Estimation of heat and mass transfer coefficients in cross-flow sliding bed drying of grains

Oswaldo Curty da Motta Lima

Departamento de Engenharia Química, Universidade Estadual de Maringá, Av. Colombo, 5790, 87020-900, Maringá, Paraná, Brasil. e-mail: oswaldo@deq.uem.br

ABSTRACT. The convective two-phase model for grain drying in a cross-flow sliding bed dryer is used for the analysis and estimation of global volumetric mass transfer coefficient and global interphase heat transfer coefficient at different corn drying experimental conditions. Code MAXIMA, based on maximum likelihood estimation techniques, was used for parameter estimation. An equation, analogous to "Newton's law of cooling", and Nu = *f* (Re,Pr) correlation were used to describe mass transfer rate and global interphase heat transfer coefficient, respectively. There were two distinct objectives for parameter estimation: minimization of average outlet solid moisture deviations and minimization of outlet gas temperature profile. Results allowed good parameter estimation in both cases. Estimated values were better when average solid moisture was chosen as the single objective variable of the estimation process.

Key words: parameter estimation, grain drying, sliding bed, heat and mass transfer.

RESUMO. Estimação dos coeficientes de transferência de calor e massa na secagem de grãos em leito deslizante e fluxos cruzados. A secagem de milho em um secador do tipo leito deslizante e fluxos cruzados é estudada através da utilização do modelo a duas fases, em conjunto com um pacote computacional de estimação paramétrica (MAXIMA), de modo a permitir a estimação do coeficiente volumétrico global (efetivo) de transferência de massa e do coeficiente global de transferência de calor interfases. Dentro desta abordagem, são adotadas e analisadas uma equação análoga à "*lei de resfriamento de Newton*", para a taxa de transferência de massa, e uma correlação do tipo Nu = *f* (Re,Pr), para o coeficiente global de transferência de calor interfases. São estudadas duas situações distintas para as variáveis objetivo da estimação: a umidade média do sólido e, no segundo caso, também o perfil de temperatura do gás, ambos na saída do secador. Os resultados obtidos mostram uma estimação adequada dos parâmetros avaliados nas duas situações estudadas, observando-se uma melhor qualidade dos valores alcançados com a umidade média do sólido como única variável objetivo da estimação.

Palavras-chave: estimação de parâmetros, secagem, leito deslizante, transferência de calor e masssa, modelagem.

Drying is an important operation in food and, more specifically, in grain processing industries. As convective air dryers are often used in most industrial situations, many Brazilian convective drying researches have been developed. Papers like those of Freire (1991), Brunello (1992), Massarani and Silva Telles (1992), Mayta Sarmiento (1994), Calçada (1994), Barrozo (1995), Mancini *et al*. (1995), and others, have analyzed different aspects of convective grain drying.

Within this context, a study of cross-flow sliding bed drying process is carried out in this research. It is assumed that convective heat and mass transfer largely overcome the conductive and diffusive ones (Massarani and Silva Telles, 1992). Therefore, an equation analogous to "Newton's law of cooling" is used to describe the mass transfer rate, while an Nu = *f* (Re,Pr) correlation is adopted to describe global interphase heat transfer coefficient.

Estimation of global volumetric solid-phase mass transfer coefficient and global interphase heat transfer coefficient (more specifically, the Nu correlation parameters) from corn drying experimental data is undertaken by solving a two-phase drying model and using a computational code of parameter estimation, based on the maximum likelihood principle.

Modelling

Squematic Model of the System

The convective cross-flow sliding bed dryer may be schematically represented as shown in Figure 1.

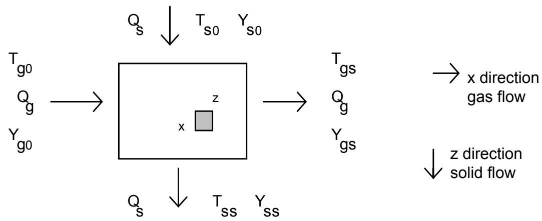

Figure 1. Schematic dryer model

Two-Phase Model

The dryer may be described by the system of partial differential equations presented below (Massarani and Silva Telles, 1992). The main hypotheses and simplifications of the model are:

- solid (dry solid + water) and fluid (dry air +vapor) are ideal mixtures;
- conductive and diffusive heat and mass transfer, heat loss to ambient and radiant heat are discarded;
- constant physical properties for phase components;
- steady-state regime and unidirectional constant plug flow for solid and fluid phases.

Mass Balance

$$G_g \cdot \frac{\partial Y_g}{\partial x} = f \qquad , Y_g(0,z) = Y_{g0} \tag{1}$$

$$G_s \cdot \frac{\partial Y_s}{\partial z} = -f \qquad , Y_S(x,0) = Y_{S0} \tag{2}$$

Energy Balance

$$G_g \cdot (c_{pg} + c_{pv}.Y_g)\frac{\partial T_g}{\partial x} = -[(h.a) + c_{pv}.f] \cdot (T_g - T_s)$$

and $T_g(0,z) = T_{g0}$ (3)

$$G_s \cdot (c_{ps} + c_{pl}.Y_s)\frac{\partial T_s}{\partial z} = (h.a) \cdot (T_g - T_s) - (f \cdot \lambda)$$

and $T_S(x,0) = T_{S0}$ (4)

The drying rate is modeled by an equation analogous to "Newton's law of cooling", which states that grain mass transfer resistance is basically superficial. It is assumed that the drying rate is proportional to the difference between the solid moisture and the equilibrium humidity (with gas). Global volumetric mass transfer coefficient (K_S.a) is the proportionality factor.

Equation may be expressed as:

$$f = (K_S.a).[Y_S - Y_{Se}(T_g, Y_g)] \,. \tag{5}$$

Henderson's isotherm (Henderson, 1952) was used to calculate the equilibrium humidity $Y_{Se}(T_g,Y_g)$, as recommended by Mayta Sarmiento (1994).

A Nu = f (Re,Pr) dimensionless correlation, as proposed by Mayta Sarmiento (1994), was employed for the calculation and parametric analysis of global interphase heat transfer coefficient (h).

Solution of the equation system

To solve the convective two-phase model, Mayta Sarmiento (1994) discretized the system of partial differential equations which describe the solid phase through a modified implicit method. The right-hand function is calculated with trapezoidal approximation of the variables between points ***i*** and ***i-1***, allowing the equations to be satisfied simultaneously. Gas equations are then integrated numerically with the code LSODAR, which uses a backward-differentiation-formula (BDF) method to integrate stiff systems of ordinary equations. The resulting system becomes:

$$\frac{d\,Y_{g_i}}{d\,x} = \frac{(K_s.a)}{G_g} \cdot (Y_{g_i} - Y_{se_i}) \tag{6}$$

$$\frac{dT_{g_i}}{dx} = -\frac{[(h.a) + (c_{pv})(K_s.a)(Y_{s_i} - Y_{se_i})](T_{g_i} - T_{s_i})}{G_g(c_{pg})} \tag{7}$$

$$Y_{s_i} = \frac{Y_{s_{i-1}}\{1 - [(K_s.a)\Delta z / 2G_s]\} + [Y_{se_m}(K_s.a)\Delta z / G_s]}{1 + [(K_s.a)\Delta z / 2G_s]} \tag{8}$$

$$T_{s_i} = \frac{T_{s_{i-1}}[G_s(c_{ps} + c_{pl}Y_{s_m}) - (h.a)(\Delta z/2)] + (h.a)(T_{g_m})\Delta z - \lambda(K_s.a)\Delta z[Y_{s_m} - Y_{se_m})}{G_s(c_{ps} + c_{pl}Y_{s_m}) + (h.a)(\Delta z/2)} \tag{9}$$

where :

$Y_{gi}(0) = Y_{g0}$; $T_{gi}(0) = T_{g0}$;

$Y_{Si}(0) = Y_{S0}$; $T_{Si}(0) = T_{S0}$; (10)

I = 1,...,N ;

and the following restriction: $c_{pv}.Y_g << c_{pg}$.

Results and discussion

The maximum likelihood principle was used to estimate model parameters. It was assumed that the experimental errors were independent and followed normal distribution.

Based on this method, code MAXIMA (Pinto *et al.*, 1993) was used for the estimation of the following parameters:

$(K_S.a)$ - global volumetric mass transfer coefficient (solid basis);

α and β - defined by:

$$Nu = \frac{h.D_p}{k_g} = \alpha . Re^{\beta} . Pr^{1/3} \quad (11)$$

$$Re = \frac{G_g . D_p}{\mu} \; , \; Pr = \frac{\mu . c_{pg}}{k_g} \quad (12, 13)$$

The corn drying experimental data used for the estimation of parameters $(K_S.a)$, α and β were obtained by Mayta Sarmiento (1994) and listed in Tables 1 and 2.

Table 1. Inlet variables

Experiment	1	2	3	4	5	6	7	σ^2
T_{g0}	44	71.3	72.5	43.5	57	58	57.1	0.25
T_{air}	24	23	23.5	23.5	23.7	22.3	23	0.25
T_w	21.2	19.3	19.8	20.6	21.2	19.5	20	0.25
DHPIT	8.3	43.9	8.3	44	26.2	26.1	26.1	0.01
T_{S0}	25	28	25.3	24.5	28.3	26.2	27	0.25
ys_0	0.205	0.193	0.170	0.225	0.237	0.215	0.230	$3x10^{-5}$
Q_S	0.0098	0.0126	0.0169	0.0162	0.0140	0.0119	0.0125	$1x10^{-6}$
$Y_{Ss}exp$	0.208	0.171	0.167	0.238	0.235	0.195	0.215	Table 2

Table 2. Outlet variables

Experiment	1	2	3	4	5	6	7	σ^2
$Y_{Ss}exp$	0.208	0.171	0.167	0.238	0.235	0.195	0.215	
$\sigma^2(Y_{Ss}exp)$	$3.4x10^{-5}$	$2.1x10^{-5}$	$2.1x10^{-4}$	$3.2x10^{-5}$	$5.0x10^{-5}$	$1.1x10^{-5}$	$2.5x10^{-5}$	
$T_{gs}(z8)$	26	46.2	29.1	25.5	28	31	28	0.25
$T_{gs}(z16)$	26.1	57.3	31.3	28.3	29	39.4	32.5	0.25
$T_{gs}(z24)$	26.6	59.8	33.6	31	33	45.5	41.5	0.25
$T_{gs}(z32)$	27.2	60.3	39.3	32.3	37.5	46.1	42.5	0.25

OBS.: - [T] ºC , [DHPIT] ≡ cm , [QS] ≡ kg/s;
- T_{gs} (z8, z16, z24, z32) → outlet drying gas temperature in 4 points in the dryer outlet (z = 8, 16, 24, 32 cm), for a total bed height of 40 cm

Initial estimate of $(K_S.a)$, 0.35 kg/m^3s, was the value used by Mayta Sarmiento (1994). The α and β initial values, 1.31 and 0.59, are those reported by Sartori (Freire, 1991).

Parameter estimation results

Outlet variable (objective variable): YM_{Ss}

Results of estimation of parameters $(K_S.a)$, α and β are presented below (Tables 3 and 4), for the following cases:

Table 3. Error-free inlet (independent) variables , $\sigma^2_{x_{exp}} = 0$

$(K_s.a)$ [kg/m³s]	0.33 ± 0.01	correlation $(K_s.a)$ / α	52.21%
α	1.26 ± 0.03	correlation $(K_s.a)$ / β	33.94%
β	0.593 ± 0.006	correlation α / β	8.33%

YM_{Ss}	$\sigma_{YM_{Ss}} = 0.003$	$\sigma^2_{YM_{Ss}} = 6.67 \times 10^{-6}$

Table 4. Uncertain inlet (independent) variables , $\sigma^2_{x_{exp}} \neq 0$

$(K_s.a)$ [kg/m³s]	0.33 ± 0.01	correlation $(K_s.a)$ / α	-2.42%
α	1.0 ± 0.1	correlation $(K_s.a)$ / β	33.20%
β	0.62 ± 0.05	correlation α / β	-54.90%

YM_{Ss}	$\sigma_{YM_{Ss}} = 0.002$	$\sigma^2_{YM_{Ss}} = 2.63 \times 10^{-6}$

Outlet variables (objective variables): YM_{Ss} and T_{gs}

Results of estimation of parameters $(K_S.a)$, α and β are presented below (Tables 5 and 6), for the following cases:

Table 5. Error-free inlet (independent) variables , $\sigma^2_{x_{exp}} = 0$

$(K_s.a)$ [kg/m³s]	0.42 ± 0.07	correlation $(K_s.a)$ / α	45.75%
α	0.7 ± 0.3	correlation $(K_s.a)$ / β	-27.57%
β	0.6 ± 0.1	correlation α / β	-80.85% *

* very correlated parameters

YM_{Ss}	$\sigma_{YM_{Ss}} = 0.01$	$\sigma^2_{YM_{Ss}} = 1.36 \times 10^{-4}$
T_{gs}	$\sigma_{T_{gs}} = 2.5ºC$	$\sigma^2_{T_{gs}} = 6.18\ ºC^2$

Table 6. Uncertain inlet (independent) variables , $\sigma^2_{x_{exp}} \neq 0$

$(K_s.a)$ [kg/m³.s]	0.40 ± 0.07	correlation $(K_s.a)$ / α	15.49%
α	1.6 ± 0.6	correlation $(K_s.a)$ / β	22.46%
β	0.5 ± 0.1	correlation α / β	-69.76% *

* correlated parameters

YM_{Ss}	$\sigma_{YM_{Ss}} = 0.003$	$\sigma^2_{YM_{Ss}} = 6.67 \times 10^{-6}$
T_{gs}	$\sigma_{T_{gs}} = 1.2ºC$	$\sigma^2_{T_{gs}} = 1.39ºC^2$

Result analysis

Objective variable : YM_{Ss}

Results of parameter estimates (K_s.a), α and β, and those of the objective variable YM_{Ss} , were very good. Best results in parameter estimates were obtained with the error-free independent variables estimation, while those of YM_{Ss} predictions were obtained with the uncertain independent variables estimation.

Inclusion of error in the independent variables improved the YM_{Ss} estimation ($\sigma^2_{YM_{Ss}}$ reduction: $\sim$ 1.3 $\rightarrow$ 0.8 %), allowing a better fit of the dependent variables and more precision in objective variable (YM_{Ss}) calculations. On the other hand, this improvement was accomplished by a reduction of parameter estimation quality, mainly for α and β. However, even for α and β, the estimation could be considered of good quality, as the greater uncertainty was smaller than 10 % (σ_α).

It may be also verified that, in both cases, the correlation indexes between parameters were relatively small.

Parameters α and β obtained in both cases are very close to values presented by Sartori (Freire, 1991). Thus, Sartori's correlation could be safely used to calculate Nu numbers, as suggested by Mayta Sarmiento (1994).

Objective variables : YM_{Ss} and T_{gs}

The need to adjust both T_{gs} and YM_{Ss} led in this case to a worse estimation of parameters (K_s.a), α and β, mainly the last two, because they are directly related to the calculation of the dryer's temperature profile.

The worse quality of estimation of parameters (K_s.a), α and β (uncertainties around 10 to 40 %) and objective variables YM_{Ss} and T_{gs} is probably related to the low reliability of the experimental data obtained for the outlet drying gas temperature profile (T_{gs}).

The estimated results of YM_{Ss} and T_{gs} , in the error-free independent variables situation, were good. The T_{gs} uncertainties (σ_{Tgs}) were the main problem, so that admitting 0.5 °C as an adequate error for temperature measurements and accepting, for temperature profile estimative, up to 1°C, an average uncertainty of $\sim$ 2.5°C is obtained.

Error in independent variables led to better results for estimated values of YM_{Ss} and T_{gs} (e.g.: $\sim$ 53% better, for T_{gs}) with bad results in parameter estimation. This situation repeats the above behavior in the estimation of YM_{Ss} only, with uncertainties significantly higher than those obtained in that case ($\sim$ 4.5 to 6 bigger).

The T_{gs} profile determination also increased the correlation degree between parameters α and β, contributing to the loss in estimation quality.

The main conclusions of this work are summarized in the next paragraphs.

Results of parameter estimates (K_s.a), α and β, in the determination of YM_{Ss} only, were very good. Parameter (K_s.a), with values between 0.319 and 0.344 kg/m^{3}s (mean value of 0.333 kg/m^{3}s), may be considered *constant* for the experiments and their operational conditions. Results of (K_s.a) agree perfectly with those of Mayta Sarmiento (1994).

However, in the YM_{Ss} and T_{gs} profile determination, there was a reduction of quality of both parameters estimation and the estimated values of the objective variables YM_{Ss} and T_{gs}. To improve the estimation performance, we could:

- use the code ESTIMA (Pinto, 1993 and Pinto *et al.*, 1993) for planning new experiments;
- increase the reliability of the experimental data involved in parameter estimation work, more specifically those related to temperature profiles.
- Finally, we strongly recommend not to include temperature profiles in the estimation proceeding. This could be done in the case of getting a rigid control of the process energy loss, the main cause of the reduction in estimation quality, when these profiles are involved.

Notation

A	- specific area	[1/L]
c_p	- constant pressure specific heat	[E/(M.$\Delta\theta$]
d.b.	- dry basis	
DHPIT	- Pitot tube height delta (gas flow measurement)	[L]
D_p	- diameter of the sphere with same volume as particle	[L]
exp	- experimental	
f	- drying rate (volumetric basis)	[M/(L^{3}T)]
G	- mass velocity	[M/(L^{2}T)]
h	- interphases heat transfer coefficient	[E/(L^{2}T$\Delta\theta$)]
k_g	- gas phase thermal conductivity	[E/(LT$\Delta\theta$)]
K_s	- mass transfer global coefficient (solid phase basis)	[M/(L^{2}T)]
N	- number of discretization points	
Nu	- Nusselt number	[-]
Pr	- Prandtl number	[-]

Q	-	mass flow	[M/T]
Re	-	Reynolds number	[-]
T	-	temperature	[θ]
x	-	space variable	[L]
w.b.	-	wet basis	
Y	-	space variable	[L]
ys	-	solid water mass fraction, (w.b.)	[-]
Y	-	water mass ratio , (d.b.)	[-]
YM	-	water mass ratio - mean, (d.b.)	[-]
z	-	space variable	[L]

Greek letters

α, β	-	parameters of equation (11) (Nu and h determination)	[-]
λ	-	water boiling heat in the solid phase temperature	[E/M]
μ	-	viscosity	[M/(LT)]
σ	-	standard deviation	[= variable]
σ^2	-	variance	[= variable^2]

Subscripts

air	-	drying air
e	-	equilibrium with gas condition
g	-	gas
i	-	discretization index
l	-	liquid water
S	-	solid
v	-	boiling water
w	-	wet bulb air temperature
0	-	inlet dryer conditions/phase alimentation
s	-	outlet dryer conditions/phase exhaustion

References

BARROZO, M.A.S *Transferência de calor e massa entre o ar e sementes de soja em leito deslizante e escoamentos cruzados.* 1995. Tese (Doutorado em Engenharia Química) - Universidade Federal de São Carlos, São Carlos, 1995.

BRUNELLO, G. Secagem no DEQ-EPUSP. *RBE-Caderno de Engenharia Química*, Rio de Janeiro, n. 4 (especial), p. 25, 39, 81 & 115, 1992.

CALÇADA, L. A. *Modelagem e simulação de secadores de leito fixo*. Dissertação (Mestrado em Engenharia Química) - COPPE, Universidade Federal do Rio de Janeiro, Rio de Janeiro, 1994.

FREIRE, J.T. Transferência de calor em sistemas particulados, *RBE-Caderno de Engenharia Química*, Rio de Janeiro, n. 3 (Especial), 1991.

HENDERSON, S.M. A basic concept of equilibrium moisture. *Agric. Eng.*, St. Joseph, p. 29-32, 1952.

MANCINI, M.C. *et al*. Transferência de massa na secagem de grãos em camada delgada. In: ENCONTRO NACIONAL DE ESCOAMENTO EM MEIOS POROSOS (ENEMP), 22, Florianópolis, 1994. *Anais…* Florianópolis: DEQ/UFSC, v. 2, p. 769-786, 1995.

MASSARANI, G.; SILVA TELLES, A. Aspectos da secagem de material sólido particulado. In: FREIRE, J.T.; SARTORI, D.J.M. (Ed.). *Tópicos especiais em secagem*. São Carlos: Universidade Federal de São Carlos, v. 1, p. 1-39, 1992.

MAYTA SARMIENTO, M.A. *Estudo do coeficiente de transferência de massa na secagem de grãos em leito deslizante e fluxos cruzados.* 1994. Dissertação (Mestrado em Engenharia Química) - COPPE, Universidade Federal do Rio de Janeiro, Rio de Janeiro, 1994.

PINTO, J.C. *Estimação de parâmetros e planejamento de experimentos*. Internal Publication, Rio de Janeiro: PEQ/ /COPPE-UFRJ, 1993.

PINTO, J.C. *et al*. *ESTIMA: Um pacote computacional para estimação de parâmetros e projeto de experimentos*. User's Manual. Rio de Janeiro: PEQ/COPPE-UFRJ, 1993.

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