Acta Scientiarum

http://www.uem.br/acta
ISSN printed: 1806-2563
ISSN on-line: 1807-8664
Doi: 10.4025/actascitechnol.v36i1.16206

On two new types of statistical convergence and a summability method

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ABSTRACT. In this paper, we introduce and investigate relationship among $I_{\Delta_r^s}$-statistically convergent, $I_{\Delta_r^s}^{\lambda}$-statistically convergent and $I-[V,\lambda,\Delta_r^s]-$summable sequences respectively over normed linear spaces.

Keywords: difference operator, ideal, filter, statistical convergence, summability.

Dois novos tipos de convergência estatística e o método de sumabilidade

RESUMO. Introduzem-se e investigam-se a relação entre $I_{\Delta_r^s}$-estatisticamente convergente, $I_{\Delta_r^s}^{\lambda}$-estatisticamente convergente e $I-[V,\lambda,\Delta_r^s]-$sequências sumáveis respectivamente sobre espaços lineares normatizados.

Palavras-chave: operador diferencial, ideal, filtro, convergência estatística, sumabilidade.

Introduction

The idea of convergence of a real sequence had been extended to statistical convergence by Fast (1951) and can also be found in Schoenberg (1959) If N denotes the set of natural numbers and $K \subset N$ then $K(m,n)$ denotes the cardinality of $K \cap [m,n]$. The upper and the lower natural density of the subset are K defined by:

$$\bar{d}(K) = \limsup_{n\to\infty} \frac{K(1,n)}{n} \text{ and } \underline{d}(K) = \liminf_{n\to\infty} \frac{K(1,n)}{n}$$

If $\bar{d}(K) = \underline{d}(K)$ then we say that the natural density of K exists and it is denoted by $d(K)$. Clearly $d(K) = \lim_{n\to\infty} \frac{K(1,n)}{n}$.

A sequence (x_n) of real numbers is said to be statistically convergent to L if for arbitrary $\varepsilon > 0$, the set $K(\varepsilon) = \{n \in N : |x_n - L| \geq \varepsilon\}$ has natural density zero. Statistical convergence turned out to be one of the most active areas of research in summability theory mainly due to Fridy (1985) and Šalàt (1980).

As a generalization of statistical convergence, the notion of ideal convergence was introduced first by Kostyrko et al. (2000/2001). This was further studied in topological spaces by Lahiri and Das (2005), Das et al. (2008) and many others. Mursaleen (2000) introduced and studied the idea of $\lambda-$convergence as an extension of the $[V,\lambda]-$summability introduced by Leindler (1965). $\lambda-$statistical convergence is a special case of more general $I-$statistical convergence studied by Kolk (1991).

The notion of difference sequence space was introduced by Kizmaz (1981), who studied the difference sequence spaces $\ell_\infty(\Delta)$, $c(\Delta)$ and $c_0(\Delta)$. The notion was further generalized by Et and Çolak (1995) by introducing the spaces $\ell_\infty(\Delta^s)$, $c(\Delta^s)$ and $c_0(\Delta^s)$. Another type of generalization of the difference sequence spaces is due to Tripathy and Esi (2006), who studied the spaces $\ell_\infty(\Delta_r)$, $c(\Delta_r)$ and $c_0(\Delta_r)$. Tripathy et al. (2005) generalized the above notions and unified these as follows:

Let r, s be non- negative integers, then for Z a given sequence space we have

$$Z(\Delta_r^s) = \{x = (x_k) \in w : (\Delta_r^s x_k) \in Z\},$$

where:

$\Delta_r^s x = (\Delta_r^s x_k) = (\Delta_r^{s-1} x_k - \Delta_r^{s-1} x_{k+r})$ and

$\Delta_r^0 x_k = x_k$ for all $k \in N$, which is equivalent to the following binomial representation:

$$\Delta_r^s x_k = \sum_{\upsilon=0}^{s} (-1)^{\upsilon} \binom{s}{\upsilon} x_{k+r\upsilon}$$

Taking $r = 1$, we get the spaces $\ell_\infty(\Delta^s)$, $c(\Delta^s)$ and $c_0(\Delta^s)$ studied by Et and Çolak (1995). Taking $s = 1$, we get the spaces $\ell_\infty(\Delta_r)$, $c(\Delta_r)$ and $c_0(\Delta_r)$ studied by Tripathy and Esi (2006). Taking $r = s = 1$, we get the spaces $\ell_\infty(\Delta)$, $c(\Delta)$ and $c_0(\Delta)$ introduced and studied by Kizmaz (1981). Some other works on difference sequences may be found in Karakaya and Dutta (2011), Tripathy and Dutta (2010), Tripathy and Dutta (2012), and many others.

Recently, Savas and Das (2011) made a new approach to the notions of $[V,\lambda]$-summability and $\lambda-$statistical convergence by using ideals and introduce new notions, namely $I-[V,\lambda]-$summability and I^{λ}-statistical convergence. In this paper, our intension is to generalize the results of Savas and Das (2011) by considering difference sequences.

Throughout $(X, \|.\|)$ will stand for a real normed linear space and by a sequence $x = (x_n)$ we shall mean a sequence of elements of X. N will stand for the set of natural numbers.

Main results

A family $I \subset 2^Y$ of subsets a non empty set Y is said to be an ideal in Y if (*i*) $\Phi \in I$ (*ii*) $A, B \in I$ imply $A \cup B \in I$ (*iii*) $A \in I$, $B \subset A$ imply $B \in I$, while an admissible ideal I of Y further satisfies $\{x\} \in I$ for each $x \in Y$. If I is an ideal in Y then the collection $F(I) = \{M \subset Y : M^c \in I\}$ forms a filter in Y which is called the filter associated with I.

Let $I \subset 2^N$ be a nontrivial ideal in N. The sequence $(x_n)_{n\in N}$ in X is said to be $I-$convergent to $x \in X$, if for each $\varepsilon > 0$ the set $A(\varepsilon) = \{n \in N : \|x_n - x\| \geq \varepsilon\}$ belongs to I. For details, we refer to Kostyrko et al. (2000/2001).

Definition 2.1: A sequence $x = (x_k)$ is said to be $I_{\Delta_r^s}$-statistically convergent to $L \in X$, if for every $\varepsilon > 0$, and every $\delta > 0$,

$$\left\{n \in N : \frac{1}{n}\left|\left\{k \leq n : \left\|\Delta_r^s x_k - L\right\| \geq \varepsilon\right\}\right| \geq \delta\right\} \in I .$$

For $I = I_{fin}$, $s = 0$, $r = 1$, $I_{\Delta_r^s}$-statistical convergence coincides with statistical convergence.

Let $\lambda = (\lambda_n)$ be a non-decreasing sequence of positive numbers tending to ∞ such that

$\lambda_{n+1} \leq \lambda_n + 1$, $\lambda_1 = 1$. The collection of such a sequence λ will be denoted by Ω.

The generalized de la Valée-Pousin mean is defined by

$$t_n(x) = \frac{1}{\lambda_n}\sum_{k\in I_n} x_k \text{, where } I_n = [n - \lambda_n + 1, n].$$

Definition 1.2: A sequence $x = (x_k)$ is said to be I-$[V, \lambda, \Delta_r^s]-$ summable to $L \in X$, if

$$I - \lim_n t_n(\Delta_r^s, x) \to L,$$

where $t_n(\Delta_r^s, x) = \frac{1}{\lambda_n}\sum_{k\in I_n} \Delta_r^s x_k$

i.e., for any $\delta > 0$,

$\left\{n \in N : \left|t_n(\Delta_r^s, x) - L\right| \geq \delta\right\} \in I$.

If $I = I_{fin}$, $s = 0$, $r = 1$, $I-[V,\lambda]-$summability becomes $[V,\lambda]-$summability (LEINDLER, 1965).

Definition 1.3: A sequence $x = (x_k)$ is said to be $I^{\lambda}_{\Delta_r^s}$-statistically convergent or $I^{S_\lambda}_{\Delta_r^s}$-convergent to L, if for every $\varepsilon > 0$ and $\delta > 0$,

$$\left\{n \in N : \frac{1}{\lambda_n}\left|\left\{k \in I_n : \left\|\Delta_r^s x_k - L\right\| \geq \varepsilon\right\}\right| \geq \delta\right\} \in I$$

In this case we write $I^{S_\lambda}_{\Delta_r^s} - \lim x = L$ or $x_k \to L(I^{S_\lambda}_{\Delta_r^s})$. We also write $I_{\Delta_r^s} - \lim\|x_k\| = \|L\|$. For $I = I_{fin}$, $r = 0$, $s = 1$, $I^{S_\lambda}_{\Delta_r^s}$-convergence again coincides with $\lambda-$statistical convergence.

We shall denote by $S(I, \Delta_r^s)$, $S_\lambda(I, \Delta_r^s)$ and $[V, \lambda, \Delta_r^s](I)$ the collections of all $I_{\Delta_r^s}$-statistically convergent, $I^{S_\lambda}_{\Delta_r^s}$-convergent and $I-[V, \lambda, \Delta_r^s]-$ summable sequences respectively.

Theorem 2.1: *Let* $\lambda = (\lambda_n) \in \Omega$.

(*i*) $x_k \to L[V,\lambda,\Delta_r^s](I) \Rightarrow x_k \to L(S_\lambda(I,\Delta_r^s))$.

(*ii*) If $x \in m(X)$, the space of all bounded sequences of X and $x_k \to L(S_\lambda(I,\Delta_r^s))$ then $x_k \to L[V,\lambda,\Delta_r^s](I)$.

(*iii*) $S_\lambda(I,\Delta_r^s) \cap m(X) = [V,\lambda,\Delta_r^s](I) \cap m(X)$.

Proof. (*i*) Let $\varepsilon > 0$ and $x_k \to L[V,\lambda,\Delta_r^s](I)$. We have

$$\sum_{k\in I_n}\left\|\Delta_r^s x_k - L\right\| \geq \sum_{k\in I_n \& \|x_k - L\| > \varepsilon}\left\|\Delta_r^s x_k - L\right\| \geq \varepsilon.\left|\left\{k \in I_n : \left\|\Delta_r^s x_k - L\right\| \geq \varepsilon\right\}\right|$$

So for a given $\delta > 0$,

$$\frac{1}{\lambda_n}\left|\left\{k \in I_n : \left\|\Delta_r^s x_k - L\right\| \geq \varepsilon\right\}\right| \geq \delta \Rightarrow \frac{1}{\lambda_n}\sum_{k\in I_n}\left\|\Delta_r^s x_k - L\right\| \geq \varepsilon\delta$$

i.e.,

$$\left\{n \in N : \frac{1}{\lambda_n}\left|\left\{k \in I_n : \left\|\Delta_r^s x_k - L\right\| \geq \varepsilon\right\}\right| \geq \delta\right\} \subset \left\{n \in N : \frac{1}{\lambda_n}\left\{\sum_{k\in I_n}\left\|\Delta_r^s x_k - L\right\| \geq \varepsilon\right\} \geq \varepsilon\delta\right\}$$

Since $x_k \to L[V,\lambda,\Delta_r^s](I)$, so the set on the right hand side belongs to I and so it follows that $x_k \to L(S_\lambda(I,\Delta_r^s))$.

(*ii*) Suppose that $x_k \to L(S_\lambda(I,\Delta_r^s))$ and $x \in m(X)$. We can choose $\left\|\Delta_r^s x_k - L\right\| \leq M$, $\forall$ *k*. Let $\varepsilon > 0$ be given. Now

$$\frac{1}{\lambda_n}\sum_{k\in I_n}\left\|\Delta_r^s x_k - L\right\| = \frac{1}{\lambda_n}\sum_{k\in I_n \& \|x_k - L\| \geq \varepsilon}\left\|\Delta_r^s x_k - L\right\| + \frac{1}{\lambda_n}\sum_{k\in I_n \& \|x_k - L\| < \varepsilon}\left\|\Delta_r^s x_k - L\right\|$$

$$\leq \frac{M}{\lambda_n}\left|\left\{k \in I_n : \left\|\Delta_r^s x_k - L\right\| \geq \varepsilon\right\}\right| + \varepsilon$$

Note that

$$\left\{n \in N : \frac{1}{\lambda_n}\left|\left\{k \in I_n : \left\|\Delta_r^s x_k - L\right\| \geq \varepsilon\right\}\right| \geq \frac{\varepsilon}{M}\right\} = A(\varepsilon) \in I$$

(say).

If $n \in (A(\varepsilon))^c$, then $\frac{1}{\lambda_n}\sum_{k\in I_n}\left\|\Delta_r^s x_k - L\right\| < 2\varepsilon$.

Hence

$$\left\{n \in N : \frac{1}{\lambda_n}\sum_{k\in I_n}\left\|\Delta_r^s x_k - L\right\| \geq 2\varepsilon\right\} \subset A(\varepsilon)$$ and so belongs to I.

This shows that $x_k \to L[V,\lambda,\Delta_r^s](I)$.

(*iii*) The proof follows from (*i*) and (*ii*).

Theorem 2.2: *If* $\liminf_{n\to\infty}\frac{\lambda_n}{n} > 0$, *then* the following hold

$$S(I,\Delta_r^s) \subset S_\lambda(I,\Delta_r^s)$$

Proof. For given $\varepsilon > 0$,

$$\frac{1}{n}\left|\left\{k \leq n : \left\|\Delta_r^s x_k - L\right\| \geq \varepsilon\right\}\right| \geq \frac{1}{n}$$

$$\left|\left\{k \in I_n : \left\|\Delta_r^s x_k - L\right\| \geq \varepsilon\right\}\right| \geq \frac{\lambda_n}{n}\frac{1}{\lambda_n}\left|\left\{k \in I_n : \left\|\Delta_r^s x_k - L\right\| \geq \varepsilon\right\}\right|$$

If $\liminf_{n\to\infty}\frac{\lambda_n}{n} = a$ then from definition $\left\{n \in N : \frac{\lambda_n}{n} < \frac{a}{2}\right\}$ is finite.

For $\delta > 0$,

$$\left\{n \in N : \frac{1}{\lambda_n}\left|\left\{k \in I_n : \left\|\Delta_r^s x_k - L\right\| \geq \varepsilon\right\}\right| \geq \delta\right\}$$
$$\subset \left\{n \in N : \frac{1}{n}\left|\left\{k \in I_n : \left\|\Delta_r^s x_k - L\right\| \geq \varepsilon\right\}\right| \geq \frac{a}{2}\delta\right\}$$
$$\cup \left\{n \in N : \frac{\lambda_n}{n} < \frac{a}{2}\right\}.$$

Since *I* is admissible, the set on the right hand side belongs to *I* and the proof follows.

Theorem 2.3: *If* $\lambda \in \Omega$ *be such that* $\lim_n \frac{\lambda_n}{n} = 1$, *then* $S_\lambda(I,\Delta_r^s) \subset S(I,\Delta_r^s)$.

Proof. Let $\delta > 0$ be given. Since $\lim_n \frac{\lambda_n}{n} = 1$, we can choose $m \in N$ such that $\left|\frac{\lambda_n}{n} - 1\right| < \frac{\delta}{2}$, for all $n \geq m$.

Now observe that, for $\varepsilon > 0$

$$\frac{1}{n}\left|\left\{k \leq n : \left\|\Delta_r^s x_k - L\right\| \geq \varepsilon\right\}\right| = \frac{1}{n}$$

$$\left|\left\{k \leq n - \lambda_n : \left\|\Delta_r^s x_k - L\right\| \geq \varepsilon\right\}\right| + \frac{1}{n}\left|\left\{k \in I_n : \left\|\Delta_r^s x_k - L\right\| \geq \varepsilon\right\}\right|$$

$$\leq \frac{n - \lambda_n}{n} + \frac{1}{n}\left|\left\{k \in I_n : \left\|\Delta_r^s x_k - L\right\| \geq \varepsilon\right\}\right|$$

$$\leq 1 - \left(1 - \frac{\delta}{2}\right) + \frac{1}{n}\left|\left\{k \in I_n : \left\|\Delta_r^s x_k - L\right\| \geq \varepsilon\right\}\right|$$

$$\leq \frac{\delta}{2}+\frac{1}{n}\left|\left\{k \in I_n : \left\|\Delta_r^s x_k - L\right\| \geq \varepsilon\right\}\right| \text{ for all } n \geq m.$$

Hence

$$\left\{n \in N : \frac{1}{n}\left|\left\{k \leq n : \left\|\Delta_r^s x_k - L\right\| \geq \varepsilon\right\}\right| \geq \delta\right\}$$

$$\subset \left\{n \in N : \frac{1}{n}\left|\left\{k \in I_n : \left\|\Delta_r^s x_k - L\right\| \geq \varepsilon\right\}\right| \geq \frac{\delta}{2}\right\} \cup \{1,2,\ldots,m\}$$

If $I - S_\lambda - \Delta_r^s - \lim x = L$ then the set on the right hand side belongs to *I* and so the set on the left hand side also belongs to *I*. This shows that $x = (x_k)$ is I -statistically $\Delta_r^s -$ convergent to *L*.

Theorem 2.4: If X is a Banach space, then $S_\lambda(I, \Delta_r^s) \cap m(X)$ is a closed subset of $m(X)$.

Proof. Suppose that (x^n) is a convergent sequence in $S_\lambda(I, \Delta_r^s) \cap m(X)$ and converges to $x \in m(X)$. The proof follows if we can show that $x \in S_\lambda(I, \Delta_r^s) \cap m(X)$ using the fact that every bounded sequence is also Δ_r^s -bounded.

Assume that $x^n \to L_n(S_\lambda(I, \Delta_r^s))$ $\forall$ $n \in N$. Take a sequence $\{\varepsilon_n\}_{n \in N}$ of strictly decreasing positive numbers converging to zero. We can find $n \in N$ as such that $\left\|x - x^j\right\|_\infty < \frac{\varepsilon_n}{4}$ for all $j \geq n$. Choose $0 < \delta < \frac{1}{5}$.

Now

$$A = \left\{m \in N : \frac{1}{\lambda_m}\left|\left\{k \in I_m : \left\|\Delta_r^s x_k^n - L_n\right\| \geq \frac{\varepsilon_n}{4}\right\}\right| < \delta\right\} \in F(I)$$

and

$$B = \left\{m \in N : \frac{1}{\lambda_m}\left|\left\{k \in I_m : \left\|\Delta_r^s x_k^{n+1} - L_{n+1}\right\| \geq \frac{\varepsilon_n}{4}\right\}\right| < \delta\right\} \in F(I)$$

Since $A \cap B \in F(I)$ and $\Phi \notin F(I)$, we can choose $m \in A \cap B$. Then

$$\frac{1}{\lambda_m}\left|\left\{\begin{array}{l} k \in I_m : \left\|\Delta_r^s x_k^n - L_n\right\| \geq \frac{\varepsilon_n}{4} \vee \\ \left\|\Delta_r^s x_k^{n+1} - L_{n+1}\right\| \geq \frac{\varepsilon_n}{4} \end{array}\right\}\right| \leq 2\delta < 1.$$

Since $\lambda_m \to \infty$ and $A \cap B \in F(I)$ is infinite, we can choose the above *m* so that $\lambda_m > 5$ (say). Hence there must exist a $k \in I_m$ for which we have simultaneously,

$$\left\|\Delta_r^s x_k^n - L_n\right\| < \frac{\varepsilon_n}{4} \text{ and } \left\|\Delta_r^s x_k^{n|+1} - L_{n+1}\right\| < \frac{\varepsilon_n}{4}.$$

Then it follows that

$$\left\|L_n - L_{n+1}\right\| \leq \left\|L_n - \Delta_r^s x_k^n\right\| +$$
$$\left\|\Delta_r^s x_k^n - \Delta_r^s x_k^{n+1}\right\| + \left\|\Delta_r^s x_k^{n+1} - L_{n+1}\right\|$$

$$\leq \left\|\Delta_r^s x_k^n - L_n\right\| + \left\|\Delta_r^s x_k^{n+1} - L_{n+1}\right\| + \left\|x - x^n\right\|_\infty + \left\|x - x^{n+1}\right\|_\infty$$

$$< \frac{\varepsilon_n}{4} + \frac{\varepsilon_n}{4} + \frac{\varepsilon_n}{4} + \frac{\varepsilon_n}{4} = \varepsilon_n$$

This implies that $\{L_n\}_{n \in N}$ is a Cauchy sequence in *X*, which is complete. Let $L_n \to L \in X$ as $n \to \infty$.

We shall prove that $x \to L(S_\lambda(I, \Delta_r^s))$. Choose $\varepsilon > 0$ and choose $n \in N$ such that

$$\varepsilon_n < \frac{\varepsilon}{4},\ \left\|x - x^n\right\|_\infty < \frac{\varepsilon}{4},\ \left\|L_n - L\right\| < \frac{\varepsilon}{4}.$$

Now

$$\frac{1}{\lambda_\gamma}\left|\left\{k \in I_\gamma : \left\|\Delta_r^s x_k - L\right\| \geq \varepsilon\right\}\right| \leq \frac{1}{\lambda_\gamma}$$
$$\left|\left\{k \in I_\gamma : \left\|\Delta_r^s x_k - \Delta_r^s x_k^n\right\| + \left\|\Delta_r^s x_k^n - L_n\right\| + \left\|L_n - L\right\| \geq \varepsilon\right\}\right|$$

$$\leq \frac{1}{\lambda_\gamma}\left|\left\{k \in I_\gamma : \left\|\Delta_r^s x_k^n - L_n\right\| \geq \frac{\varepsilon}{2}\right\}\right|,$$

It follows that, for any given $\delta > 0$

$$\left\{\gamma \in N : \frac{1}{\lambda_\gamma}\left|\left\{k \in I_\gamma : \left\|\Delta_r^s x_k - L\right\| \geq \varepsilon\right\}\right| \geq \delta\right\} \subset$$
$$\left\{\gamma \in N : \frac{1}{\lambda_\gamma}\left|\left\{k \in I_\gamma : \left\|\Delta_r^s x_k^n - L_n\right\| \geq \frac{\varepsilon}{2}\right\}\right| \geq \delta\right\}$$

This shows that $x \to L(S_\lambda(I, \Delta_r^s))$ and completes the proof of the theorem.

Conclusion

The paper defines and studies two types of statistical convergence and a summability method for difference sequences over a normed space. Although we are able to extend some results of Savas and Das (2011), the following further suggestions remain open: Is there other conditions such that Theorem 2.2 holds? Whether the condition in Theorem 2.3 is necessary?

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Received on March 1, 2012.
Accepted on June 4, 2012.