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On weakly *b*-continuous functions in bitopological spaces

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ABSTRACT. In this article we introduce the notion of weakly *b*-continuous functions in bitopological spaces as a generalization of *b*-continuous functions. We prove several properties of these functions. AMS Classification No : 54A10; 54C10; 54C08; 54D15.

Keywords: bitopological space, (*i*, *j*)-*b*-open, (*i*, *j*)-regular open, (*i*, *j*)-θ-open, (*i*, *j*)-weakly continuous, (*i*, *j*)-weakly *b*-continuous.

Funções fracamente contínuas *b* em espaços bitopológicos

RESUMO. Neste artigo apresentamos o conceito de funções fracamente contínuas *b* em espaços bitopológicos como uma generalização de funções contínuas *b*. Comprovamos várias propriedades destas funções. Classificação AMS N°: 54A10; 54C10; 54C08; 54D15.

Palavras-chave: espaço bitopológico, (i, j)-*b*-aberto, (i, j)-aberto regular, (i, j)- θ-aberto, (i, j)-fracamente contínuo, (i, j)-fracamente contínuo *b*.

Introduction

The concept of bitopological spaces (X,τ_1,τ_2) was first introduced by Kelly (1963), where X is a non-empty set and τ_1, τ_2 are topologies on X. The notion of *b*-open sets is due to Andrijevic (1996) plays a significant role in general topology. A subset A of (X,τ) is called *b*-open, if $A \subset Int(Cl(A)) \cup Cl(Int(A))$ and called *b*-closed if $X - A$ is *b*-open. Sengul (2008) and Sengul (2009) defined the notion of almost *b*-continuous functions and weakly *b*-continuous functions in topological spaces. T. Noiri and N. Rajesh have investigated some properties of the concept of *b*-open sets and *b*-continuous functions in bitopological spaces. Recently Sarsak and Rajesh (2009), Banerjee (1987), Bose and Sinha (1981) and Tripathy and Sarma (2011, 2012) have done some works on bitopological spaces. Bitopological spaces has been istudied bt Bose (1981), Bose and Sinha (1982), Jelic (1992), Kariofillies (1986), Kelly (1963), Khedr et al. (1992), Noiri and Popa (2007), Popa and Noiri (2004) and others.

In this paper, we introduce the notion of weakly *b*-continuous functions in bitopological spaces and investigate their different properties.

Preliminaries

Throughout the present paper (X,τ) denotes a topological space and (X,τ_1,τ_2) denotes a bitopological space on which no separation axioms are assumed. Let (X,τ_1,τ_2) be a bitopological space and A be a subset of X. The closure(resp. interior) of A with respect to the topology τ_i ($i = 1, 2$) will be denoted by $i\ Cl(A)$ (resp. $i\ Int(A)$).

Now we list some known definitions and results those will be used throughout this article.

Definition 1. A subset A of a bitopological space (X,τ_1,τ_2) is said to be

(*i*) (i,j)-*b*-open if $A \subset i\ Int(j\ Cl(A)) \cup j\ Cl(i\ Int(A))$.

(*ii*) (i,j)-regular open if $A = i\ Int(j\ Cl(A))$.

(*iii*) (i,j)-regular closed if $A = i\ Cl(j\ Int(A))$.

The complement of (i,j)-*b*-open set is said to be (i,j)-*b*-closed.

Definition 2. Let A be a subset of a bitopological space (X,τ_1,τ_2). Then:

(*i*) The (i, j)-*b*-closure of A denoted by (i, j)-*b* $Cl(A)$, is defined by the intersection of all (i, j)-*b*-closed sets containing A.

(*ii*) The (i, j)-*b*-interior of A denoted by (i, j)-*b* $Int(A)$, is defined by the union of all (i,j)-*b*-open sets contained in A.

Lemma 2.1. Let (X,τ_1,τ_2) be a bitopological space. Then:

(i) the arbitrary union of (i, j)-b-open sets is (i, j)-b-open.

(ii) the arbitrary intersection of (i, j)-b-closed sets is (i, j)-b-closed.

Lemma 2.2. Let (X,τ_1,τ_2) be a bitopological space and A be a subset of X.

(i) (i, j)-b Int(A) is (i, j)-b-open.

(ii) (i, j)-b Cl(A) is (i, j)-b-closed.

(iii) A is (i, j)-b-open if and only if A = (i, j)-b Int(A).

(iv) A is (i, j)-b-closed if and only if A = (i, j)-b Cl(A).

Lemma 2.3. Let (X,τ_1,τ_2) be a bitopological space and A be a subset of X. Then

(i) X – (i, j)-b Int(A) = (i, j)-b Cl(X – A).

(ii) X – (i, j)-b Cl(A) = (i, j)-b Int(X – A).

Lemma 2.4. For any subset A of a bitopological space (X,τ_1,τ_2), $x \in$ (i, j)-b Cl(A) if and only if $B \cap A \neq \emptyset$ for every (i, j)-b-open set B containing x.

Definition 3. A bitopological space (X,τ_1,τ_2) is said to be (i, j)-regular if for each $x\in X$ and each τ_i-open set A containing x, there exists a τ_i-open set B such that $x\in B \subset j\,Cl(B)\subset A$.

Definition 4. Let (X,τ_1,τ_2) be a bitopological space and A be a subset of X. A point x of X is said to be in the (i, j)-θ-closure of A, denoted by (i, j)-$Cl_\theta(A)$, if $A\cap j\,Cl(B) \neq \emptyset$ for every τ_i-open set B containing x, where i, j = 1, 2 and $i \neq j$.

A subset *A* of *X* is said to be (*i*, *j*)-θ-closed if $A = (i, j)\text{-}Cl_\theta(A)$. A subset *A* of *X* is said to be (*i*, *j*)-θ-open if *X* – *A* is (*i*, *j*)-θ-closed. The (*i*, *j*)-θ-interior of *A*, denoted by $(i, j)\text{-}Int_\theta(A)$ is defined as the union of all (*i*, *j*)-θ-open sets contained in *A*. Hence $x\in (i, j)\text{-}Int_\theta(A)$ if and only if there exists a τ_i-open set *B* containing *x* such that $x\in B\subset j\,Cl(B) \subset A$.

Lemma 2.5. For a *subset A of a bitopological space* (X,τ_1,τ_2), *the following properties hold*:

(i) X – (i, j)-Int_θ(A) = (i, j)-Cl_θ(X – A).

(ii) X – (i, j)-Cl_θ(A) = (i, j)-Int_θ(X – A).

Lemma 2.6. Let (X,τ_1,τ_2) be a bitopological space. If B is a τ_j-open set of X, then (i, j)-Cl_θ(B) = i Cl(B).

Definition 5. A function $f: (X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2)$ is said to be

(1) (*i*, *j*)-semi-continuous if $f^{-1}(A)$ is (*i*, *j*)-semi-open in *X* for each σ_i-open set *A* of *Y*.

(2) (*i*, *j*)-weakly continuous if for each $x\in X$ and each σ_i-open set *B* of *Y* containing *f*(*x*), there exists a τ_i-open set *A* containing *x* such that $f(A) \subset j\,Cl(B)$.

(3) (*i*, *j*)-*b*-continuous if $f^{-1}(A)$ is (*i*, *j*)-*b*-open in *X* for each σ_i-open set *A* of *Y*.

Now we introduce the following definition in this article.

Definition 6. A function $f: (X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2)$ is said to be (*i*, *j*)-weakly *b*-continuous if for each $x\in X$ and each σ_i-open set *V* of *Y* containing *f*(*x*), there exists an (*i*, *j*)-*b*-open set *U* containing *x* such that $f(U) \subset j\,Cl(V)$.

Main results

Theorem 1. A mapping f: $(X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2)$ is (i, j)-weakly b-continuous if and only if for every open set V in Y, $f^{-1}(V) \subset$ (i, j)-b Int(f^{-1}(j Cl(V))), for all i, j =1, 2.

Proof: Let $x\in X$ and V be a σ_i-open set containing f(x). Then $x\in f^{-1}(V) \subset$ (i, j)-b Int(f^{-1}(j Cl(V))). Let U = (i, j)-b Int(f^{-1}(j Cl(V))). Then U is (i, j)-b-open set and f(U) ⊂ j Cl(V). Hence *f* is (i, j)-weakly b-continuous.

Conversely let V be a σ_i-open subset of Y and $x\in f^{-1}(V)$. Since f is (i, j)-weakly b-continuous, so there exists an (i, j)-b-open set U in X such that $x\in U$ and $f(U)\subset j\,Cl(V)$. Therefore we have $x\in U \subset f^{-1}$(j Cl(V)) and hence $x\in$(i, j)-b Int(f^{-1}(j Cl(V))). This proves that $f^{-1}(V) \subset$ (i, j)-b Int(f^{-1}(j Cl(V))).

Theorem 2. For a function f : $(X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2)$, the following statements are equivalent:

(1) f is (i, j)-weakly b-continuous;

(2) $f^{-1}(V) \subset$ (i, j)-b Int(f^{-1}(j Cl(V))) for every σ_i-open set V of Y;

(3) (i, j)-b Cl(f^{-1}(j Cl(V))) $\subset f^{-1}(V)$ for every σ_i-closed set V of Y, for all i, j =1, 2.

Proof : (1) ⇒ (2) The proof follows from Theorem 3.1.

(2) ⇒ (3) Let *V* be a σ_i-closed subset of *Y*. Then *Y* – *V* is an σ_i-open subset of *Y*.

By hypothesis we have

$$f^{-1}(Y - V) \subset (i, j)\text{-b } Int(f^{-1}(j\,Cl(Y - V)))$$
$$= (i, j)\text{-}b\ Int(f^{-1}(Y - j\,Int(V)))$$
$$= (i, j)\text{-}b\ Int(X - f^{-1}(j\,Int(V)))$$
$$= X - (i, j)\text{-}b\ Cl(f^{-1}(j\,Int(V))).$$

Thus $f^{-1}(Y - V) \subset X - (i, j)\text{-}b\ Cl(f^{-1}(j\,Int(V)))$.

$\Rightarrow (i, j)\text{-}b\ Cl(f^{-1}(j\,Int(V)))\subset f^{-1}(V)$.

(3) ⇒(1) Let $x\in X$ and let *V* be σ_i-open set of *Y* containing *f*(*x*). So *Y* – *V* is a σ_i-closed. Then by hypothesis it follows $(i, j)\text{-}b\ Cl(f^{-1}(j\,Int(Y - V))) \subset f^{-1}(Y - V) = X - f^{-1}(V)$. Thus $x\notin (i, j)\text{-}b\ Cl(f^{-1}(j\,Int(Y - V)))$. Then by Lemma 2.4, there exists a (*i*, *j*)-*b*-open set *U* such that $x\in U$ and $U\cap f^{-1}(j\,Int(Y - V)) = \emptyset$. Which implies $f(U) \cap j\,Int(Y - V) = \emptyset$.

$\Rightarrow f(U) \subset Y - j\,Int(V)$.

$\Rightarrow f(U) \subset j\,Cl(V)$.

This shows that *f* is (*i*, *j*)-weakly *b*-continuous.

Theorem 3. For a function f : $(X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2)$, the following statements are equivalent :

(1) f is (i, j)-weakly b-continuous;

(2) (i, j)-b Cl(f^{-1}(j Int(i Cl(B)))) $\subset f^{-1}$(i Cl(B)) for every subset B of Y;

(3) (i, j)-b Cl(f^{-1}(j Int(A))) $\subset f^{-1}(A)$ for every (i, j)-regular closed set A of Y;

(4) (i, j)-b $Cl(f^{-1}(V)) \subset f^{-1}(i\ Cl(V))$ for every σ_j-open set V of Y;

(5) $f^{-1}(V) \subset$ (i, j)-b $Int(f^{-1}(j\ Cl(V)))$ for every σ_i-open set V of Y, for all i, j = 1, 2.

Proof : (1) ⇒(2) Let $B \subset Y$ and $x \in X - f^{-1}(i\ Cl(B))$. Then $f(x) \in Y - i\ Cl(B)$. This implies there exists a σ_i-open set A of Y containing $f(x)$ such that $A \cap B = \varnothing$. Therefore $A \cap j\ Int(i\ Cl(B)) = \varnothing$ and hence $j\ Cl(A) \cap j\ Int(i\ Cl(B)) = \varnothing$. Since f is (i, j)-weakly b-continuous, so there exists an (i, j)-b-open set C such that $x \in C$ and $f(C) \subset j\ Cl(A)$. Thus $f(C) \cap j\ Int(i\ Cl(B)) = \varnothing$. This implies $C \cap f^{-1}(j\ Int(i\ Cl(B)) = \varnothing$. Therefore by Lemma 2.4, we have $x \in X - (i, j)\text{-}b\ Cl(f^{-1}(j\ Int(i\ Cl(B))))$. Hence $(i, j)\text{-}b\ Cl(f^{-1}(j\ Int(i\ Cl(B))) \subset f^{-1}(i\ Cl(B))$.

(2) ⇒(3) Let A be any (i, j)-regular closed set in Y. Therefore $A = i\ Cl(j\ Int(A))$. Now $(i, j)\text{-}b\ Cl(f^{-1}(j\ Int(A))) = (i, j)\text{-}b\ Cl(f^{-1}(j\ Int(i\ Cl(j\ Int(A)))))$

$$\subset f^{-1}(i\ Cl(j\ Int(A)))$$
$$= f^{-1}(A).$$

Thus $(i, j)\text{-}b\ Cl(f^{-1}(j\ Int(A))) \subset f^{-1}(A)$.

(3) ⇒(4) Let V be σ_j-open subset of Y. Since $i\ Cl(V)$ is (i, j)-regular closed in Y, $(i, j)\text{-}b\ Cl(f^{-1}(V)) \subset (i, j)\text{-}b\ Cl(f^{-1}(j\ Int(i\ Cl(V)))) \subset f^{-1}(i\ Cl(V))$.

(4) ⇒(5) Let V be σ_i-open subset of Y. Since $Y - j\ Cl(V)$ is σ_j-open in Y, so by hypothesis we have

$(i, j)\text{-}b\ Cl(f^{-1}(Y - j\ Cl(V))) \subset f^{-1}(i\ Cl(Y - j\ Cl(V)))$.

$\Rightarrow (i, j)\text{-}b\ Cl(X - f^{-1}(j\ Cl(V))) \subset X - f^{-1}(i\ Int(j\ Cl(V)))$.

$\Rightarrow X - (i, j)\text{-}b\ Int\ (f^{-1}(j\ Cl(V))) \subset X - f^{-1}(i\ Int(j\ Cl(V))) \subset X - f^{-1}(V)$.

Therefore $f^{-1}(V) \subset (i, j)\text{-}b\ Int(f^{-1}(j\ Cl(V)))$.

(5) ⇒(1) Follows from Theorem 1.

Theorem 4. *Let* $f : (X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ be a function.Then the following properties are equivalent :

(1) f is (i, j)-weakly b-continuous.

(2) $x \in (i, j)\text{-}b\ Int(f^{-1}(j\ Cl(A)))$ *for each* σ_i-*neighbourhood A of f(x), for all i, j = 1, 2.*

Proof : (1) ⇒(2) Let A be a σ_i-neighbourhood of $f(x)$ and $x \in X$. Since f is (i, j)-weakly b-continuous, so there exists an (i, j)-b-open set B containing x such that $f(B) \subset j\ Cl(A)$. Further $B \subset f^{-1}(j\ Cl(A))$ and B is (i, j)-b-open implies $x \in B \subset (i, j)\text{-}b\ Int(B) \subset (i, j)\text{-}b\ Int(f^{-1}(j\ Cl(A)))$.

(2) ⇒(1) Let $x \in (i, j)\text{-}b\ Int(f^{-1}(j\ Cl(A))$ for each σ_i-neighbourhood A of $f(x)$. Let $B = (i, j)\text{-}b\ Int(f^{-1}(j\ Cl(A))) \subset f^{-1}(j\ Cl(A))$. Then $f(B) \subset j\ Cl(A)$ and B is (i, j)-b-open. Hence f is (i, j)-weakly b-continuous.

Theorem 5. Let $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ be a function. Then the following statements are equivalent :

(1) f is (i, j)-weakly b-continuous;

(2) $f((i, j)\text{-}b\ Cl(A)) \subset (i, j)\text{-}Cl_\theta(f(A))$ for every subset A of X;

(3) $(i, j)\text{-}b\ Cl(f^{-1}(B)) \subset f^{-1}((i, j)\text{-}Cl_\theta(B))$ for every subset B of Y;

(4) $(i, j)\text{-}b\ Cl(f^{-1}(j\ Int((i, j)\text{-}Cl_\theta(B)))) \subset f^{-1}((i, j)\text{-}Cl_\theta(B))$ for every subset B of Y, for all i, j = 1, 2.

Proof : (1) ⇒(2) Let $A \subset X$ and $x \in (i, j)\text{-}b\ Cl(A)$. Let V be a σ_i-open subset of Y containing $f(x)$. Since f is (i, j)-weakly b-continuous, so there exists an (i, j)-b-open set U containing x such that $f(U) \subset j\ Cl(V)$. Since $x \in (i, j)\text{-}b\ Cl(A)$, we have by Lemma 2.4, $U \cap A \neq \varnothing$. Thus $\varnothing \neq f(U) \cap f(A) \subset j\ Cl(V) \cap f(A)$. Therefore $f(x) \in (i, j)\text{-}Cl_\theta(f(A))$. Hence $(i, j)\text{-}b\ Cl(A) \subset f^{-1}((i, j)\text{-}Cl_\theta(f(A)))$.This implies $f((i, j)\text{-}b\ Cl(A) \subset (i, j)\text{-}Cl_\theta(f(A))$.

(2) ⇒(3) Let $B \subset Y$. Then $f^{-1}(B)$ is a subset of X. By hypothesis we have

$f((i, j)\text{-}b\ Cl(f^{-1}(B))) \subset (i, j)\text{-}Cl_\theta(f(f^{-1}(B)))$.

$\Rightarrow f((i, j)\text{-}b\ Cl(f^{-1}(B))) \subset (i, j)\text{-}Cl_\theta(B)$.

$\Rightarrow (i, j)\text{-}b\ Cl(f^{-1}(B)) \subset f^{-1}((i, j)\text{-}Cl_\theta(B))$.

(3) ⇒(4) Let $B \subset Y$. Since $(i, j)\text{-}Cl_\theta(B)$ is σ_i-closed in Y, so by hypothesis we have

$(i, j)\text{-}b\ Cl(f^{-1}(j\ Int((i, j)\text{-}Cl_\theta(B)))) \subset f^{-1}((i, j)\text{-}Cl_\theta(j\ Int((i, j)\text{-}Cl_\theta(B))))$

$= f^{-1}(i\ Cl(j\ Int((i, j)\text{-}Cl_\theta(B))))$ [By Lemma 2.6]

$\subset f^{-1}(i\ Cl((i, j)\text{-}Cl_\theta(B)))$

$= f^{-1}((i, j)\text{-}Cl_\theta(B))$.

Hence $(i, j)\text{-}b\ Cl(f^{-1}(j\ Int((i, j)\text{-}Cl_\theta(B)))) \subset f^{-1}((i, j)\text{-}Cl_\theta(B))$.

(4) ⇒(1) Let V be any σ_j-open subset of Y. Then $V \subset j\ Int\ (i\ Cl(V)) = j\ Int((i, j)\text{-}Cl_\theta(V))$, by Lemma 2.6. Now by hypothesis we have $(i, j)\text{-}b\ Cl(f^{-1}(V)) \subset (i, j)\text{-}b\ Cl(f^{-1}(j\ Int((i, j)\text{-}Cl_\theta(V)))) \subset f^{-1}((i, j)\text{-}Cl_\theta(V)) = f^{-1}(i\ Cl(V))$. Thus $(i, j)\text{-}b\ Cl(f^{-1}(V)) \subset f^{-1}(i\ Cl(V))$. Hence by Theorem 3.3, we have f is (i, j)-weakly b-continuous.

Theorem 6. Let $f : (X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ be a function. If f is (i, j)-weakly b-continuous then inverse image of every (i, j)-θ-closed set of Y is (i, j)-b-closed in X, for all i, j = 1, 2.

Proof : Let A be any (i, j)-θ-closed subset of Y. Since f is (i, j)-weakly b-continuous, so by Theorem 3.5, we have $(i, j)\text{-}b\ Cl(f^{-1}(A)) \subset f^{-1}((i, j)\text{-}Cl_\theta(A)) = f^{-1}(A)$. Therefore by Lemma 2.2, $f^{-1}(A)$ is (i, j)-b-closed in X.

Theorem 7. Let $f : (X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ be a function. If f is (i, j)-weakly b-continuous then inverse image of every (i, j)-θ-open set of Y is (i, j)-b-open in X, for all i, j=1, 2.

Proof : Let A be any (i, j)-θ-open subset of Y. Then $Y - A$ is (i, j)-θ-closed subset of Y. Then $f^{-1}(Y - A) = X - f^{-1}(A)$ is (i, j)-b-closed in X, by Theorem 3.6. Hence $f^{-1}(A)$ is (i, j)-b-open set in X.

Some further properties

Definition 1. A function $f : (X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2)$ is said to have (i, j)-b Interiority condition if (i, j)-b $Int(f^{-1}(j\ Cl(V))) \subset f^{-1}(V)$ for every σ_i-open subset V of Y.

Theorem 1. If a function $f : (X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2)$ *is* (i, j)-weakly b-continuous and satisfies the (i, j)-b Interiority condition, then f *is* (i, j)-b-continuous, for all $i, j = 1, 2$.

Proof : Let A be any σ_i-open set of Y. Since f is (i, j)-weakly b-continuous, then by Theorem 3.1, we have $f^{-1}(A) \subset (i, j)$-b $Int(f^{-1}(j\ Cl(A)))$. By (i, j)-b Interiority condition of f, we have (i, j)-b $Int(f^{-1}(j\ Cl(A))) \subset f^{-1}(A)$ and hence $f^{-1}(A) = (i, j)$-b $Int(f^{-1}(j\ Cl(A)))$. By Lemma 2.2, $f^{-1}(A)$ is (i, j)-b-open in X and hence f is (i, j)-b-continuous.

Definition 2. Let (X,τ_1,τ_2) be a bitopological space and A be a subset of X. Then (i, j)-b-frontier of A is defined as follows:

$$(i,j)\text{-}b\ Fr(A) = (i,j)\text{-}b\ Cl(A) \cap (i,j)\text{-}b\ Cl(X - A) = (i,j)\text{-}b\ Cl(A) - (i,j)\text{-}b\ Int(A).$$

Theorem 2. The set of all points x of X at which a function $f : (X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2)$ is not (i, j)-weakly b-continuous is identical with the union of the (i, j)-b-frontiers of the inverse images of the σ_j-closure of σ_i-open set of Y containing f(x), for all i, j=1, 2.

Proof: Let x be a point of X at which f(x) is not (i, j)-weakly b-continuous. Then there exists a σ_i-open subset B of Y containing f(x) such that $A \cap (X - (f^{-1}(j\ Cl(B))) \neq \varnothing$ for every (i, j)-b-open subset A of X containing x. By Lemma 2.4, we have $x \in (i, j)$-b $Cl(X - (f^{-1}(j\ Cl(B)))$. Since $x \in f^{-1}(j\ Cl(B))$, we have $x \in (i, j)$-b $Cl(f^{-1}(j\ Cl(B)))$. Therefore we have $x \in (i, j)$-b $Cl(f^{-1}(j\ Cl(B))) \cap (i, j)$-b $Cl(X - (f^{-1}(j\ Cl(B)))$. Hence $x \in (i, j)$-b $Fr(f^{-1}(j\ Cl(B)))$.

Conversely if f is (i, j)-weakly b-continuous at x, then for each σ_i-open set B of Y containing f(x), there exists an (i, j)-b-open subset A containing x such that $f(A) \subset j\ Cl(B)$ and hence $x \in A \subset f^{-1}(j\ Cl(B))$. Therefore $x \in (i, j)$-b $Int(f^{-1}(j\ Cl(B)))$. Which is a contradiction to $x \in (i, j)$-b $Fr(f^{-1}(j\ Cl(B)))$.

Definition 3. A bitopological space (X,τ_1,τ_2) is said to be pairwise b-T_2 if for each pair of distinct points x and y of X, there exists a (i, j)-b-open set U containing x and a (j, i)-b-open set V containing y such that $U \cap V = \varnothing$ for $i \neq j$ and $i, j = 1, 2$.

Definition 4. A bitopological space (X,τ_1,τ_2) is said to be pairwise Urysohn (BOSE; SINHA 1981) if for each distinct points x, y of X there exists a τ_i-open set U and a τ_j-open set V such that $x \in U$, $y \in V$ and $j\ Cl(U) \cap i\ Cl(V) = \varnothing$ for $i \neq j$ and $i, j = 1, 2$.

Theorem 3. *If* (Y,σ_1,σ_2) *is a pairwise Urysohn and* $f : (X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2)$ *is* (i, j)- *weakly b-continuous injection, then* (X,τ_1,τ_2) *is pairwise b-*T_2, *for all* $i, j = 1, 2$.

Proof : Let x and y be any two distinct points of X, then $f(x) \neq f(y)$. Since Y is pairwise Urysohn, there exists a τ_i-open set U and a τ_j-open set V such that $f(x) \in U$, $f(y) \in V$ and $j\ Cl(U) \cap i\ Cl(V) = \varnothing$. Hence $f^{-1}(j\ Cl(U)) \cap f^{-1}(i\ Cl(V)) = \varnothing$. Therefore (i, j)-b $Int(f^{-1}(j\ Cl(U))) \cap (j, i)$-$b$ $Int(f^{-1}(i\ Cl(V))) = \varnothing$. Since f is (i, j)-weakly b-continuous, so by Theorem 3.1 we have $x \in f^{-1}(U) \subset (i, j)$-$b$ $Int(f^{-1}(j\ Cl(U)))$ and $y \in f^{-1}(V) \subset (j, i)$-$b$ $Int(f^{-1}(i\ Cl(V)))$. This implies (X,τ_1,τ_2) is pairwise b-T_2.

Theorem 4. *If a function* $f : (X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2)$ *is* (i, j)*-weakly b-continuous and* (j, i)*-semi-continuous then* f *is* (i, j)*-weakly continuous, for all* $i, j = 1, 2$.

Proof: Let V be a σ_i-open subset of (Y,σ_1,σ_2). Since f is (i, j)-weakly b-continuous, then by Theorem 3.1 we have $f^{-1}(V) \subset (i, j)$-b $Int(f^{-1}(j\ Cl(V)))$. By Lemma 2.2, (i, j)-b $Int(f^{-1}(j\ Cl(V)))$ is (i, j)-b-open, therefore we have (i, j)-b $Int(f^{-1}(j\ Cl(V))) \subset i\ Int(j\ Cl((i, j)\text{-}b\ Int(f^{-1}(j\ Cl(V))))) \subset i\ Int(j\ Cl(f^{-1}(j\ Cl(V))))$. Since $j\ Cl(V)$ is σ_j-closed and f is (j, i)-semi-continuous, $f^{-1}(j\ Cl(V))$ is (j, i)-semi-closed and $i\ Int(j\ Cl(f^{-1}(j\ Cl(V)))) \subset f^{-1}(j\ Cl(V))$. Hence $f^{-1}(V) \subset i\ Int(j\ Cl(f^{-1}(j\ Cl(V)))) \subset i\ Int(f^{-1}(j\ Cl(V)))$. By Lemma 3.1 of Bose and Sinha (1981), f is (i, j)-weakly continuous.

Lemma 1. (KHEDR, 1992) If $f : (X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2)$ *is* (i, j)*-weakly continuous and* (Y,σ_1,σ_2) *is* (i, j)*-regular, then* f *is i-continuous, for all* $i, j = 1, 2$.

Corollary 1. *If* $f : (X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2)$ *is* (i, j)*-weakly b-continuous,* (j, i)*-semi-continuous and* (Y,σ_1,σ_2) *is* (i, j)*-regular, then* f *is i-continuous, for all* $i, j = 1, 2$.

Proof : The proof follows from Theorem 4.4 and Lemma 4.1.

Conclusion

The notion of b-continuous functions in a bitopological space has been generalized and the notion of weakly b-continuous functions has been introduced. The notion of b-frontier of a subset in a bitopological space has been introduced. It is shown, if (Y,σ_1,σ_2) is a pairwise Urysohn and f : $(X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2)$ is (i, j)- weakly b-continuous injection, then (X,τ_1,τ_2) is pairwise b-T_2 . These notions can be applied for investigating many other properties and some properties relative to separation axioms.

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