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Almost statistical convergence of order α

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ABSTRACT. In this paper, we introduce the concept $\hat{S}_{\lambda}^{\alpha}$ – statistical convergence of order α. Also some relations between $\hat{S}_{\lambda}^{\alpha}$ - statistical convergence of order α and strong $\hat{w}_{p}^{\alpha}(\lambda)$- summability of order α are given. Furthermore some relations between the spaces $\hat{w}_{(p)}^{\alpha}[\lambda, M]$ and $\hat{S}_{\lambda}^{\alpha}$ are examined.

Keywords: statistical convergence, almost convergence, Cesàro summability.

Quase convergência estatística da order α

RESUMO. Apresenta-se o conceito $\hat{S}_{\lambda}^{\alpha}$ – a convergência estatística da ordem α - e fornecem-se alguns relacionamentos entre $\hat{S}_{\lambda}^{\alpha}$ - a convergência estatística da ordem α e a forte $\hat{w}_{p}^{\alpha}(\lambda)$- sumabilidade da ordem α. Algumas relações entre os espaços $\hat{w}_{(p)}^{\alpha}[\lambda, M]$ e $\hat{S}_{\lambda}^{\alpha}$ são investigadas.

Palavras-chave: convergência estatística, quase-convergência, sumabilidade de Cesàro.

Introduction

The idea of statistical convergence was given by Zygmund (1979) in the first edition of his monograph published in Warsaw in 1935. The concept of statistical convergence was introduced by Steinhaus (1951) and Fast (1951) and later reintroduced by Schoenberg (1959) independently. Over the years and under different names statistical convergence has been discussed in the theory of Fourier analysis, Ergodic theory, Number theory, Measure theory, Trigonometric series, Turnpike theory and Banach spaces. Later on it was further investigated from the sequence space point of view and linked with summability theory by Šalát (1980), Fridy (1985), Connor (1988), Rath and Tripathy (1994), Savaş (2000), Mursaleen (2000), Miller and Orhan (2001), Et and Nuray (2001), Mursaleen et al. (2001, 2003, 2009), Mursaleen and Edely (2009), Mohiuddine and Lohani (2009), Çolak (2010), Çolak and Bektaş (2011), Bhunia et al. (2012), Kumar and Mursaleen (2011) and Savaş and Mohiuddine (2012) and many others. In recent years, generalizations of statistical convergence have appeared in the study of strong integral summability and the structure of ideals of bounded continuous functions on locally compact spaces. Statistical convergence and its generalizations are also of the natural numbers.

Moreover, statistical convergence is closely related to the concept of convergence in probability.

Let w denote the set of all real sequences $x = (x_n)$. By ℓ_∞ and c, we denote the Banach spaces of bounded and convergent sequences $x = (x_n)$ normed by $\|x\| = \sup_n |x_n|$, respectively. A linear functional $\mathbf{L}$ on ℓ_∞ is said to be a Banach limit if it has the properties, i) $\mathbf{L}(x) \geq 0$ if $x \geq 0$ (i.e. $x_n \geq 0$ for all n), ii) $\mathbf{L}(e) = 1$, where $e = (1,1,\ldots)$, iii) $\mathbf{L}(Dx) = \mathbf{L}(x)$, where D is the shift operator defined by $(Dx_n) = (x_{n+1})$ Banach (1955).

Let B be the set of all Banach limits on ℓ_∞. A sequence x is said to be almost convergent to a number L if $\mathbf{L}(x) = L$ for all $\mathbf{L} \in \mathrm{B}$. Lorentz (1948) has shown that x is almost convergent to L if and only if

$$\lim_{k\to\infty} t_{km}(x) = \lim_{k\to\infty} \frac{x_m + x_{m+1} + \ldots + x_{m+k}}{k+1} = L,$$
uniformly in m.

Let f denote the set of all almost convergent sequences. We write $f - \lim x = L$ if x is almost convergent to L. Maddox (1978) and (independently)

Freedman et al. (1978) has defined $x=(x_k)$ to be strongly almost convergent to a number L if

$$\lim_{k\to\infty}\frac{1}{k+1}\sum_{i=0}^{k}|x_{i+m}-L|=0, \text{uniformly in } m.$$

Let $[f]$ denote the set of all strongly almost convergent sequences. If x is strongly almost convergent to L, we write $[f]-\lim x=L$. It is easy to see that $[f]\subset f\subset \ell_\infty$. Das and Sahoo (1992) defined the sequence space

$$[\hat{w}(p)]=\left\{\begin{array}{l} x\in w : \frac{1}{n+1}\sum_{k=0}^{n}|t_{km}(x)-L|^{p_k}\to 0 \\ \text{as } n\to\infty, \text{ uniformlyin } m\end{array}\right\}$$

and investigated some of its properties.

The order of statistical convergence of a sequence of numbers was given by Gadjiev and Orhan (2002) and after then statistical convergence of order α and strongly $p-$Cesàro summability of order α studied by Çolak (2010) and generalized by Çolak and Bektas (2011).

The statistical convergence of order α is defined as follows. Let $0<\alpha\leq 1$ be given. The sequence $x=(x_k)$ is said to be statistically convergent of order α if there is a real number L such that

$$\lim_{n\to\infty}\frac{1}{n^\alpha}\left|\{k\leq n : |x_k-L|\geq\varepsilon\}\right|=0,$$

for every $\varepsilon>0$, in which case we say that x is statistically convergent of order α, to L. In this case we write $S^\alpha-\lim x_k=L$. The set of all statistically convergent sequences of order α will be denoted by S^α.

The generalized de la Vallée-Poussin mean is defined by

$$t_n(x)=\frac{1}{\lambda_n}\sum_{k\in I_n}x_k,$$

where $\lambda=(\lambda_n)$ is a non-decreasing sequence of positive numbers such that $\lambda_{n+1}\leq\lambda_n+1$, $\lambda_1=1$, $\lambda_n\to\infty$ as $n\to\infty$ and $I_n=[n-\lambda_n+1,n]$.

A sequence $x=(x_k)$ is said to be $(V,\lambda)-$summable to a number L if $t_n(x)\to L$ as $n\to\infty$, Leindler (1965). $(V,\lambda)-$summability reduces to $(C,1)$ summability when $\lambda=(\lambda_n)=(n)$.

We write

$$[C,1]=\left\{x=(x_k) : \lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}|x_k-L|=0 \text{ for some } L\right\},$$

$$[V,\lambda]=\left\{x=(x_k) : \lim_{n\to\infty}\frac{1}{\lambda_n}\sum_{k\in I_n}|x_k-L|=0 \text{ for some } L\right\}$$

for the sets of strongly Cesàro summable and strongly $(V,\lambda)-$summable sequences, respectively. Strong $(V,\lambda)-$summability reduces to strong $(C,1)$ summability when $\lambda=(\lambda_n)=(n)$.

Main results

In this section we give the main results of the paper. In Theorem 2.4 we give the inclusion relations between the sets of $\hat{S}_\lambda^\alpha-$statistical convergent sequences of order α for different α' s. In Theorem 2.8 we give the relationship between the strong $\hat{w}_p^\alpha(\lambda)-$summability of order α and the strong $\hat{w}_p^\beta(\lambda)-$summability of order β. In Theorem 2.11 we give the relationship between the strong $\hat{w}_p^\alpha(\lambda)-$summability of order α and the $\hat{S}_\lambda^\beta-$statistical convergence of order β.

Definition 2.1

Let the sequence $\lambda=(\lambda_n)$ of real numbers be defined as above and $0<\alpha\leq 1$ be given. The sequence $x=(x_k)\in w$ is said to be $\hat{S}_\lambda^\alpha-$ statistically convergent of order α if there is a real number L such that

$$\lim_{n\to\infty}\frac{1}{\lambda_n^\alpha}\left|\{k\in I_n : |t_{km}(x)-L|\geq\varepsilon\}\right|=0,$$

uniformly in m,

where $I_n=[n-\lambda_n+1,n]$ and λ_n^α denote the α^{th} power $(\lambda_n)^\alpha$ of λ_n, that is $\lambda^\alpha=(\lambda_n^\alpha)=(\lambda_1^\alpha,\lambda_2^\alpha,...,\lambda_n^\alpha,...)$. We write $\hat{S}_\lambda^\alpha-\lim x_k=L$ in case $x=(x_k)$ is $\hat{S}_\lambda^\alpha-$ statistically convergent to L of order α. The set of all $\hat{S}_\lambda^\alpha-$statistically convergent sequences of order α will be denoted by $\hat{S}_\lambda^\alpha$. We shall write $\hat{S}^\alpha$ in the special case $\lambda_n=n$ for all

$n \in \mathbb{N}$; $\hat{S}_\lambda$ in the special case $\alpha = 1$ and $\hat{S}$ in the special case $\lambda_n = n$, $\alpha = 1$ instead of $\hat{S}_\lambda^\alpha$.

The $\hat{S}_\lambda^\alpha -$ statistical convergence of order α is well defined for $0 < \alpha \le 1$, but it is not well defined for $\alpha > 1$ in general. For this let $x = (x_k)$ be fixed. Then for an arbitrary number L and $\varepsilon > 0$ we have

$$\lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} \left| \{ k \in I_n : |t_{km}(x) - L| \ge \varepsilon \} \right| \le \lim_{n\to\infty} \frac{[\lambda_n] + 1}{\lambda_n^\alpha} = 0,$$

uniformly in m

Thus $\hat{S}_\lambda^\alpha - \lim x_k$ is not uniquely determined for $\alpha > 1$, where $[a]$ denotes the integer part of the real number a.

Theorem 2.2

Let $0 < \alpha \le 1$ and $x = (x_k)$, $y = (y_k)$ be sequences of complex numbers, then (i) If $\hat{S}_\lambda^\alpha - \lim x_k = x_0$ and $c \in \mathbb{C}$, then $\hat{S}_\lambda^\alpha - \lim(cx_k) = cx_0$, (ii) If $\hat{S}_\lambda^\alpha - \lim x_k = x_0$ and $\hat{S}_\lambda^\alpha - \lim y_k = y_0$, then $\hat{S}_\lambda^\alpha - \lim(x_k + y_k) = x_0 + y_0$.

Proof.

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Definition 2.3

Let the sequence $\lambda = (\lambda_n)$ be as above, $\alpha \in (0,1]$ be any real number and let p be a positive real number. A sequence x is said to be strongly $\hat{w}_p^\alpha(\lambda) -$ summable of order α, if there is a real number L such that

$$\lim_{n\to\infty} \frac{1}{\lambda_n^\alpha} \sum_{k\in I_n} |t_{km}(x) - L|^p = 0$$

uniformly in m,

where $I_n = [n - \lambda_n + 1, n]$. The strong $\hat{w}_p^\alpha(\lambda) -$ summability of order α reduces to the strong $\hat{w}_p(\lambda) -$ summability for $\alpha = 1$. The set of all strongly $\hat{w}_p^\alpha(\lambda) -$ summable sequences of order α will be denoted by $\hat{w}_p^\alpha(\lambda)$.

Theorem 2.4

If $0 < \alpha \le \beta \le 1$ then $\hat{S}_\lambda^\alpha \subseteq \hat{S}_\lambda^\beta$.

Proof.

If $0 < \alpha \le \beta \le 1$ then we may write

$$\frac{1}{\lambda_n^\beta} \left| \{ k \in I_n : |t_{km}(x) - L| \ge \varepsilon \} \right| \le \frac{1}{\lambda_n^\alpha} \left| \{ k \in I_n : |t_{km}(x) - L| \ge \varepsilon \} \right|$$

for every $\varepsilon > 0$ and this gives that $\hat{S}_\lambda^\alpha \subseteq \hat{S}_\lambda^\beta$.

From Theorem 2.4 we have the following.

Corollary 2.5

If a sequence is $\hat{S}_\lambda^\alpha -$ statistically convergent of order α, to L, then it is $\hat{S}_\lambda -$ statistically convergent to L, that is $\hat{S}_\lambda^\alpha \subseteq \hat{S}_\lambda$ for each $\alpha \in (0,1]$.

Theorem 2.6

$\hat{S}^\alpha \subseteq \hat{S}_\lambda^\alpha$ if

$$\lim_{n\to\infty} \inf \frac{\lambda_n^\alpha}{n^\alpha} > 0.$$

Proof.

For given $\varepsilon > 0$ we have

$$\{ k \le n : |t_{km}(x) - L| \ge \varepsilon \} \supset \{ k \in I_n : |t_{km}(x) - L| \ge \varepsilon \}$$

and so

$$\frac{1}{n^\alpha} \left| \{ k \le n : |t_{km}(x) - L| \ge \varepsilon \} \right| \ge \frac{\lambda_n^\alpha}{n^\alpha} \frac{1}{\lambda_n^\alpha} \left| \{ k \in I_n : |t_{km}(x) - L| \ge \varepsilon \} \right|.$$

Taking the limit as $n \to \infty$ and using (1), we get $\hat{S}^\alpha - \lim x_k = L \Rightarrow \hat{S}_\lambda^\alpha - \lim x_k = L$.

Corollary 2.7

If $\liminf_{n\to\infty} \frac{\lambda_n^\alpha}{n} > 0$, then $\hat{S} \subseteq \hat{S}_\lambda^\alpha$.

Theorem 2.8

Let $0 < \alpha \le \beta \le 1$ and p be a positive real number, then $\hat{w}_p^\alpha(\lambda) \subseteq \hat{w}_p^\beta(\lambda)$.

Proof.

Let $x=(x_k)\in \hat{w}_p^{\alpha}(\lambda)$. Then given α and β such that $0<\alpha\le\beta\le 1$ and a positive real number p, we may write

$$\frac{1}{\lambda_n^{\beta}}\sum_{k\in I_n}|t_{km}(x)-L|^p \le \frac{1}{\lambda_n^{\alpha}}\sum_{k\in I_n}|t_{km}(x)-L|^p$$

and this gives that $\hat{w}_p^{\alpha}(\lambda)\subseteq \hat{w}_p^{\beta}(\lambda)$.

The following result is a consequence of Theorem 2.8.

Corollary 2.9

Let $0<\alpha\le 1$ and p be a positive real number. Then $\hat{w}_p^{\alpha}(\lambda)\subseteq \hat{w}_p(\lambda)$.

Theorem 2.10

Let $0<\alpha\le 1$ and $0<p<q<\infty$. Then $\hat{w}_q^{\alpha}(\lambda)\subset \hat{w}_p^{\beta}(\lambda)$.

Proof is seen from Hölder inequality.

Theorem 2.11

Let α and β be fixed real numbers such that $0<\alpha\le\beta\le 1$ and $0<p<\infty$. If a sequence is strongly $\hat{w}_p^{\alpha}(\lambda)-$summable of order α, to L, then it is $\hat{S}_{\lambda}^{\alpha}-$statistically convergent of order β, to L, i.e $\hat{w}_p^{\alpha}(\lambda)\subset \hat{S}_{\lambda}^{\beta}$.

Proof.

For any sequence $x=(x_k)$ and $\varepsilon>0$, we have

$$\begin{aligned}\sum_{k\in I_n}|t_{km}(x)-L|^p &= \sum_{\substack{k\in I_n\\ |t_{km}(x)-L|\ge\varepsilon}}|t_{km}(x)-L|^p + \sum_{\substack{k\in I_n\\ |t_{km}(x)-L|<\varepsilon}}|t_{km}(x)-L|^p \\ &\ge \sum_{\substack{k\in I_n\\ |t_{km}(x)-L|\ge\varepsilon}}|t_{km}(x)-L|^p \\ &\ge \left|\{k\in I_n : |t_{km}(x)-L|\ge\varepsilon\}\right|\varepsilon^p\end{aligned}$$

and so that

$$\begin{aligned}&\frac{1}{\lambda_n^{\alpha}}\sum_{k\in I_n}|t_{km}(x)-L|^p \\ &\ge \frac{1}{\lambda_n^{\alpha}}\left|\{k\in I_n : |t_{km}(x)-L|\ge\varepsilon\}\right|\varepsilon^p \\ &\ge \frac{1}{\lambda_n^{\beta}}\left|\{k\in I_n : |t_{km}(x)-L|\ge\varepsilon\}\right|\varepsilon^p.\end{aligned}$$

From this it follows that if $x=(x_k)$ is strongly $\hat{w}_p^{\alpha}(\lambda)-$summable of order α, to L, then it is $\hat{S}_{\lambda}^{\beta}-$statistically convergent of order β, to L.

Corollary 2.12

Let α be a fixed real number such that $0<\alpha\le 1$ and $0<p<\infty$. Then, the following statements hold: *i*) If a sequence is strongly $\hat{w}_p^{\alpha}(\lambda)-$summable of order α, to L, then it is $\hat{S}_{\lambda}^{\alpha}-$ statistically convergent of order α, to L, i.e $\hat{w}_p^{\alpha}(\lambda)\subset \hat{S}_{\lambda}^{\alpha}$, *ii*) $\hat{w}_p^{\alpha}(\lambda)\subset \hat{S}_{\lambda}$.

Results related to orlicz function

In this section we give the inclusion relations between the sets of $\hat{S}_{\lambda}^{\alpha}-$ statistical convergent sequences of order α and strongly $\hat{w}_{(p)}^{\alpha}[\lambda,M]-$summable sequences of order α with respect to an Orlicz function M.

An Orlicz function is a function $M:[0,\infty)\to[0,\infty)$, which is continuous, non-decreasing and convex with $M(0)=0$, $M(x)>0$ for $x>0$ and $M(x)\to\infty$ as $x\to\infty$.

Lindenstrauss and Tzafriri (1971) used the idea of Orlicz function to construct the sequence space

$$\ell_M=\left\{x\in w : \sum_{k=1}^{\infty}M\left(\frac{|x_k|}{\rho}\right)<\infty \text{ for some } \rho>0\right\}.$$

The space ℓ_M with the norm

$$\|x\|=\inf\left\{\rho>0 : \sum_{k=1}^{\infty}M\left(\frac{|x_k|}{\rho}\right)\le 1\right\}$$

becomes a Banach space, called an Orlicz sequence space. The Orlicz space ℓ_M is reduced in the special case $M(x)=|x|^p$ to the classical space ℓ_p of absolutely $p-$summable sequences, where $1\le p<\infty$.

Recently Orlicz sequence spaces have been studied by Bhardwaj and Singh (2000), Mursaleen et al. (2001), Savaş and Rhoades (2002), Çolak et al. (2006), Et et al. (2006), Güngör et al. (2004), Tripathy et al. (2008), Dutta and Başar (2011) and many others.

Definition 3.1

Let M be an Orlicz function, $p=(p_k)$ be a sequence of strictly positive real numbers and let $\alpha\in(0,1]$ be any real number. Now we define

$$\hat{w}^{\alpha}_{(p)}[\lambda,M]=\left\{\begin{array}{l} x=(x_k)\ :\ \lim\limits_{n\to\infty}\frac{1}{\lambda_n^{\alpha}} \\ \sum\limits_{k\in I_n}\left[\frac{M(|t_{km}(x)-L|)}{\rho}\right]^{p_k}=0 \\ \text{uniformly in } m, \text{ for some } L \text{ and } \rho>0\end{array}\right\}.$$

If $x\in\hat{w}^{\alpha}_{(p)}[\lambda,M]$, then we say that x is strongly $\hat{w}^{\alpha}_{(p)}[\lambda,M]-$summable of order α with respect to the Orlicz function M.

In the following theorems we shall assume that the sequence $p=(p_k)$ is bounded and $0<h=\inf_k p_k\le p_k\le\sup_k p_k=H<\infty$.

Theorem 3.2

Let $\alpha,\beta\in(0,1]$ be real numbers such that $\alpha\le\beta$ and M be an Orlicz function, then $\hat{w}^{\alpha}_{(p)}[\lambda,M]\subset\hat{S}^{\beta}_{\lambda}$.

Proof.

Let $x\in\hat{w}^{\alpha}_{(p)}[\lambda,M]$, $\varepsilon>0$ be given and $\sum_1$ and $\sum_2$ denote the sums over $k\in I_n$, $|t_{km}(x)-L|\ge\varepsilon$ and $k\in I_n$, $|t_{km}(x)-L|<\varepsilon$ respectively. Since $\lambda_n^{\alpha}\le\lambda_n^{\beta}$ for each n we may write

$$\begin{aligned}\frac{1}{\lambda_n^{\alpha}}\sum_{k\in I_n}\left[M\left(\frac{|t_{km}(x)-L|}{\rho}\right)\right]^{p_k} &=\frac{1}{\lambda_n^{\alpha}}\left[\begin{array}{l}\sum_1\left[M\left(\frac{|t_{km}(x)-L|}{\rho}\right)\right]^{p_k}\\ +\sum_2\left[M\left(\frac{|t_{km}(x)-L|}{\rho}\right)\right]^{p_k}\end{array}\right]\\ &\ge\frac{1}{\lambda_n^{\beta}}\left[\begin{array}{l}\sum_1\left[M\left(\frac{|t_{km}(x)-L|}{\rho}\right)\right]^{p_k}\\ +\sum_2\left[M\left(\frac{|t_{km}(x)-L|}{\rho}\right)\right]^{p_k}\end{array}\right]\\ &\ge\frac{1}{\lambda_n^{\beta}}\sum_1[f(\varepsilon)]^{p_k}\\ &\ge\frac{1}{\lambda_n^{\beta}}\sum_1\min\left([M(\varepsilon_1)]^h,[M(\varepsilon_1)]^H\right),\\ &\quad\varepsilon_1=\frac{\varepsilon}{\rho}\\ &\ge\frac{1}{\lambda_n^{\beta}}\left|\{k\in I_n\ :\ |t_{km}(x)-L|\ge\varepsilon\}\right|\\ &\quad\min\left([f(\varepsilon_1)]^h,[f(\varepsilon_1)]^H\right).\end{aligned}$$

Since $x\in\hat{w}^{\alpha}_{(p)}[\lambda,M]$, the left hand side of the above inequality tends to zero as $n\to\infty$ uniformly in m. Therefore the right hand side tends to zero as $n\to\infty$ uniformly in m and hence $x\in\hat{S}^{\beta}_{\lambda}$, because $\min([f(\varepsilon_1)]^h,[f(\varepsilon_1)]^H)>0$.

Corollary 3.3

Let $0<\alpha\le1$ and M be an Orlicz function, then $\hat{w}^{\alpha}_{(p)}[\lambda,M]\subset\hat{S}^{\alpha}_{\lambda}$.

Theorem 3.4

Let M be an Orlicz function and $x=(x_k)$ be a bounded sequence, then $\hat{S}^{\alpha}_{\lambda}\subset\hat{w}^{\alpha}_{(p)}[\lambda,M]$.

Proof.

Suppose that $x\in\ell_\infty$ and $\hat{S}^{\alpha}_{\lambda}-\lim x_k=L$. Since $x\in\ell_\infty$, then there is a constant $T>0$ such that $|t_{km}(x)|\le T$. Given $\varepsilon>0$ we have

$$\begin{aligned}&\frac{1}{\lambda_n^{\alpha}}\sum_{k\in I_n}\left[M\left(\frac{|t_{km}(x)-L|}{\rho}\right)\right]^{p_k}=\frac{1}{\lambda_n^{\alpha}}\sum\nolimits_1\left[M\left(\frac{|t_{km}(x)-L|}{\rho}\right)\right]^{p_k}\\ &+\frac{1}{\lambda_n^{\alpha}}\sum\nolimits_2\left[M\left(\frac{|t_{km}(x)-L|}{\rho}\right)\right]^{p_k}\\ &\le\frac{1}{\lambda_n^{\alpha}}\sum\nolimits_1\max\left\{\left[M\left(\frac{T}{\rho}\right)\right]^h,\left[M\left(\frac{T}{\rho}\right)\right]^H\right\}\\ &+\frac{1}{\lambda_n^{\alpha}}\sum\nolimits_2\left[M\left(\frac{\varepsilon}{\rho}\right)\right]^{p_k}\\ &\le\max\{[M(K)]^h,[M(K)]^H\}\frac{1}{\lambda_n^{\alpha}}\left|\{k\in I_n\ :\ |t_{km}(x)-L|\ge\varepsilon\}\right|\\ &+\max\{[M(\varepsilon_1)]^h,[M(\varepsilon_1)]^H\},\ \frac{T}{\rho}=K,\ \frac{\varepsilon}{\rho}=\varepsilon_1.\end{aligned}$$

Hence $\hat{S}^{\alpha}_{\lambda}\subset\hat{w}^{\alpha}_{(p)}[\lambda,M]$.

Conclusion

We introduced an studied the λ - statistical convergence of order α, λ - strong p- Cesàro summability of order α. Some of inclusion relations of given sets are established.

Also using an Orlicz function establish some other sets of sequences related to the subset.

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