symmetry has been used to explain the features of deformed nuclei (BOHR et al., 1982), super-deformation (DUDEK et al., 1987), and to establish an effective nuclear shell-model scheme (ARIMA et al., 1969; HECHT; ADLER, 1969). Summary, such symmetry, the near equality of an attractive scalar potential with a repulsive vector potential is well know in the literature (GINOCCHIO, 1997; 2005) of the Dirac equation and has been proved very useful in describing the motion of nucleons in the relativistic mean fields resulting form nucleon-meson interactions, nucleon-nucleon Skyrme-type interactions and QCD sum rules.

According to the report given in the researcher (ESHGHI; MEHRABAN, 2011a), the q-deformed hyperbolic Scarf potential is defined by $\alpha r > \ln\sqrt{q}$ as following

$$V_q(r) = V_0 + V_1 \coth_q^2 \alpha r + V_2 \frac{\coth_q \alpha r}{\sinh_q \alpha r} \cdot \quad (1)$$

In this present work, we give the approximate solutions, and corresponding wave functions of the Dirac equation for the q-deformed Hyperbolic Scarf potential (1) under the case of spin that including a coulomb-like tensor potential (AKCAY, 2009; ESHGHI; MEHRABAN, 2011c)

$$U(r) = -\frac{H}{r}, \quad H = \frac{Z_a Z_b e^2}{4\pi\varepsilon_0}, \quad r \geq R_c , \quad (2)$$

(where $R_c = 7.78\, fm$ is the Coulomb radius, Z_a and Z_b denote the charges of the projectile *a* and the target nuclei *b*, respectively.). We obtain the energy equation and the normalized corresponding spinor wave functions. In order to find the spectrum we use the parametric generalization of the Nikiforov-Uvarov (NU) is a powerful tool to solve of the second order linear differential equations with special orthogonal functions.

NU method

We give a brief description of the conventional NU method (NIKIFOROV; UVAROV, 1988). This method is based on solving the second order differential equation of hypergeometric-type by means of special orthogonal functions

$$\psi_n''(s) + \frac{\tilde{\tau}(s)}{\sigma(s)}\psi_n'(s) + \frac{\tilde{\sigma}(s)}{\sigma^2(s)}\psi_n(s) = 0 , \quad (3)$$

where: $\sigma(s)$ and $\tilde{\sigma}(s)$ are polynomials, at the most of the second degree, and $\tilde{\tau}(s)$ is a polynomials, at most of the first degree. In this method, if we take the following factorization $\psi_n(s) = \phi(s) y_n(s)$, (3) becomes

$$\sigma(s) y_n''(s) + \tau(s) y_n'(s) + \lambda y_n(s) = 0 , \quad (4)$$

where:

$$\sigma(s) = \pi(s) \frac{d}{ds}(\ln \phi(s)) , \quad (5)$$

and

$$\tau(s) = \tilde{\tau}(s) + 2\pi(s), \qquad \tau'(s) < 0 , \quad (6)$$

where:

$\pi(r)$ is a polynomial of order at most one, and $y_n(s)$ can be written as

$$y_n(s) = \frac{a_n}{\rho(s)} \frac{d^n}{ds^n}\left[\sigma^n(s)\rho(s)\right], \quad (7)$$

where: a_n is a normalization constant and the weight function $\rho(s)$ must satisfy the differential equation

$$\omega'(s) - \left(\frac{\tau(s)}{\sigma(s)}\right)\omega(s) = 0, \quad \omega(s) = \sigma(s)\rho(s). \quad (8)$$

The function $\pi(s)$ and the parameter λ in the above equation are defined as follows

$$\pi(s) = \frac{\sigma'(s) - \tilde{\tau}(s)}{2} \pm \sqrt{\left(\frac{\sigma'(s) - \tilde{\tau}(s)}{2}\right)^2 - \tilde{\sigma}(s) + q\sigma(s)} , \quad (9)$$

and

$$\lambda = q + \pi'(s) . \quad (10)$$

The determination of *q* is the essential point in the calculation of $\pi(s)$. It is simply defined by setting the discriminate of the square root which must be zero. The eigenvalues equation have calculated from the above equation

$$\lambda = \lambda_n = -n\tau'(s) - \frac{n(n-1)}{2}\sigma''(s). \quad n = 0,1,2,... \quad (11)$$

For a more simple application of the method, we develop a parametric generalization of the NU method valid for any potential under consideration by an appropriate coordinate transformation $s = s(r)$. The following equation is a general form of the Schrodinger equation written for any potentials (HAMZAVI et al., 2011; ESHGHI, 2011) as

$$s^2(1-\alpha_3 s)^2 \frac{d^2\psi_n(s)}{ds^2} + s(1-\alpha_3 s)(\alpha_1-\alpha_2 s)\frac{d\psi_n(s)}{ds} + [-\xi_1 s^2 + \xi_2 s - \xi_3]\psi_n(s) = 0. \quad (12)$$

We may solve this as follows. Comparing (12) with (3), yields

$$\tilde{\tau}(s) = \alpha_1 - \alpha_2 s \quad , \quad \sigma(s) = s(1-\alpha_3 s) \quad , \quad \tilde{\sigma}(s) = -\xi_1 s^2 + \xi_2 s - \xi_3 . \quad (13)$$

Substituting these into (9), we find

$$\pi(s) = \alpha_4 + \alpha_5 s \pm \left[(\alpha_6 - k\alpha_3)s^2 + (\alpha_7 + k)s + \alpha_8\right]^{\frac{1}{2}}, \quad (14)$$

with the following parameters

$$\alpha_4 = \frac{1}{2}(1-\alpha_1), \quad \alpha_5 = \frac{1}{2}(\alpha_2 - 2\alpha_3), \quad \alpha_6 = \alpha_5^2 + \xi_1, \quad \alpha_7 = 2\alpha_4\alpha_5 - \xi_2, \quad \alpha_8 = \alpha_4^2 + \xi_3. \quad (15)$$

In Equation (14), the function under the square root should be the square of a polynomial according to the NU method. so that

$$k_{1,2} = -(\alpha_7 + 2\alpha_3\alpha_8) \pm 2\sqrt{\alpha_8\alpha_9}, \quad (16)$$

where:

$$\alpha_9 = \alpha_3\alpha_7 + \alpha_3^2\alpha_8 + \alpha_6 . \quad (17)$$

For each k the following π's are obtained. The function $\pi(s)$ becomes

$$\pi(s) = \alpha_4 + \alpha_5 s - [(\sqrt{\alpha_9} + \alpha_3\sqrt{\alpha_8})s - \sqrt{\alpha_8}], \quad (18)$$

for the k-value

$$k = -(\alpha_7 + 2\alpha_3\alpha_8) - 2\sqrt{\alpha_8\alpha_9} . \quad (19)$$

We also have from $\tau(s) = \tilde{\tau}(s) + 2\pi(s)$,

$$\tau(s) = \alpha_1 + 2\alpha_4 - (\alpha_2 - 2\alpha_5)s - 2[(\sqrt{\alpha_9} + \alpha_3\sqrt{\alpha_8})s - \sqrt{\alpha_8}]. \quad (20)$$

Thus, we impose the following condition to fix the k-value

$$\tau'(s) = -(\alpha_2 - 2\alpha_5) - 2(\sqrt{\alpha_9} + \alpha_3\sqrt{\alpha_8}) = -2\alpha_3 - 2(\sqrt{\alpha_9} + \alpha_3\sqrt{\alpha_8}) < 0. \quad (21)$$

When (10) is used with (20) and (21) the following equation is derived

$$n[(n-1)\alpha_3 + \alpha_2 - 2\alpha_5] - \alpha_5 + (2n+1)(\sqrt{\alpha_9} + \alpha_3\sqrt{\alpha_8}) + \alpha_7 + 2\alpha_3\alpha_8 + 2\sqrt{\alpha_8\alpha_9} = 0 . \quad (22)$$

This equation gived the energy spectrum of a given problem. By using (8)

$$\rho(s) = s^{\alpha_{10}-1}(1-\alpha_3 s)^{\frac{\alpha_{11}}{\alpha_3} - \alpha_{10} - 1}, \quad (23)$$

and together with (7), we have

$$y_n(s) = P_n^{\left(\alpha_{10}-1 \ , \ \frac{\alpha_{11}}{\alpha_3} - \alpha_{10} - 1\right)}(1 - 2\alpha_3 s), \quad (24)$$

where:

$$\alpha_{10} = \alpha_1 + 2\alpha_4 + 2\sqrt{\alpha_8}, \quad (25)$$

and

$$\alpha_{11} = \alpha_2 - 2\alpha_5 + 2(\sqrt{\alpha_9} + \alpha_3\sqrt{\alpha_8}), \quad (26)$$

and $P_n^{(\alpha,\beta)}$ are Jacobi polynomials. By using (5), we get

$$\phi(s) = s^{\alpha_{12}}(1-\alpha_3 s)^{-\alpha_{12} - \frac{\alpha_{13}}{\alpha_3}}, \quad (27)$$

and the total wave function become

$$\Psi(s) = s^{\alpha_{12}}(1-\alpha_3 s)^{-\alpha_{12} - \frac{\alpha_{13}}{\alpha_3}} \times P_n^{\left(\alpha_{10}-1 \ , \ \frac{\alpha_{11}}{\alpha_3} - \alpha_{10} - 1\right)}(1 - 2\alpha_3 s), \quad (28)$$

where: $\alpha_{12}=\alpha_4+\sqrt{\alpha_8}$ and $\alpha_{13}=\alpha_5-(\sqrt{\alpha_9}+\alpha_3\sqrt{\alpha_8})$.

In some problems the situation appears where $\alpha_3=0$. For such problems, the solution given in (28) becomes as

$$\Psi(s)=s^{\alpha_{12}}e^{\alpha_{13}s}L_n^{\alpha_{10}-1}(\alpha_{11}s) \quad (29)$$

In some cases, one may need a second solution of (12). In this case, if the same procedure is followed, by using

$$k=-(\alpha_7+2\alpha_3\alpha_8)+2\sqrt{\alpha_8\alpha_9}\ , \quad (30)$$

the solution becomes

$$\Psi(s)=s^{\alpha_{12}^*}(1-\alpha_3 s)^{-\alpha_{12}^*-\frac{\alpha_{13}^*}{\alpha_3}} \times P_n^{\left(\alpha_{10}-1\ ,\ \frac{\alpha_{11}^*}{\alpha_3}-\alpha_{10}^*-1\right)}(1-2\alpha_3 s)\,, \quad (31)$$

and the energy spectrum is

$$\begin{aligned} &n[(n-1)\alpha_3+\alpha_2-2\alpha_5] \\ &+(2n+1)(\sqrt{\alpha_9}-\alpha_3\sqrt{\alpha_8}) \\ &+\alpha_7+2\alpha_3\alpha_8-2\sqrt{\alpha_8\alpha_9}+\alpha_5=0\,. \end{aligned} \quad (32)$$

Pre-defined α parameters are:

$$\begin{aligned} \alpha_{10}^* &=\alpha_1+2\alpha_4-2\sqrt{\alpha_8}\ , \\ \alpha_{11}^* &=\alpha_2-2\alpha_5+2(\sqrt{\alpha_9}-\alpha_3\sqrt{\alpha_8})\,, \\ \alpha_{12}^* &=\alpha_4-\sqrt{\alpha_8}\ , \\ \alpha_{13}^* &=\alpha_5-(\sqrt{\alpha_9}-\alpha_3\sqrt{\alpha_8})\,. \end{aligned} \quad (33)$$

Solution of the Dirac equation

According to the report given in the researcher (AKCA, 2009; AYDOGDU; SEVER, 2010; HAMZAVI et al., 2010a and b; HAMZAVI et al., 2011; ESHGHI; MEHRABAN, 2011a and c; ZARRINKAMAR et al., 2010), the Dirac equation with the attractive scalar potential $V_s(r)$, repulsive vector potential $V_v(r)$ and a tensor potential $U(r)$ is $(\hbar=c=1)$:

$$\begin{aligned} &\left[\alpha\cdot P+\beta\left(M+V_s(r)\right)-i\,\beta\alpha\cdot\hat{r}U(r)\right]\psi_{nk}(\vec{r}) \\ &=\left[E-V_v(r)\right]\psi_{nk}(\vec{r}), \end{aligned} \quad (34)$$

where E is the relativistic energy of the system, $\vec{P}=-i\vec{\nabla}$ is the three-dimensional momentum operator, α and β are the 4×4 matrices which have the following forms (GRINER, 2000), respectively

$$\alpha=\begin{pmatrix}0 & \sigma\\ \sigma & 0\end{pmatrix},\ \beta=\begin{pmatrix}I & 0\\ 0 & I\end{pmatrix}, \quad (35)$$

where I denotes the 2×2 identity matrix and σ are three-vector Pauli spin matrices

$$\sigma_1=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix},\ \sigma_2=\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix},\ \sigma_3=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}. \quad (36)$$

For a particle in a spherical (central) field, the total angular momentum $\vec{J}$, and the spin-orbit matrix operator $\hat{K}=-\beta(\hat{\sigma}.\hat{L}+1)$ commute with the Dirac Hamiltonian, where L is orbital angular momentum operator. For a given total angular momentum j, the eigenvalues of $\hat{K}$ are $k=-(j+1/2)$ for aligned spin $(s_{1/2},p_{3/2},etc)$ and $k=j+1/2$ for unaligned spin $(p_{1/2},d_{3/2},etc)$. The Dirac spinor can be written using the Pauli-Dirac representation

$$\psi_{nk}(r)=\frac{1}{r}\begin{bmatrix}F_{nk}(r)Y_{jm}^{\,l}(\theta,\varphi)\\ iG_{nk}(r)Y_{jm}^{\,\tilde{l}}(\theta,\varphi)\end{bmatrix} \quad k=(j+\frac{1}{2}),\ , \quad (37)$$

where:

$Y_{jm}^{\,l}(\theta,\phi)$ and $Y_{jm}^{\,\tilde{l}}(\theta,\phi)$ are the spin and pseudo-spin spherical harmonics functions, respectively, $F_{nk}(r)$ and $G_{nk}(r)$ are the radial wave functions of the upper- and the lower-spinor components respectively, m is the projection of the total angular momentum on the z-axis, n is the radial quantum number. The orbital and the pseudo-orbital angular momentum quantum numbers for spin symmetry l and pseudo-spin symmetry $\tilde{l}$ refer to the upper- and lower-component respectively. For a given spin-orbit quantum number $k=\pm1,\pm2,\ldots$, the orbital angular momentum and pseudo-orbital angular momentum are given by $l=|k+1/2|-1/2$ and $l=|k-1/2|-1/2$, respectively.

Substituting (37) into (34) and using the following relations (BJORKEN; DRELL, 1964) as

$$(\vec{\sigma}.\vec{A})(\vec{\sigma}.\vec{B}) = \vec{A}.\vec{B} + i\vec{\sigma}.(\vec{A}\times\vec{B}), \tag{38}$$

$$(\vec{\sigma}.\vec{P}) = \vec{\sigma}.\hat{r}(\hat{r}.\vec{P} + i\frac{\vec{\sigma}.\vec{L}}{r}), \tag{39}$$

and properties

$$(\vec{\sigma}.\vec{L})Y_{jm}^{\tilde{l}}(\theta,\phi) = (k-1)Y_{jm}^{\tilde{l}}(\theta,\phi), \tag{40}$$

$$(\vec{\sigma}.\vec{L})Y_{jm}^{l}(\theta,\phi) = -(k-1)Y_{jm}^{l}(\theta,\phi), \tag{41}$$

$$(\vec{\sigma}.\hat{r})Y_{jm}^{\tilde{l}}(\theta,\phi) = -Y_{jm}^{l}(\theta,\phi), \tag{42}$$

$$(\vec{\sigma}.\hat{r})Y_{jm}^{l}(\theta,\phi) = -Y_{jm}^{\tilde{l}}(\theta,\phi). \tag{43}$$

Splitting off the angular part and leaving the radial wave function satisfy the following equations

$$\left(\frac{d}{dr}+\frac{k}{r}-U(r)\right)F_{nk}(r) = [E_{nk}+M-\Delta(r)]G_{nk}(r), \tag{44}$$

$$\left(\frac{d}{dr}-\frac{k}{r}+U(r)\right)G_{nk}(r) = [M-E_{nk}+\Sigma(r)]F_{nk}(r), \tag{45}$$

to eliminating $G_{nk}(r)$ in (43) and $F_{nk}(r)$ in (45), one obtains Schrodinger-like equations for the upper and lower components, respectively

$$\left\{ \frac{d^2}{dr^2} - \frac{k(k+1)}{r^2} + \frac{2k}{r}U(r) - \frac{dU(r)}{dr} - U^2(r) + (E_{nk}+M-\Delta(r))(E_{nk}-M-\Sigma(r)) + \frac{\frac{d\Delta(r)}{dr}}{(M+E_{nk}-\Delta(r))}\left(\frac{d}{dr}+\frac{k}{r}-U(r)\right) \right\} F_{nk}(r) = 0, \tag{46}$$

and

$$\left\{ \frac{d^2}{dr^2} - \frac{k(k-1)}{r^2} + \frac{2k}{r}U(r) + \frac{dU(r)}{dr} - U^2(r) + (E_{nk}+M-\Delta(r))(E_{nk}-M-\Sigma(r)) + \frac{\frac{d\Sigma(r)}{dr}}{(M-E_{nk}+\Sigma(r))}\left(\frac{d}{dr}-\frac{k}{r}+U(r)\right) \right\} G_{nk}(r) = 0, \tag{47}$$

where:

$$k(k+1) = l(l+1) \quad \text{and} \quad k(k-1) = \tilde{l}(\tilde{l}+1) .$$

Equations (45) and (46) can not be solved exactly for $k = 0,-1$ and $k = 0,1$, because of the spin-orbit centrifugal term. We applied deform hyperbolic functions introduced for the first time by Arai in (ARAI, 1991)

$$\sinh_q x = \frac{e^x - qe^{-x}}{2}, \cosh_q x = \frac{e^x + qe^{-x}}{2}, \quad \tanh_q x = \frac{\sinh_q x}{\cosh_q x}, \operatorname{sech}_q x = \frac{1}{\cosh_q x}, \tag{48}$$

where q is real parameter and $q > 0$.

Substituting (1) and (2) into (46) and considering spin symmetry (the condition of spin symmetry $d\Delta(r)/dr = 0$ or $\Delta(r) = const = C_s$) (MENG et al., 1998), we have

$$\left\{ \frac{d^2}{dr^2} - \frac{(k+H)(k+H+1)}{r^2} - (M-E_{nk})(E_{nk}+M-C_s) - (E_{nk}+M-C_s) \times \left(V_0 + V_1 \coth_q^2 \alpha r + V_2 \frac{\coth_q \alpha r}{\sinh_q \alpha r} \right) \right\} F_{nk}(r) = 0 \cdot \tag{49}$$

This equation is describes a particle of spin-1/2 such as the electron in the Dirac theory with q-deformed hyperbolic Scarf potential including a tensor coupling.

We apply the approximation for the centrifugal term of the form as

$$\frac{1}{r^2} \approx 4\alpha^2 \left[C_0 + \frac{e^{-2\alpha r}}{\left(1-qe^{-2\alpha r}\right)^2} \right], \tag{50}$$

where the dimensionless constant $C_0 = 1/12$ is reported in Ref. (ANTIA et al., 2010). However, when $C_0 = 0$ then new improved approximation Scheme become the conventional approximation Scheme suggested by Greene and Aldrich in Ref. (GREENE; ALDRICH, 1976).

By using the approximation in (50) and transformation of the form $s = \cosh \alpha r$, we rewrite (48) as follows

$$\left\{ \frac{d^2}{ds^2} + \frac{-s}{q-s^2}\frac{d}{ds} + \frac{1}{(q-s^2)^2} \times \left[-\left(4b_1C_0 + b_2b_3 + b_2\tilde{V}_0 + b_2\tilde{V}_1\right)s^2 + (-b_2\tilde{V}_2)s - (b_1 - b_2b_3q - 4b_1C_0q + b_2\tilde{V}_0 q) \right] \right\} F_{nk}(s) = 0, \tag{51}$$

by comparing (51) with (12) we have obtained the parameter set as

$$
\begin{aligned}
&\xi_1 = 4b_1C_0 + b_2b_3 + b_2\tilde{V}_0 + b_2\tilde{V}_1\,,\\
&\xi_2 = -b_2\tilde{V}_2\,,\\
&\xi_3 = b_1 - b_2b_3q - 4b_1C_0q + b_2\tilde{V}_0q\,,\\
&\alpha_1 = 0\,,\ \alpha_2 = 1\,,\ \alpha_3 = 1\\
&\alpha_4 = \frac{1}{2}\,,\ \alpha_5 = -\frac{1}{2}\,,\ \alpha_6 = \frac{1}{4} + \xi_1\,,\\
&\alpha_7 = -\frac{1}{2} - \xi_2\,,\ \alpha_8 = \frac{1}{4} + \xi_3\,,\\
&\alpha_9 = (4b_1C_0 + b_2b_3 + b_2\tilde{V}_0)(1-q) + b_2(\tilde{V}_1 - \tilde{V}_2) + b_2\,,\\
&\alpha_{10} = 1 + 2\sqrt{\frac{1}{4} + \xi_3}\,,\\
&\alpha_{11} = 2 + 2\sqrt{(4b_1C_0 + b_2b_3 + b_2\tilde{V}_0)(1-q) + b_2(\tilde{V}_1 - \tilde{V}_2) + b_2} + 2\sqrt{\frac{1}{4} + \xi_3}\\
&\alpha_{12} = \frac{1}{2} + \sqrt{\frac{1}{4} + \xi_3}\,,\\
&\alpha_{13} = -\frac{1}{2} - \sqrt{(4b_1C_0 + b_2b_3 + b_2\tilde{V}_0)(1-q) + b_2(\tilde{V}_1 - \tilde{V}_2) + b_2} - \sqrt{\frac{1}{4} + \xi_3}\,.
\end{aligned}
\tag{52}
$$

Using (14), (16) and (52), we calculate the parameters required for the method

$$
\pi(s) = \frac{1-s}{2} \pm \left\{ \left(\frac{1}{4} + 4b_1C_0 + b_2b_3 + b_2\tilde{V}_0 + b_2\tilde{V}_1 - k \right) s^2 + \left(-\frac{1}{2} + b_2\tilde{V}_2 + k \right) s + \frac{1}{4} + b_1 - b_2b_3q - 4b_1C_0q + b_2\tilde{V}_0q \right\}^{\frac{1}{2}}, \tag{53}
$$

where:

$$
k_{1,2} = b_2\tilde{V}_2 - 2\left(\frac{1}{4} + b_1 - b_2b_3q - 4b_1C_0q + b_2\tilde{V}_0q \right) \pm 2\left\{ \left[\frac{1}{4} + b_1 - b_2b_3q - 4b_1C_0q + b_2\tilde{V}_0q \right] \times \left[\begin{matrix} (4b_1C_0 + b_2b_3 + b_2\tilde{V}_0)(1-q) \\ + b_2(\tilde{V}_1 - \tilde{V}_2) + b_2 \end{matrix} \right] \right\}^{\frac{1}{2}}. \tag{54}
$$

Different k's lead to the different π's. For

$$
k = b_2\tilde{V}_2 - 2\left(\frac{1}{4} + b_1 - b_2b_3q - 4b_1C_0q + b_2\tilde{V}_0q \right) - 2\left\{ \begin{matrix} \left[\frac{1}{4} + b_1 - b_2b_3q - 4b_1C_0q + b_2\tilde{V}_0q \right] \\ \times \left[\begin{matrix} (4b_1C_0 + b_2b_3 + b_2\tilde{V}_0)(1-q) \\ + b_2(\tilde{V}_1 - \tilde{V}_2) + b_2 \end{matrix} \right] \end{matrix} \right\}^{\frac{1}{2}}, \tag{55}
$$

$\pi(s)$ become

$$
\pi(s) = \frac{1-s}{2} - \left[\begin{matrix} \sqrt{(4b_1C_0 + b_2b_3 + b_2\tilde{V}_0)(1-q) + b_2(\tilde{V}_1 - \tilde{V}_2) + b_2} \\ + \sqrt{\left[\frac{1}{4} + b_1 - b_2b_3q - 4b_1C_0q + b_2\tilde{V}_0q \right]} \end{matrix} \right] s - \sqrt{\left[\frac{1}{4} + b_1 - b_2b_3q - 4b_1C_0q + b_2\tilde{V}_0q \right]}, \tag{56}
$$

and

$$
\tau(s) = 1 - 2s - 2\left[\begin{matrix} \sqrt{(4b_1C_0 + b_2b_3 + b_2\tilde{V}_0)(1-q) + b_2(\tilde{V}_1 - \tilde{V}_2) + b_2} \\ + \sqrt{\left[\frac{1}{4} + b_1 - b_2b_3q - 4b_1C_0q + b_2\tilde{V}_0q \right]} \end{matrix} \right] s - \sqrt{\left[\frac{1}{4} + b_1 - b_2b_3q - 4b_1C_0q + b_2\tilde{V}_0q \right]}, \tag{57}
$$

where: $\tau'(s) < 0$.

Using (22) and (52), the Energy eigenvalue equation for the potential under the consideration following as

$$
\begin{aligned}
&n^2 + n\left(\begin{matrix} 1 - 2\sqrt{(4b_1C_0 + b_2b_3 + b_2\tilde{V}_0)(1-q) + b_2(\tilde{V}_1 - \tilde{V}_2) + b_2} \\ -2\sqrt{\left[\frac{1}{4} + b_1 - b_2b_3q - 4b_1C_0q + b_2\tilde{V}_0q \right]} \end{matrix} \right)\\
&+\left(-1 + 2\sqrt{\frac{1}{4} + b_1 - b_2b_3q - 4b_1C_0q + b_2\tilde{V}_0q} \right)\\
&\times\left(\sqrt{(4b_1C_0 + b_2b_3 + b_2\tilde{V}_0)(1-q) + b_2(\tilde{V}_1 - \tilde{V}_2) + b_2} + \sqrt{\frac{1}{4} + b_1 - b_2b_3q - 4b_1C_0q + b_2\tilde{V}_0q} \right) + b_2\tilde{V}_2 = 0\,.
\end{aligned}
\tag{58}
$$

The corresponding normalized eigen-functions are obtained in terms of the functions,

$$\rho(s)=s^{2\sqrt{\frac{1}{4}+b_1-b_2b_3q-4b_1C_0q+b_2\tilde{V}_0q}}$$
$$\times(1-s)^{2\sqrt{(4b_1C_0+b_2b_3+b_2\tilde{V}_0)(1-q)+b_2(\tilde{V}_1-\tilde{V}_2)+b_2}}, \tag{59}$$

and

$$y_n(s)=P_n^{\left(2\sqrt{\frac{1}{4}+b_1-b_2b_3q-4b_1C_0q+b_2\tilde{V}_0q},\,2\sqrt{(4b_1C_0+b_2b_3+b_2\tilde{V}_0)(1-q)+b_2(\tilde{V}_1-\tilde{V}_2)+b_2}\right)}(1-2s), \tag{60}$$

and

$$\varphi(s)=s^{\frac{1}{2}+\sqrt{\frac{1}{4}+b_1-b_2b_3q-4b_1C_0q+b_2\tilde{V}_0q}}$$
$$\times(1-s)^{\sqrt{(4b_1C_0+b_2b_3+b_2\tilde{V}_0)(1-q)+b_2(\tilde{V}_1-\tilde{V}_2)+b_2}}, \tag{61}$$

now, let us give the corresponding lower Dirac spinor. Using (28), the corresponding wave functions to be

$$F(s)=a_{nk}s^{\frac{1}{2}+\sqrt{\frac{1}{4}+b_1-b_2b_3q-4b_1C_0q+b_2\tilde{V}_0q}}$$
$$\times(1-s)^{\sqrt{(4b_1C_0+b_2b_3+b_2\tilde{V}_0)(1-q)+b_2(\tilde{V}_1-\tilde{V}_2)+b_2}} \tag{62}$$
$$\times P_n^{\left(2\sqrt{\frac{1}{4}+b_1-b_2b_3q-4b_1C_0q+b_2\tilde{V}_0q},\,2\sqrt{(4b_1C_0+b_2b_3+b_2\tilde{V}_0)(1-q)+b_2(\tilde{V}_1-\tilde{V}_2)+b_2}\right)}(1-2s).$$

Where a_{nk} is a normalization constant that the wave functions satisfy the normalization condition (IKHDAIR; SEVER, 2009)

$$\int_0^\infty |F_{nk}(r)|^2\,dr=\int_0^1 s^{-1}|F_{nk}(s)|^2\,ds=1, \tag{63}$$

and a_{nk} can be determined via

$$ba_{nk}^2\int_0^1 s^{2b_4-1}(1-s)^{2b_5+2}\left[P_n^{(2b_4,2b_5+1)}(1-2s)\right]^2ds=1. \tag{64}$$

Therefore, we have

$$a_{nk}=\frac{1}{\sqrt{z(n)}},$$
$$z(n)=b(-1)^n\frac{\Gamma(n+2b_5+1)\Gamma(n+2b_4+1)}{\Gamma(n+2b_4+2b_5+1)}$$
$$\times\sum_{p,r=0}^{n}\left(\frac{(-1)^{p+r}\Gamma(n+2b_4+r-p+1)}{p!r!(n-p)!(n-r)!\Gamma(n+2b_4+r+2b_5+1)}\right. \tag{65}$$
$$\left.\times\frac{(n+2b_5+1)}{\Gamma(n+2b_4-p+1)\Gamma(2b_4+r+1)}\right),$$

where

$$b_1=(k+H)(k+H+1),$$
$$b_2=E_{nk}+M-C_s,\ b_3=\frac{M-E_{nk}}{\alpha^2},$$
$$b_4=\sqrt{\frac{1}{4}+b_1-b_2b_3q-4b_1C_0q+b_2\tilde{V}_0q}, \tag{66}$$
$$b_5=\sqrt{(4b_1C_0+b_2b_3+b_2\tilde{V}_0)(1-q)+b_2(\tilde{V}_1-\tilde{V}_2)+b_2},$$
$$\tilde{V}_0=\frac{V_0}{\alpha^2},\ \tilde{V}_1=\frac{V_1}{\alpha^2},\ \tilde{V}_2=\frac{V_2}{\alpha^2}.$$

The lower component of Dirac spinor can be calculated by using (12) as

$$G_{nk}(r)=\frac{1}{M+E_{nk}-\Delta(r)}\left(\frac{d}{dr}+\frac{k}{r}-U(r)\right)F_{nk}(r). \tag{67}$$

Conclusion

The Dirac equation have solved with q-deformed hyperbolic Scarf potential including Coulomb-like tensor coupling in the case of spin symmetry. By using the approximation Scheme and the parameteric generalization of the Nikiforov-Uvarov method, we have obtained the energy eigenvalues equation and the normalized wave functions.

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