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Matrix transformations of paranormed sequence spaces through de la Vallée-Pousin mean

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ABSTRACT. In this paper we define the sequence space $V_\sigma^\infty(\lambda)$ related to the concept of invariant mean and de la Vallée-Pousin mean. We also determine the necessary and sufficient conditions to characterize the matrices which transform paranormed sequence spaces into the spaces $V_\sigma(\lambda)$ and $V_\sigma^\infty(\lambda)$, where $V_\sigma(\lambda)$ denotes the space of all (σ,λ)-convergent sequences.

Keywords: invariant mean, de la Vallée-Pousin mean, σ-convergence, matrix transformation.

Transformações matriciais de espaços sequenciais paranormatizados pela média de la Vallée-Pousin

RESUMO. Definiu-se a sequência espacial $V_\sigma^\infty(\lambda)$ relacionada ao conceito de média invariável e à media de de la Vallée-Pousin. Determinaram-se também as condições necessárias e suficiente para caracterizar as matrizes que transformam os espaços sequenciais paranormatizados nos espaços $V_\sigma(\lambda)$ and $V_\sigma^\infty(\lambda)$, onde $V_\sigma(\lambda)$ é o espaço de todas as sequências convergentes (σ,λ).

Palavras-chave: média invariável, media de de la Vallée-Pousin, convergência σ, transformação matricial.

Definitions, notations and preliminaries

By w, we shall denote the space of all real-valued sequences. Any vector subspace of w is called a 'sequence space'. If $x \in w$, then we write $x=(x_k)$ instead of $x=(x_k)_{k=0}^{\infty}$. We shall write ϕ, ℓ_∞, c and c_0 for the spaces of all finite, bounded, convergent and null sequences, respectively. Further, we shall use the conventions that $e=(1,1,1,\ldots)$ and $e^{(k)}$ is the sequence whose only non-zero term is 1 in the kth place for each $k \in \mathbb{N}$, where $\mathbb{N}=\{0,1,2,\ldots\}$.

A sequence space X with a linear topology is called a K-space if each of the maps $p_i: X \to \mathbb{C}$ defined by $p_i(x)=x_i$ is continuous for all $i \in \mathbb{N}$. A K-space is called an FK-space if X is complete linear metric space; a BK-space is a normed FK-space.

A linear topological space X over the real field $\mathbb{R}$ is said to be a paranormed space if there is a subadditive function $g: X \to \mathbb{R}$ such that $g(\theta)=0$, $g(x)=g(-x)$, and scalar multiplication is continuous, i.e., $|\alpha_n-\alpha| \to 0$ and $g(x_n-x) \to 0$ imply $g(\alpha_n x_n - \alpha x) \to 0$ for all x's in X and α's in $\mathbb{R}$, where θ is the zero vector in the linear space X. Assume here and after that $x=(x_k)$ be a sequence such that $x_k \neq 0$ for all $k \in \mathbb{N}$ and (p_k) be the bounded sequence of strictly positive real numbers with $\sup p_k = H$ and $M=\max\{1,H\}$. Then, the sequence spaces

$$c_0(p):=\{x=(x_k)\in\omega: \lim_{k\to\infty}|x_k|^{p_k}=0\},\ c(p):=\{x=(x_k)\in\omega: x-le\in c_0(p) \text{ for some } l\in\mathbb{C}\},$$
$$\ell_\infty(p):=\{x=(x_k)\in\omega: \sup_k|x_k|^{p_k}<\infty\}$$

and

$$\ell(p):=\left\{x=(x_k)\in\omega: \sum_{k=0}^{\infty}\left|x_k\right|^{p_k}<\infty\right\}$$

were defined and studied by Maddox (1968) and Simons (1965). If $p_k=p$ $(k=0,1,\ldots)$ for some constant $p>0$, then these sets reduce to c_0, c, ℓ_∞ and ℓ_p respectively. Note that $c_0(p)$ is a linear metric space paranormed by $h(x)=\sup_k|x_k|^{\frac{p_k}{M}}$. $\ell_\infty(p)$ and $c(p)$ fail to be linear metric spaces because the continuity of scalar multiplication does not hold for them but these two turn out to be linear metric spaces if and only if $\inf_k p_k>0$. $\ell(p)$ is linear metric space paranormed by $g(x)=(\sum_k|x_k|^{p_k})^{1/M}$. The sequence spaces $c_0(p), c(p), \ell_\infty(p)$ and $\ell(p)$ are complete paranormed by $h(x)$ iff $\inf_k p_k>0$ and $g(x)$, respectively. However, these are not normed spaces in general, see (AYDIN; BAŞAR, 2004) and (KARAKAYA et al., 2011).

Let X and Y be two sequence spaces and $A = (a_{nk})_{n;k=1}^{\infty}$ be an infinite matrix of real or complex numbers. We write $Ax = (A_n(x))$, $A_n(x) = \sum_k a_{nk}x_k$ provided that the series on the right converges for each n. If $x = (x_k) \in X$ implies that $Ax \in Y$, then we say that A defines a matrix transformation from X into Y and by (X, Y) we denote the class of such matrices.

Let σ be a one-to-one mapping from the set ℕ of natural numbers into itself. A continuous linear functional φ on the space ℓ_∞ is said to be an *invariant mean* or a σ - mean if and only if (i) $\varphi(x) \geq 0$ if $x \geq 0$ (i.e. $x_k \geq 0$ for all k), (ii) $\varphi(e) = 1$, where e=(1,1,1,...), (iii) $\varphi(x) = \varphi((x_{\sigma(k)}))$ for all $x \in \ell_\infty$.

Throughout this paper we consider the mapping σ which has no finite orbits, that is, $\sigma^p(k) \neq k$ for all integer $k \geq 0$ and $p \geq 1$, where $\sigma^p(k)$ denotes the pth iterate of σ at k. Note that, a σ-mean extends the limit functional on the space c in the sense that $\varphi(x) = \lim x$ for all $x \in c$, (MURSALEEN, 1983). Consequently, $c \subset V_\sigma$, the set of bounded sequences all of whose σ-means are equal. We say that a sequence $x = (x_k)$ is *σ-convergent* if and only if $x \in V_\sigma$. Using this concept, Schaefer (1972) defined and characterized σ-conservative, σ-regular and σ-coercive matrices. If σ is translation then V_σ is reduced to the set f of almost convergent sequences (LORENTZ, 1948). As an application of almost convergence, Mohiuddine (2011) established some approximation theorems for sequences of positive linear operators through this concept. The idea of σ-convergence for double sequences was introduced in (ÇAKAN et al., 2006) and further studied recently in (MURSALEEN; MOHIUDDINE, 2007). Çakan et al. (2009), Mohiuddine and Alotaibi (2013; 2014), Mursaleen and Mohiuddine (2008; 2009b; 2010a; 2010b; 2010c; 2012), studied various classes of four dimensional matrices, e.g. σ-regular, σ-conservative, regularly σ-conservative, boundedly σ-conservative and σ-coercive matrices.

De la Vallée-Pousin mean

Let $\lambda = (\lambda_m)$ be a non-decreasing sequence of positive numbers tending to ∞ such that $\lambda_{m+1} \leq \lambda_m + 1$, $\lambda_1 = 0$, $\rho_m(x) = \frac{1}{\lambda_m}\sum_{j\in I_m} x_j$ is called the generalized *de la Vallée-Pousin* mean, where $I_m = [m - \lambda_m + 1, m]$.

A sequence $x = (x_k)$ of real numbers is said to be (σ, λ) – convergent (MURSALEEN et al., 2009a) to a number L if and only if $\lim_{m\to\infty} \frac{1}{\lambda_m}\sum_{j\in I_m} x_{\sigma^j(n)} = L$, uniformly in n, and let $V_\sigma(\lambda)$ denote the set of all such sequences, i.e.

$$V_\sigma(\lambda) = \left\{ x \in \ell_\infty : \lim_{m\to\infty} \frac{1}{\lambda_m}\sum_{j\in I_m} x_{\sigma^j(n)} = L \text{ uniformly in } n \right\}.$$

Note that a convergent sequence is (σ,λ) - convergent but converse need hold. We remark that

(i) if $\sigma(n) = n + 1$, then $V_\sigma(\lambda)$ is reduced to the space f_λ (MURSALEEN et al., 2009a),

(ii) if $\lambda_m = m$, then $V_\sigma(\lambda)$ is reduced to the space V_σ,

(iii) if $\sigma(n) = n + 1$ and $\lambda_m = m$, then $V_\sigma(\lambda)$ is reduced to the space f,

(iv) $c \subset V_\sigma(\lambda) \subset \ell_\infty$.

A sequence $x = (x_k)$ of real numbers is said to be (σ,λ) - bounded if and only if

$$\sup_{m,n} \left| \frac{1}{\lambda_m}\sum_{j\in I_m} x_{\sigma^j(n)} \right| < \infty,$$

and let $V_\sigma^\infty(\lambda)$ denote the set of all such sequences, i.e.

$$V_\sigma^\infty(\lambda) = \{x \in \ell_\infty : \sup_{m,n} |t_{mn}(x)| < \infty\},$$

where:

$$t_{mn}(x) = \frac{1}{\lambda_m}\sum_{j\in I_m} x_{\sigma^j(n)}.$$

We remark that $c \subset V_\sigma(\lambda) \subset V_\sigma^\infty(\lambda) \subset \ell_\infty$.

Theorem 2.1. The spaces $V_\sigma(\lambda)$ and $V_\sigma^\infty(\lambda)$ are BK space with the norm

$$\| x \| = \sup_{m,n\geq 0} |t_{mn}(x)| \tag{2.1.1}$$

Proof. It can be easily verified that (2.1.1) defines a norm on $V_\sigma(\lambda)$. Now, we show that $V_\sigma(\lambda)$ is complete. Let $(x^{(i)})$ be a Cauchy sequence in $V_\sigma(\lambda)$. Then it is a Cauchy sequence in ℝ, and hence convergent in ℝ (since ℝ is complete). That is for each k, $x_k^{(i)} \to x_k$, say, as $i \to \infty$. Let $x = (x_k)_{k=1}^\infty$. Then by the definition of $V_\sigma(\lambda)$, we have $||x^{(i)}-x|| = \sup_{m,n} |t_{mn}(x^{(i)}-x)| \to 0$ $(i \to \infty)$ since $x_n^{(i)} \to x_n$.

Now, we have to show that $x \in V_\sigma(\lambda)$. Since $(x^{(i)})$ is a Cauchy sequence in $V_\sigma(\lambda)$, we have that for a given $\varepsilon > 0$, there is a positive integer N depending upon ε such that, for each $i, r > N$,

$\left\|x^{(i)} - x^{(r)}\right\| < \varepsilon.$

Hence by (2.1.1) we have

$$\sup_{m,n}|t_{mn}(x^{(i)} - x^{(r)})| < \varepsilon.$$

This implies that

$$\left|t_{mn}\left(x^{(i)} - x^{(r)}\right)\right| < \varepsilon, \qquad (2.1.2)$$

for each m,n; or

$$\left|L^{(i)} - L^{(r)}\right| < \varepsilon, \qquad (2.1.3)$$

where: $L^{(i)} = \sigma\text{-lim } x^{(i)}$. Let $L = \lim_{r\to\infty} L^{(r)}$. Then the σ-mean of x is $\varphi(x) = \lim_i \varphi(x^{(i)})$ (since $x = \lim_i x^{(i)}$ and φ is continuous and linear). Further $\lim_i \varphi(x^{(i)}) = \lim_i L^{(i)} = L$ (since $\varphi(x^{(i)})$ means σ-$\lim x^{(i)}$). Now letting $r\to\infty$ in (2.1.2) and (2.1.3), we get

$$\left|t_{mn}\left(x^{(i)} - x\right)\right| < \varepsilon \qquad (2.1.4)$$

for each m,n (since $x = \lim_r x^{(r)}$); and

$$\left|L^{(i)} - L\right| < \varepsilon, \left(\text{since} \lim_r L^{(r)} = L\right). \qquad (2.1.5)$$

for $i > N$. Now, fix i in the above inequalities. Since $x^{(i)} \in V_\sigma(\lambda)$ for fixed i, we get

$$\lim_{m\to\infty} t_{mn}\left(x^{(i)}\right) = L^{(i)} \text{ uniformly in } n.$$

Hence, for given $\varepsilon > 0$ there exis.t,s, a positive integer m_0 (depending upon i and ε but not on n) such that

$$\left|t_{mn}\left(x^{(i)}\right) - L^{(i)}\right| < \varepsilon, \qquad (2.1.6)$$

for each m,n (since $x = \lim_r x^{(r)}$); for all $m \geq m_0$ and for all n. Now by (2.1.4), (2.1.5) and (2.1.6), we get

$$|t_{mn}\varepsilon(x) - L| \leq |t_{mn}(x) - t_{mn}(x^{(i)}) + t_{mn}\left(x^{(i)}\right) - L^{(i)} + L^{(i)} - L \leq \left|t_{mn}(x) - t_{mn}\left(x^{(i)}\right)\right| + |t_{mn}(x^{(i)}) - L^{(i)} + |L^{(i)} - L| < \varepsilon + \varepsilon + \varepsilon = 3,$$

for all $m \geq m_0$ and for all n. Hence $x \in V_\sigma(\lambda)$, which proves the completeness of $V_\sigma(\lambda)$.

Now, let $\| x^{(r)} - x \| \to 0$ as $r \to \infty$. Then for given $\varepsilon > 0$, there is $r_0 \in \mathbb{N}$ such that

$$\| x^{(r)} - x \| < \varepsilon \text{ for all } r \geq r_0,$$

which implies

$$\sup_{m,n}\left|t_{mn}\left(x^{(r)} - x\right)\right| < \varepsilon \text{ for all } r \geq r_0,$$

and so that

$$\left|L^{(r)} - L\right| < \varepsilon \text{ for all } r \geq r_0,$$

as above in (2.1.5). Hence we easily get

$$\left|x_k^{(r)} - x_k\right| < \varepsilon \text{ for all } r \geq r_0, \text{and for all },$$

that is $|x_k^{(r)} - x_k| \to 0$ as $r \to \infty$, and this proves the continuity of the coordinate projection. Hence $V_\sigma(\lambda)$ is a BK space. The case $V_\sigma^\infty(\lambda)$ can be proved similarly.

This completes the proof of the theorem.

Matrix transformations

In this section we characterize the matrix class $(\ell(p), V_\sigma(\lambda))$ and $(\ell(p), V_\sigma^\infty(\lambda))$.

Theorem 3.1. Let $1 < p_k < \sup_k p_k = H < \infty$ for every k. Then $A \in (\ell(p), V_\sigma^\infty(\lambda))$ if and only if there exists an integer $N > 1$ such that

$$\sup_{m,n}\sum_k \left|\frac{1}{\lambda_m}\sum_{j\in I_m} a_{\sigma^j(n),k}\right|^{q_k} N^{-q_k} < \infty \qquad (3.1.1)$$

Proof. Sufficiency. Let (3.1.1) hold and that $x \in \ell(p)$ using the following inequality (MADDOX, 1969)

$$|ab| \leq C(|a|^q C^{-q} + |b|^p)$$

for $C > 0$ and a,b two complex numbers $(q^{-1} + p^{-1} = 1)$, we have

$$|t_{mn}(Ax)| = \sum_k \left|\frac{1}{\lambda_m}\sum_{j\in I_m} a_{\sigma^j(n),k} x_k\right|$$

$$\sum_k N\left[\left|\frac{1}{\lambda_m}\sum_{j\in I_m} a_{\sigma^j(n),k}\right|^{q_k} N^{-q_k} + |x_k|^{p_k}\right]$$

where $q_k^{-1} + p_k^{-1} = 1$.

Taking the supremum over m,n on both sides and using (3.1.1), we get $Ax \in V_\sigma^\infty(\lambda)$ for $x \in \ell(p)$, i.e. $A \in (\ell(p), V_\sigma^\infty(\lambda))$.

Necessity. Let $A \in (\ell(p), V_\sigma^\infty(\lambda))$. Write $q_n(x) = \sup_m |t_{mn}(Ax)|$. It is easy to see that for $n \geq 0$, q_n is a continuous seminorm on $\ell(p)$ and (q_n) is pointwise bounded on $\ell(p)$. Suppose that (3.1.1) is not true. Then there exists $x \in \ell(p)$ with $\sup_n q_n(x) = \infty$. By the principle of condensation of singularities (YOSIDA, 1966), the set

$$\left\{x \in \ell(p): \sup_n q_n(x) = \infty\right\}$$

is of second category in $\ell(p)$ and hence nonempty, that is, there is $x \in \ell(p)$ with $\sup_n q_n(x) = \infty$. But this contradicts the fact that (q_n) is pointwise bounded on $\ell(p)$. Now by the Banach-Steinhaus theorem, there is constant M such that

$$q_n(x) \le Mg(x). \tag{3.1.2}$$

Now define a sequence $x = (x_k)$ by

$$x_k = \begin{cases} \delta^{\frac{M}{p_k}}\left(\operatorname{sgn}\frac{1}{\lambda_m}\sum_{j\in I_m} a_{\sigma^j(n),k}\right) \\ \left|\frac{1}{\lambda_m}\sum_{j\in I_m} a_{\sigma^j(n),k}\right|^{q_k-1} S^{-1} \\ N^{\frac{q_k}{p_k}} \quad \text{for } 1\le k\le k_0, \\ 0 \quad \text{for } k>k_0, \end{cases}$$

where $0 < \delta < 1$ and

$$S = \sum_{k=1}^{k_0}\left|\frac{1}{\lambda_m}\sum_{j\in I_m} a_{\sigma^j(n),k}\right|^{q_k} N^{-q_k}.$$

Then it is easy to see that $x \in \ell(p)$ and $g(x) \le \delta$. Applying this sequence to (3.1.2) we get the condition (3.1.1).

This completes the proof of the theorem.

In the following theorem we characterize the matrix class $(\ell(p), V_\sigma(\lambda))$.

Theorem 3.2. Let $1<p_k\le\sup_k p_k=H<\infty$ for every k. Then $A \in (\ell(p), V_\sigma(\lambda))$ if and only if

(i) condition (3.1.1) of Theorem 3.1 holds

(ii) $\lim_m \frac{1}{\lambda_m}\sum_{j\in I_m} a_{\sigma^j(n),k} = \alpha_k$ uniformly in n, for every k.

Proof. Sufficiency. Let (i) and (ii) hold and $x \in \ell(p)$. For $j \ge 1$

$$\sum_{k,=1}^{j}\left|\frac{1}{\lambda_m}\sum_{j\in I_m} a_{\sigma^j(n),k}\right|^{q_k} N^{-q_k} \le \sup_m \sum_k \left|\frac{1}{\lambda_m}\sum_{j\in I_m} a_{\sigma^j(n),k}\right|^{q_k} N^{-q_k} < \infty,$$

for every n. Therefore

$$\sum_k |\alpha_k|^{q_k} N^{-q_k} = \lim_j \lim_m \sum_{k=1}^{j}\left|\frac{1}{\lambda_m}\sum_{j\in I_m} a_{\sigma^j(n),k}\right|^{q_k}$$

$$N^{-q_k} \le \sup_m \sum_k \left|\frac{1}{\lambda_m}\sum_{j\in I_m} a_{\sigma^j(n),k}\right|^{q_k} N^{-q_k} < \infty,$$

where $q_k^{-1}+p_k^{-1}=1$. Consequently reasoning as in the proof of the sufficiency of Theorem 3.1, the series $\frac{1}{\lambda_m}\sum_k \sum_{j\in I_m} a_{\sigma^j(n),k}x_k$ and $\sum_k \alpha_k x_k$ converge for every n, m; and for every $x \in \ell(p)$. For a given $\varepsilon > 0$ and $x \in \ell(p)$, choose k_0 such that

$$\left(\sum_{k=k_0+1}^{\infty} |x_k|^{p_k}\right)^{\frac{1}{H}} < \varepsilon, \tag{3.2.1}$$

where $H=\sup p_k$. Condition (ii) implies that there exists m_0 such that

$$\left\|\sum_{k=1}^{k_0}\left[\frac{1}{\lambda_m}\sum_{j\in I_m} a_{\sigma^j(n),k} - \alpha_k\right]\right\| < \varepsilon/2$$

for all $m \ge m_0$ and uniformly in n. Now, since $\frac{1}{\lambda_m}\sum_k \sum_{j\in I_m} a_{\sigma^j(n),k}x_k$ and $\sum_k \alpha_k x_k$ converges (absolutely) uniformly in m, n and for every $x \in \ell(p)$; we have that

$$\sum_{k=k_0+1}^{\infty}\left[\frac{1}{\lambda_m}\sum_{j\in I_m} a_{\sigma^j(n),k} - \alpha_k\right] x_k$$

converges uniformly in m, n for every $x \in \ell(p)$. Hence by conditions (i) and (ii)

$$\left\|\sum_{k=k_0+1}^{\infty}\left[\frac{1}{\lambda_m}\sum_{j\in I_m} a_{\sigma^j(n),k} - \alpha_k\right]\right\| < \varepsilon/2$$

for all $m \ge m_0$ and uniformly in n. Therefore

$$\left|\sum_{k=k_0+1}^{\infty}\left[\frac{1}{\lambda_m}\sum_{j\in I_m} a_{\sigma^j(n),k} - \alpha_k\right] x_k\right| \to 0 \ (m \to \infty)$$

uniformly in n, i.e.

$$\lim_m \sum_k \frac{1}{\lambda_m}\sum_{j\in I_m} a_{\sigma^j(n),k}x_k = \sum_k \alpha_k x_k \tag{3.2.2}$$

uniformly in n.

Necessity. Let $A \in (\ell(p), V_\sigma(\lambda))$. Since $V_\sigma(\lambda) \subset V_\sigma^\infty(\lambda)$, condition (i) follows by Theorem 3.1. Since $e^{(k)} = (0,0,\cdots,1(k^{th}\text{ place }),0,0,\cdots) \in l(p)$, condition (ii) follows immediately by (3.2.2).

This completes the proof of the theorem.

Conclusion

Two notions - one of σ-mean and the other of de *la Vallée-Pousin* mean - play a very active role in recent research on matrix transformations. With the help of these two notions, author has defined the concept of (σ, λ)-bounded sequence, denoted by $V_\sigma^\infty(\lambda)$. He also characterized the matrix classes $(\ell(p), V_\sigma(\lambda))$ and $(\ell(p), V_\sigma^\infty(\lambda))$, where $\ell(p)$ and $V_\sigma(\lambda)$ are defined in Section 1 and Section 2, respectively.

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