



## Prediction of modulus of elasticity and compressive strength of concrete specimens by means of artificial neural networks

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**ABSTRACT.** Currently, artificial neural networks are being widely used in various fields of science and engineering. Neural networks have the ability to learn through experience and existing examples, and then generate solutions and answers to new problems, involving even the effects of non-linearity in their variables. The aim of this study is to use a feed-forward neural network with back-propagation technique, to predict the values of compressive strength and modulus of elasticity, at 28 days, of different concrete mixtures prepared and tested in the laboratory. It demonstrates the ability of the neural networks to quantify the strength and the elastic modulus of concrete specimens prepared using different mix proportions.

**Keywords:** modulus of elasticity, compressive strength, concrete, neural networks, artificial intelligence.

## Previsão do módulo de elasticidade e da resistência à compressão de corpos de prova de concreto por meio de redes neurais artificiais

**RESUMO.** Atualmente, redes neurais artificiais são largamente utilizadas em vários campos da ciência e da engenharia. Redes neurais possuem a habilidade de aprender por meio de experiências anteriores, e, posteriormente, gerar soluções e respostas para novos problemas, envolvendo até a não linearidade em suas variáveis. Este trabalho visa à utilização de uma rede neural, com a técnica da retropropagação, para prever os valores do módulo de elasticidade e da resistência à compressão do concreto, aos 28 dias, advindos de dosagens preparadas e ensaiadas em laboratório. Demonstra-se, a partir dos resultados obtidos, a capacidade das Redes Neurais em quantificar a resistência e o módulo de elasticidade de corpos de prova de concreto elaborados a partir das dosagens escolhidas.

**Palavras-chave:** módulo de elasticidade, resistência à compressão, concreto, redes neurais, inteligência artificial.

### Introduction

In recent years, there has been a significant development of new computer technologies and information based on artificial intelligence (Bender, 1996). This technology has given a significant contribution to the solution of a great number of problems in various areas of human knowledge. Engineers always face formulations or inconsistent data that does not fully describe the involved phenomenon.

For many practical situations, the mathematical formulation may become difficult to be conceived, especially when the functional relationships between dependent and independent variables are poorly understood, meaning that they are subjected to uncertainties.

Artificial neural networks can be understood as a powerful family of parallel architectures which can

solve problems through the interconnection of simple computational elements, also defined as artificial neurons, arranged in layers, similar to neurons of the human brain.

Neural networks have the ability to obtain the solution to a problem based on data from previous experiences, and continue to adapt according to changing circumstances or changes of variables in the training data. This attribute allows them to be applied to solve new problems, different from those used for training, and to produce accurate and valid results.

The data used for training the network may be theoretical, experimental, empirical, or even a combination of them. During the training step, even if some incomplete or inconsistent data exist, in most cases, the neural network will generalize a solution.

Based on historical applications of neural networks, Rafiq, Bugmann & Easterbrook (2001) mention that neural networks should not be seen as an alternative tool to conventional computational techniques, but as a complement to them. According to Flood & Kartan (1994), neural networks could not be considered as a solution to computational limitations. Nowadays, many systems are being developed with a hybrid combination, using the neural networks as a solution to some computational disabilities.

Concrete is the most common structural material used in civil construction; it can be considered as the combination of Portland cement paste and aggregate. Aggregates are, basically, inert and rigid materials that form a granular skeleton. In a rising evolution, new materials have been incorporated into the concrete, such as additives and additions (e.g.: minerals, chemicals, polymers or fibers). Hence, concrete is a heterogeneous material, and its mechanical behavior is related to non-linearities.

In order to satisfy the technical requirements related to the situation in which the concrete will be used, it is necessary to define some properties related to its fresh and hardened states, and, therefore, it is necessary to use a mix proportion method that provides the adequate proportioning of its components. The mix proportion methods originate diagrams and abacuses based on results of experimental tests.

For the structural engineer, the compressive strength of concrete is the main parameter to be considered (Helene & Terzian, 1993), since other properties, such as modulus of elasticity, permeability and durability may also be related to it. The mix proportion is also responsible for the extremely relevant characteristics of fresh concrete, like its workability.

The compatibility of the optimal characteristics for the fresh and hardened states of the concrete is the aim of systematic research throughout history. Therefore, any change on its components or in proportioning can cause a change in the value of the compressive strength.

The aim of this study is to demonstrate the ability of the neural networks to quantify the strength and the elastic modulus of concrete specimens prepared using different mix proportions.

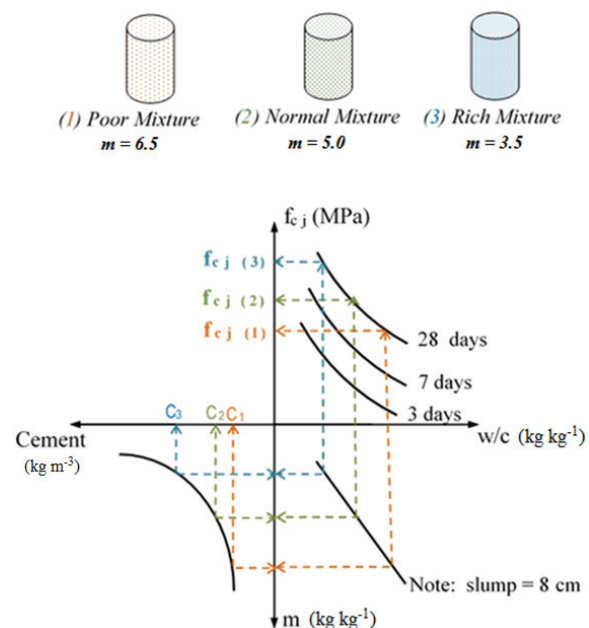
### Method of mixture

The widely mix proportion method for concrete used in Brazil is described in details by Helene & Terzian (1993). This empirical method, which is based on laboratory tests, is sensitive to the

characteristics of the regional materials used in concrete.

After defining the mortar content ( $\alpha$ ), three mixtures of concrete were prepared: the first one with an aggregate/Portland cement ratio (by weight) of 6.5 (this relationship is also named as  $m$ ), which is considered the 'poor' mixture; the second one, with  $m$  equal to 5.0, is considered the 'normal' mixture; and the last one, with  $m$  equal to 3.5, is considered 'rich' mixture due its high Portland cement consumption. Subsequently, laboratory tests were performed in order to obtain the compressive strength for all specimens.

With the values of the compressive strength of each one of the three mixes, four parameters are correlated: compressive strength at  $j$  days of age ( $f_{cj}$ ); water/cement ratio ( $w/c$ ); dry aggregate/cement ratio, given by weight in  $\text{kg kg}^{-1}$  ( $m$ ); and the content of cement per  $\text{m}^3$  ( $C$ ). Figure 1 depicts the parameters that should be used for the empirical method proposed by Helene & Terzian (1993). Using this diagram, it is possible to obtain the mix proportion related to the designed concrete and its compressive strength. It is important to consider that this diagram provides results for concrete mixtures that must have the same mortar content.



**Figure 1.** Example of a diagram of concrete mix proportion specified for a fixed content of dry mortar.

Thus, a satisfactory method to correlate a certain concrete mixture to its compressive strength can be achieved. It is important to highlight that the use of the diagram is restricted to the type of the materials used in the mixture method.

The fundamental laws of the concrete behavior, showed by Helene & Terzian (1993), can be confirmed with the mix proportion diagram results:

$$\text{Abrams' law: } f_{cj} = K_1 / (K_2)^{a/c}; \quad (1)$$

$$\text{Lyse's law: } m = K_3 + K_4 \cdot (a/c); \quad (2)$$

$$\text{Molinari's law: } C = 1000 / (K_5 + K_6 \cdot m); \quad (3)$$

$$\text{Mortar content: } \alpha = (1 + a) / (1 + m) \quad (4)$$

$$\text{Cement content } m^{-3}: C = \gamma / (1 + a + p + a/c) \quad (5)$$

$$\text{Aggregate-cement ratio, given by weight (kg kg}^{-1}\text{): } m = a + p \quad (6)$$

where:

$f_{cj}$ : compressive strength at  $j$  days of age, in MPa;

$a/c$ : water-cement ratio, given by weight (kg kg<sup>-1</sup>);

$a$ : dry fine aggregate-cement ratio, given by weight (kg kg<sup>-1</sup>);

$\alpha$ : dry mortar content (kg kg<sup>-1</sup>);

$p$ : dry coarse aggregate-cement ratio, given by weight (kg kg<sup>-1</sup>);

$K_1, K_2, K_3, K_4$  and  $K_6$ : constants that depend exclusively on materials (cement, aggregates and additives);

$C$ : cement content per cubic meter of concrete (kg m<sup>-3</sup>);

$\gamma$ : specific gravity of concrete (kg m<sup>-3</sup>).

### Defining the neural network

In order to predict the compressive strength of concrete or its modulus of elasticity, a feed-forward neural network with a back-propagation technique based on a gradient descent algorithm in its training phase was used, according to Shwartz & David (2014).

The network works with normalized data. The neural network used in this study, as shown in Figure 2, consists of a number of interconnected processing elements (artificial neurons), arranged in two or more layers, and the interaction between them is based on weights (matrix  $W_{ij}$ ). The weight matrix ( $W_{ij}$ ) determines the influence between the interconnected elements, from one layer to another, that corresponds to the intensity of the signal to be sent.

There is an input layer (the first one), where data is entered on the network, and the output layer (the last one), where the answer to the problem to be solved is stored. Between them, there is a layer (or several ones) which represents the interaction between inputs and outputs, creating a complex structure of relationships and interactions.

Each neuron also has a activation function which defines and sends stimulus values to the neurons of

the subsequent layer. It is noteworthy that the sigmoid function is one of the most used in this process, as highlighted in Figure 2.

Similar to the human brain, the network needs a process of learning (or training) which consists of adjusting the weights contained in the matrix  $W_{ij}$ , thus enabling a proper understanding of the problem to be solved. For this, a set of known inputs with their respective outputs is introduced into the network. An algorithm designed for this task analyzes the difference between the answer generated by the network and the real one. The algorithm performs the adjustment of weights based on the square error of the output neurons and such error is propagated in the opposite direction. Changes in weight between layers are determined using the gradient descent method. It is a systematic procedure for adjusting the matrix of the weights so that the squared error is minimized to an acceptable range. A variable that defines the tolerance for this error is used in the convergence process.

Once trained, the mathematical function of the network provides a simple computational algorithm. So, the network quickly generates responses when input data are provided. This is a significant feature of a neural network. It will be able to reproduce the experiments that were used for its training or generalize to other experiments, allowing satisfactory answers. The precision of the answers depends on factors such as quality of the data introduced during the training, the idealized structure for the network, which involves number of neurons and layers and the desired accuracy, achieved by the control of the variables in the training process.

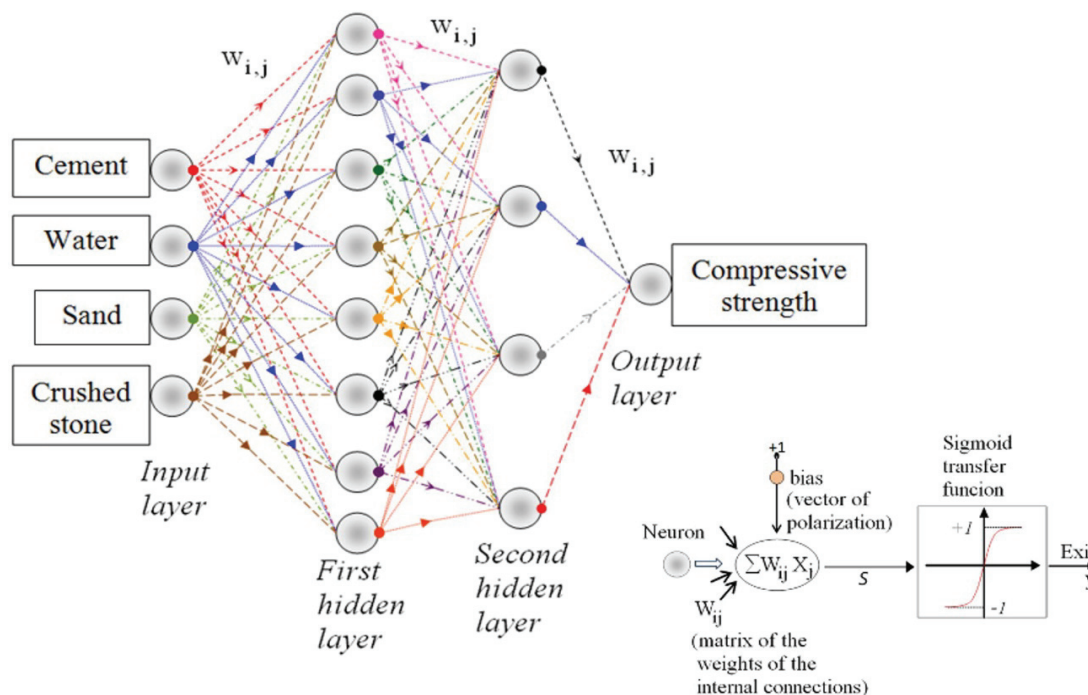
The input layer was fed by the values of the quantities of materials used for the production of one cubic meter of concrete.

It is also shown, in Figure 2, the idealized neural network used to predict the compressive strength of concrete constituted by the following components: cement, sand, crushed stone and water.

These four components form the data of the input layer of the neural network. The first and the second hidden layers had eight (8) and four (4) neurons, respectively.

The output layer had one (1) neuron that stored the value of the compressive strength of the concrete, or the modulus of elasticity, according to the designed neural network training.

The algorithm used in this study was developed in Microsoft Windows Fortran PowerStation, version 4.0, at the Department of Electrical Engineering of the Faculty of Engineering of Ilha Solteira, of the Univ Estadual Paulista-UNESP.



**Figure 2.** Layout of the neural network used, with four layers, and the representation of the function of a neuron.

In order to generalize the solution of the network, different concrete mixtures, with different contents of mortar, were assessed. The specimens were tested in order to determine their compressive strength and their modulus of elasticity. The neural network was trained using part of the available data, and the other part was used to provide its compressive strength and modulus of elasticity.

### Training by sampling

In most applications of neural networks, they are trained with a significant amount of real data, with several variations in the parameters that correlate its components, in order to obtain a wide generalization of the problem analyzed.

In the last decade, researchers such as Yeh (2006) and Noorzai, Hakim, Jaafar & Thanoon (2007) used the potential of neural networks to predict the strength of concrete. They explored, conveniently, the ability of the neural networks in questions of non-linearity, to analyze high performance concretes and others with a more complex composition. They used concrete mixtures from various research centers in Europe and Asia, in a constant exchange of laboratory data. Although the concrete is strictly dependent on the technical characteristics of the regional materials used, the researchers tended to generalize significantly the understanding of concrete over the network.

In this study, local materials were used for the manufacture of concrete. Thus neural network presents better performance and precision, enough to allow its use in everyday life.

For this research, sixty (60) mixtures of usual concrete, made with cement, sand, basaltic crushed stone and water were elaborated, and the corresponding specimens were tested to obtain their compressive strength and modulus of elasticity at 28 days. The content of mortar ranged from 0.49 to 0.59 (step 0.02); the content of aggregates, from 3.5 to 6.5; and the slump test varied between 5 cm and 11 cm. From the total number of concrete mixtures, forty-eight (48) were used to train the neural network and twelve (12) were used to compare the results obtained experimentally with the network outcomes.

### Results and discussion

Table 1 lists the concrete mixtures that were used by the network to predict the values of compressive strength and modulus of elasticity. Table 2 lists the results of compressive strength and modulus of elasticity.

The terms network-1 and network-2 are related to the same type of neural network, differing only by the level of the fixed tolerance. Considering the modulus of elasticity, only the network-1 was able to obtain outcomes, since, due to the level of tolerance used for the network-2, the process did not have convergence during the training step.

Table 3 lists the internal variables set of the networks performances. The experimental results and the predicted ones (made by the network), related to compressive strength and modulus of elasticity, are shown in Figures 3 and 4, respectively.

Table 1. Mixtures analyzed by the networks 1 and 2.

Mix	Cement (kg)	Content of materials per cubic meter of concrete			
		Water (kg)	Coarse aggregate (kg)	Fine aggregate (kg)	Admixture (kg)
T.1	332.25	212.94	964.12	954.12	10.00
T.2	332.35	200.00	1051.90	828.23	10.00
T.3	370.00	202.94	1051.90	828.23	10.00
T.4	335.88	189.41	1070.00	878.23	10.00
T.5	365.88	194.11	955.88	913.53	10.00
T.6	405.29	197.06	1000.00	900.00	10.00
T.7	365.88	182.35	1010.00	1077.62	10.00
T.8	405.29	188.23	1025.90	780.00	10.00
T.9	451.76	200.00	1090.00	634.15	10.00
T.10	451.76	188.23	1090.00	634.15	10.00
T.11	510.00	198.82	1010.00	590.00	10.00
T.12	510.00	194.12	924.12	867.64	10.00

Table 2. Results of the mixtures analyzed by the neural networks 1 and 2.

Mix	Experimental	Compressive strength (MPa)		
		Network 1	Difference (%)	Network 2
T.1	22.28	24.02	7.81	22.97
T.2	23.36	23.27	-0.39	24.30
T.3	29.43	26.38	-10.36	28.43
T.4	29.69	32.02	7.85	29.27
T.5	34.27	33.72	-1.60	33.53
T.6	34.96	36.06	3.15	35.59
T.7	37.94	37.19	-1.98	37.00
T.8	39.16	37.69	-3.75	38.31
T.9	39.52	40.96	3.64	39.97
T.10	42.24	43.88	3.88	42.34
T.11	47.90	48.53	1.32	47.01
T.12	48.15	47.77	-0.79	48.08

Table 3. Configuration of the control parameters of the networks.

Network	Number of training data	Tolerance	Rate Training	Moment	Lambda	Iterations
Network 1	48	0.02	0.70	0.70	0.0001	5288
Network 2	48	0.01	0.70	0.70	0.0001	5288

Both networks 1 and 2 were done with 4, 8, 4 and 1 neurons, in layers 1 to 4, respectively.

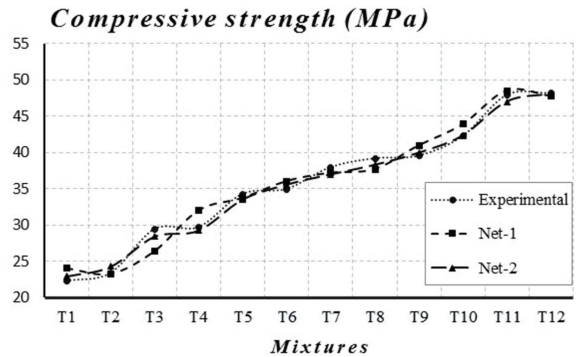


Figure 3. Comparison of the compressive strength values obtained experimentally with those obtained by using neural networks 1 and 2.

The nearest and the most distant outcomes for compressive strength given by the network-1 in comparison with the experimental results, occurred, respectively, for mixtures 2 and 3, and the differences of percentages were, respectively, 0.15 and 4.02%.

The nearest and the most distant outcomes, for modulus of elasticity given only by the network-1 in comparison with the experimental results, occurred, respectively, for mixtures 8 and 11, and the differences of percentages were, respectively, 0.18 and 5.81%.

The networks 1 and 2 had error tolerances adjusted to 0.01 and 0.02, respectively, during the training. In these two applications of the neural network, major differences were observed regarding the amount of interactions, as shown in Table 3.

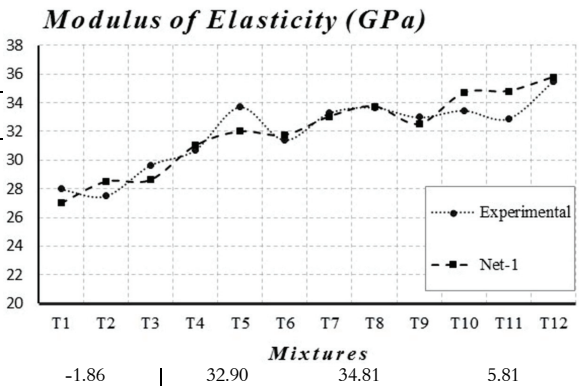


Figure 4. Comparison of the modulus of elasticity values obtained experimentally with those obtained by using neural network 1.

### Conclusion

It is considered that the laws of behavior of the concrete, expressed by the equations 1 to 6 shown in Helene & Terzian (1993), were well assimilated by the neural network, and excellent results were obtained with respect to predicting the concrete compressive strength, based on the characteristics of the materials used.

Considering the modulus of elasticity, an increase in the tolerance value had to be done. Therefore, the network could perform prediction values, which, in turn, were close to those obtained experimentally.

One difficulty found during the elaboration of this study was that, although the experimental procedures had followed the recommendations of the Brazilian standards, some factors could not be completely controlled, such as the identification of the weather parameters related to the elaboration, preparation and testing of the specimens in the laboratory.

Therefore, the main conclusion of this study is that the neural networks, for the specific conditions in what the tests were performed, were able to predict the mechanical behavior of various concrete mixtures, especially considering variations in their

content of mortar ( $\alpha$ ), unlike the diagram method that is specific for only one value of  $\alpha$ .

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