



Analysis and design of reinforced concrete deep beams using the stress fields method

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ABSTRACT. Bernoulli's hypothesis facilitates the understanding of some structural elements. However, this hypothesis cannot be applied for some special regions, denominated 'D regions' or 'Discontinuity regions'. Deep beams are usually classified into this category and, for this reason, the Brazilian structural code (Associação Brasileira de Normas Técnicas [ABNT], 2014) proposes the design of these elements using Strut-and-Tie Method (STM), which is based on forces acting in hypothetical truss models. Different from the Strut-and-Tie Method (STM), the Stress Field Method (SFM) is based on the knowledge of the stresses acting inside of a structure. Both methods have the same purpose, identify the load carrying mechanisms inside of a complex region. In order to evaluate the potential of the so-mentioned methods, the Jconc software package, which is based on the Non-Linear Elastic-Plastic Stress Fields (NL-EPSF), was applied for the design and analysis of some deep beams. Based on some validations and comparisons, it was possible to propose a solution based on the SFM for the analysis, design, and detailing of deep beams. Finally, the parameters established by the ABNT (2014) for the deep beams were evaluated based on the simulations.

Keywords: stress field method, Strut-and-Tie Method, Jconc, D regions, deep beams.

Análise e dimensionamento de vigas-parede de concreto armado utilizando o método dos campos de tensão

RESUMO. A hipótese simplificadora de Bernoulli facilita o entendimento de alguns elementos estruturais, entretanto, essa hipótese não pode ser aplicada para algumas regiões especiais, denominadas 'Regiões D' ou 'Regiões descontínuas'. As vigas-paredes são classificadas dentro dessa categoria, com isso, a Norma Brasileira de Projeto de estruturas de concreto (ABNT, 2014) propõe o dimensionamento desses elementos pelo método das Bielas (MB), que é baseado nas forças que agem em modelos hipotéticos de treliça. Diferente do método das Bielas, o método dos Campos de Tensão (MCT) é baseado no conhecimento das tensões que atuam nos elementos estruturais. Ambos os métodos têm o mesmo objetivo que é conhecer o caminhamento dos carregamentos dentro de uma região especial. De maneira a avaliar a potencialidade do MCT, o programa Jconc, que é baseado nos Campos de Tensão Elasto-Plásticos Não Linear (CTEP-NL), foi aplicado para o dimensionamento de algumas vigas-paredes. Baseado em algumas validações e comparações foi possível propor uma solução baseada no MCT para análise, dimensionamento e detalhamento de vigas-parede. Por último os parâmetros estabelecidos para vigas-parede pela ABNT (2014) foram avaliados com base nessas simulações.

Palavras-chave: método dos campos de tensão, método das Bielas, Jconc, regiões D, vigas-parede.

Introduction

The Strut-and-Tie Method (STM) was mainly developed at the University of Stuttgart, taking into account that a complex region in structural concrete may be idealized as a simple truss, where compressed elements are denominated struts and tensile elements are named ties. In this method, concrete is designed for the forces that act in the struts while reinforcement is designed for the forces that act in the ties, always providing good detail for the development of the reinforcement and adequate

level of ductility for the nodal regions. The design of discontinuous regions (D regions) using STM is recommended by the Brazilian structural code (ABNT, 2014) and also by some international codes as Comité Euro-International Du Béton (1993), Canadian Standards Association (CSA, 1994), EHE (2008) and ACI Committee 318 (2014).

Based on the Theory of Plasticity, researchers from Zürich and Copenhagen Universities have developed an alternative method for concrete structures subjected to discontinuities, named Stress

Fields Method (SFM). Among the pioneer papers published in this field, the following papers should be highlighted: Drucker (1961), Thürlimann, Grob, and Lüchinger (1975), Nielsen, Brastrup, Jensen, and Bach (1978) and Marti (1980). Nowadays, it is worth to mention the papers published by Muttoni, Schwartz, and Thürlimann (1997), Muttoni and Ruiz (2007), Kostic (2006) and Muttoni, Kostic, and Ruiz (2011). The Swiss structural code (Swiss Society of Engineers and Architects, 2003) recommends the SFM for the design of special regions in structural concrete (D Regions).

In conceptual terms, the biggest difference between STM and SFM is the magnitude investigated in each one of them. STM is based on finding the forces acting in each element of the idealized truss whereas SFM is based on finding the stresses acting in discrete area elements. Muttoni and Ruiz (2007) and Souza (2008) have shown that the addition of minimum web reinforcement, which is usually mandatory in deep beams by structural codes, may cause a great distortion of the idealized truss using STM. This issue can be better addressed through the use of Non-Linear Elastic-Plastic Stress Fields (NL-EPSF).

In this way, the NL-EPSF allows a more generic approach adjusted to the practical requirements needed to the concrete structure reinforcement, since it can take into account the stress flow considering the minimum web reinforcement. Figure 1 shows a complex deep beam analyzed using the Jconc software package (based on Elastic-Plastic Stress Fields).

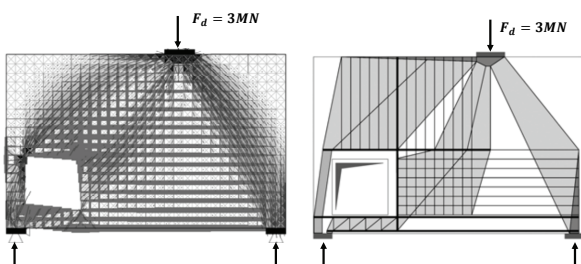


Figure 1. Stress fields for deep beam with hole using the Jconc software package. Source: Muttoni and Ruiz (2007).

Materials and methods

Deep beam definition

In order to compare the performance of the STM in regarding to the SFM, the deep beam is shown in Figure 2 is investigated. The element's dimensions were proposed to meet the requirements of the item 22.4.1 from the Brazilian Code (ABNT, 2014), i.e., the ratio between the span

(l) and the height (h) of the deep beam is maintained under two. The point load P_k is defined as the characteristic total load considering dead and live loads.

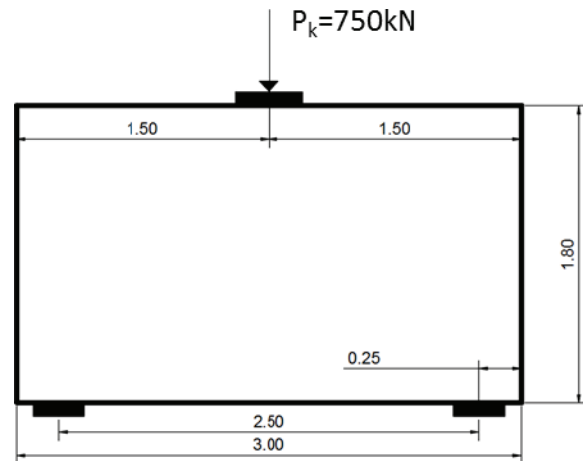


Figure 2. Investigated deep beam.

The design was conducted considering 30 mm for concrete cover and width (b) of 250 mm for the cross section. Concrete characteristic compressive strength (f_{ck}) was selected to be 30 MPa and rebars were chosen to be CA50 ($f_{yk} = 500$ MPa for the main reinforcement) and CA60 ($f_{yk} = 600$ MPa for the web reinforcement). Figure 3 shows the elastic stress fields for tension and compression obtained from the Jconc software package.

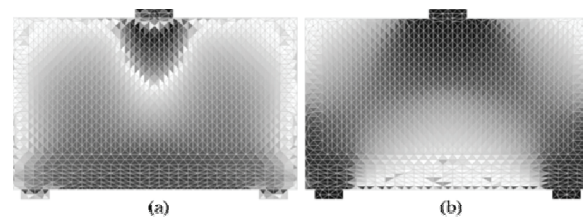


Figure 3. Elastic stress fields for (a) tension and (b) compression.

The approach solution

The Jconc software package was used to develop the elastic stress fields and then to generate a proposal of strut-and-tie model. By the other hand, also using the Jconc software package the NL-EPSF was applied in order to design the deep beam only considering the minimum reinforcement. According to Muttoni and Ruiz (2007), Jconc can be defined as a computational tool that generates the stress fields by means of the Finite Element Method, based on the main assumptions of EPSF. Another situation contemplated in this tool is the nonlinear behavior of structure, i.e., the stress and strain are updated as each increase is applied and concatenated in secant

stiffness matrix, based on the constitutive law of each material, using the Newton-Raphson complete algorithm.

The physical nonlinearity of the concrete in Jconc program is based on a small part of the Modified Compression Field Theory (MCFT), proposed by Vecchio and Collins (1986). The assumption takes into account the softening of the concrete struts due to the presence of transversal stresses which is often caused by ties crossing the struts. Therefore, it is possible to conduct a nonlinear analysis through the use of only three material parameters (yielding stress, Poisson's ratio and modulus of elasticity), making this method consistent with structures designer approaches.

Equation 1 takes into account the plastic behavior of the concrete according to Muttoni et al. (1997) as well as the interference of the ties crossing struts based on the work of Vecchio and Collins (1986). The value of $\eta(\epsilon_j)$ in Equation 2 is proposed by Vecchio and Collins (2006) and is mentioned in the Jconc software package as Eta2.

$$f_{cp} = f'_c \cdot \eta(\epsilon_j) \quad (1)$$

$$(\epsilon_j) = \frac{1}{0.8 + 170 \cdot \epsilon_j} \leq 1,0 \quad (2)$$

The steel behavior according to Muttoni and Ruiz (2007) can be modeled by an uniaxial relationship for stress and strain. The steel response is based on the strength yielding stress (f_y) and the modulus of elasticity (E_s). Further, the constitutive model can be increased to take into account the hardening of the steel as shown in Figure 4.

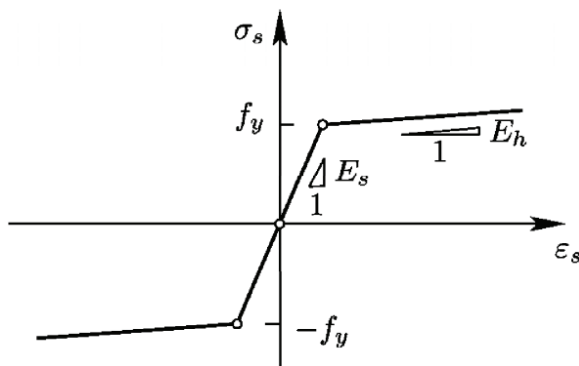


Figure 4. Elastic-plastic behavior of the steel rebar with hardening. Source: Muttoni and Ruiz (2007).

Results

Strut-and-Tie Model

Based in Figure 3 it is possible to propose a strut-and-tie model in order to design the presented deep

beam. Figure 5 is a superposition of the proposed strut-and-tie model with regarding to the relative linear elastic stresses obtained using the Jconc software package.

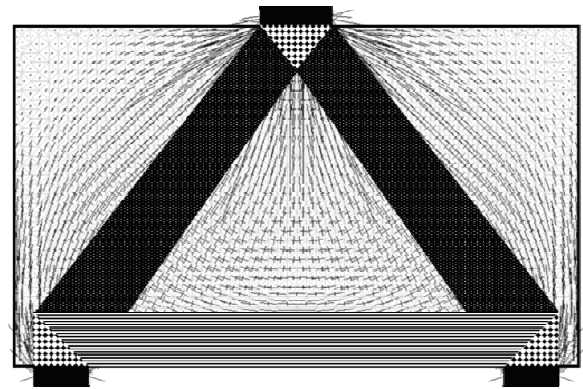


Figure 5. Strut-and-tie model overlapped to the elastic stress fields.

The designs and verifications were based on the item 22.3 from the ABNT (2014). The ties were designed in order to reach the yielding design stress in the ultimate (ULS – Ultimate Limit State) before the failure of struts and nodes (ductile behavior). Therefore, it is possible to design and detail the reinforcement of the structure under study, adding the minimum web established on the item 22.4.4.3 from ABNT (2014). Figure 6 shows the reinforcement detailed for the studied deep beam according to the Strut-and-Tie Method.

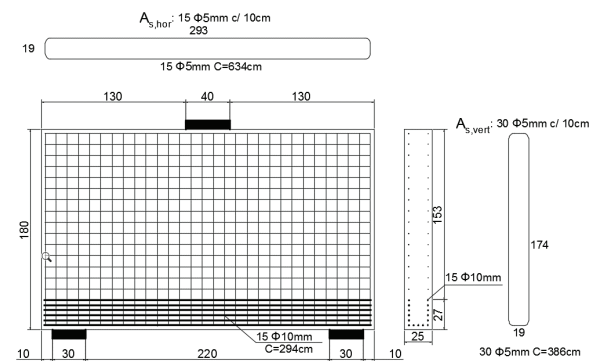


Figure 6. Detailing of the proposed deep beam according to the STM.

Simple beam model

The proposed deep beam was also investigated as a simple beam, despite the fact that element is clearly a D-Region as a whole. For the longitudinal reinforcement, simple beam model equations were used and for the transverse reinforcement, Model I recommended by ABNT (2014) was selected.

Figure 7 shows the deep beam detailing according to the beam approach.

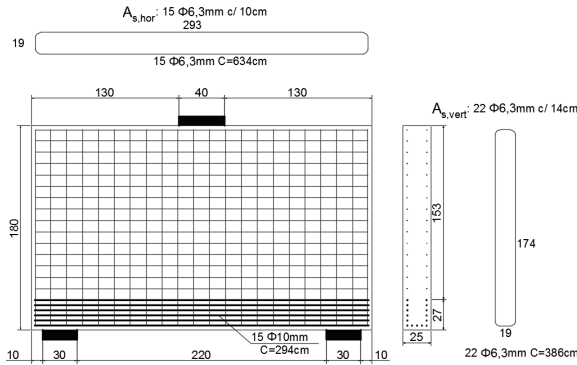


Figure 7. Detailing of the proposed deep beam according to the classic beam model.

Non-Linear Elastic-Plastic Stress Fields (NL-EPSF)

The objective of this topic is to present the procedures used to design the proposed deep beam according to the SFM using the Jconc software package. As the minimum web reinforcement is mandatory in structural codes in order to improve the concrete cracking behavior, the proposed deep beam was also investigated considering only this type of reinforcement. Based on this model it was possible to estimate the real steel area needed in the tensile region of the deep beam.

According to Muttoni, Schawartz, and Thurlimann (2006), reinforced concrete structures are internally hyperstatic and the internal forces acting on each element directly depends on the rigidity of it. In other words, analyzing two similar structures with the same load level, the one with more reinforcement bars will demand more internal forces.

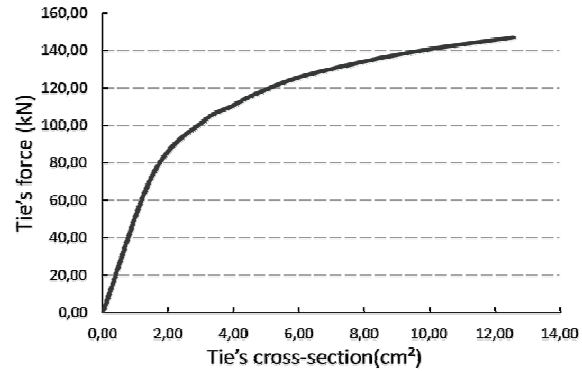
Therefore, to design the main tie of the studied deep beam it is necessary to find out the variation of the tensile force as the reinforcement tie area is increased. In order to start this investigation, the point load acting in the deep beam was increased until 504.35 kN, a load level before the yielding of the minimum web reinforcement and related to the service limit state (SLS). This load level was sustained and the main tie reinforcement area was increased in order to analyze the variation in the tie force as shown in Figure 8. As it can be seen, the relationship between the main tie area and force is non-linear.

Based on the fact that a ductile behavior is desired for the deep beam in the ultimate limit state (ULS), the yielding of the reinforcement must be reached before the occurrence of the concrete failure. Assuming this fact, the design yielding (f_{yd})

of the main tie must be reached when the design load of the deep beam (P_d) is attained. In this way, using a simple linear relationship, it is possible to estimate the force that the tie ($F_{tie1260kN} = F_{sd}$) would absorb (Equation 3):

$$\frac{F_{tie1260kN}}{P_d = 1260} = \frac{F_{tie504,35 kN}}{504.35} \rightarrow F_{sd} = F_{tie1260kN} \tag{3}$$

$$= \frac{1260 * F_{tie504.35 kN}}{504.35}$$



Applied load (kN)	Tie's force (kN)	Tie's cross-section (cm ²)
504.35	75.00	1.57
504.35	103.00	3.14
504.35	110.00	3.93
504.35	117.00	4.71
504.35	127.00	6.28
504.35	139.00	9.42
504.35	147.00	12.57

Figure 8. As it can be seen, the relationship between the main tie area and force is non-linear.

Equation 4, which is a linear relationship to estimate the main reinforcement area of a simple strut-and-tie model according to the ABNT (2014) is then combined with Equation 3 in order to obtain Equation 5:

$$A_s = \frac{F_{sd}}{f_{yd}} \tag{4}$$

$$A_s = \frac{1260 * F_{tir504,35 kN}}{504.35 * f_{yd}} = 2.489 * \frac{F_{tir504.35 kN}}{f_{yd}} \tag{5}$$

Consequently, the ideal value for the tie area will be the meeting point between the nonlinear relationship of Figure 8 and the linear relationship of the Equation 5, As it can be seen in Figure 9. In other words, the dashed line may be assumed as the loading condition of the main tie while the filled line may be understood as the strength of the main tie of the deep beam. If the curves do not touch each other the tensile force absorbed by the tie will not be realistic, neither the main reinforcement tie area.

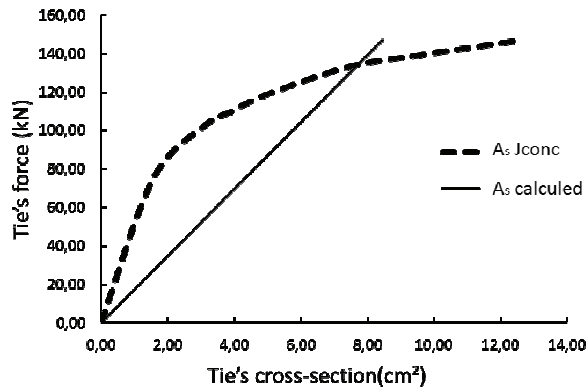


Figure 9. Tie force versus reinforcement tie area according to Jconc and ABNT (2014).

Therefore, according to the NL-EPSF numeric model, the steel area in the tie's region must be 7.73 cm² (10 ϕ 10 mm). In order to verify the ULS, the main tie of the deep beam was updated to the previous amount area and the minimum web reinforcement was kept with the same values. Therefore, the deep beam was re-analyzed for the design load of P_d = 1260 kN considering the design values for the materials and the stress in the main tie was monitored.

If the stress in the main tie is immediately under f_{yd} = 434.78 MPa, then the selected reinforcement area for the main tie will satisfy the ULS, observing that concrete stresses must be maintained under the prescribed limits recommended by the codes (condition for ductile behavior).

Once this fact was not verified, the main tie reinforcement was again increased. As a result, the arrange of the bars was changed from 10 ϕ 10 mm to 11 ϕ 10 mm, as shown in Figure 10. The simulation after this reinforcement update shows that the solution is able to attend the ULS. As it can be seen, the proposed methodology is a shortcut to design the main reinforcement of deep beams using the NL-EPSF. However, an incremental procedure for the main reinforcement area and for the point load could also be used to obtain the same reinforcement.

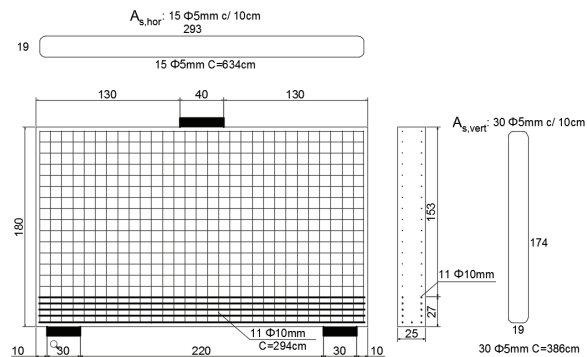


Figure 10. Detailing of the proposed deep beam according to SFM.

Discussion

In order to compare the methods and the influence of the minimum web reinforcement on the behavior of the deep beams, the performance of the presented models were compared. The Jconc software package was applied to investigate the proposed deep beam reinforced only with the main ties (STM approach) and also reinforced with the main ties plus the minimum web reinforcement (STM and SFM approaches), as shown in Table 1.

Table 1. Results of struts in each model for a design load of 1260 kN.

Model	Strain (‰)	Strut Stress (MPa)	Strength Stress (MPa)	Eta2
STM (tie)	-2,37	-14,53	14,61	0,487
STM (web + tie)	-0,83	-30,00	30,00	1,000
SFM (web + tie)	-1,49	-29,98	30,00	1,000

Table 1 offers some results comparison of the three investigated models regarding to the struts. It is possible to notice that Eta2 is different from unity in the model that does not have the minimum web reinforcement. In this case, it is possible to identify that the presence of minimum web improves considerably the behavior of the struts under compressive strength.

Figure 11 shows results of relative stresses extracted from Jconc. The relative stress is presented in grayscale and as it becomes darker, closer is the solicitation stress to the strength stress. Although the reinforcement will not be reviewed in this topic, it is important to mention that the thickness of horizontal lines is proportional to the tie stress.

Figure 12 shows the same relative stresses obtained using Jconc, however, the results are presented for an applied point loads that induce the concrete failure. In the middle of each model it is shown the load level of each model. In Figure 11 as well as in Figure 12, it is possible to observe the formation of bottle struts which have the minimum web, while prismatic struts are shown in the model without the minimum web reinforcement. In other words, concrete struts are penalized by the absence of the minimum web reinforcement and this fact may be proved by the appearance of dark elements in Figure 12a, which are greater than Figure 12b and c.

Table 2 shows that the model without minimum web reinforcement has Eta2 inferior to one, consequently the strength is smaller in this model than the others. Also, it is possible to check that the load capacity of the deep beams designed by the same method (STM), is increased from 1347.10 to 2000 kN when the minimum web reinforcement is added.

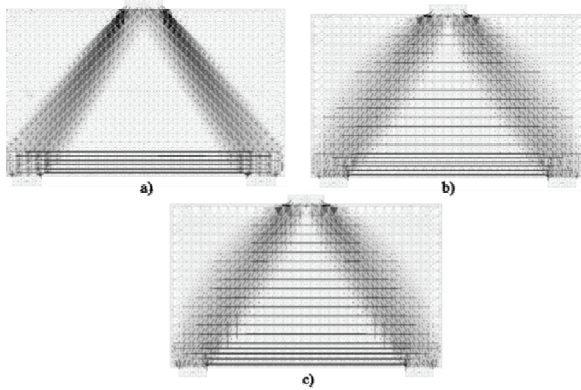


Figure 11. Relative stress to load of 1260 kN, a) STM (tie), b) STM (web + tie), c) EPSF (web + tie).

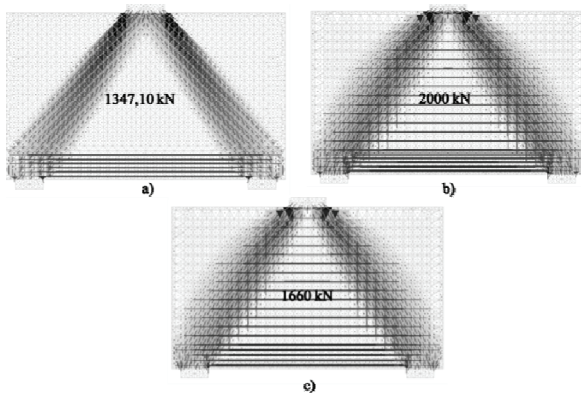


Figure 12. Relative stress when concrete reaches failure a) STM (tie), b) STM (web + tie), c) EPSF (web + tie).

Table 2. Results for the struts when concrete reaches its capacity.

Model	Load (kN)	Strut Stress (MPa)	Strength Stress (MPa)	Eta2
STM (tie)	1347,10	-11,20	11,34	0,378
STM (web + tie)	2000,00	-29,90	30,00	1,000
SFM (web + tie)	1660,00	-29,90	30,00	1,000

Muttoni et al. (2006) indicate a relationship between the load that causes the first yielding and the one that causes the failure ($Q_y Q_R^{-1}$). The same author recommends that this relationship should be less than $1 \gamma^{-1}$ as shown in Equation 6, considering γ as the safety factor. Table 3 shows a comparison of the results for each method considering Equation 6.

Table 3. Comparison between models in notorious loads.

	Notorious Loads			
	STM (tie)	STM (WEB + Tie)	SFM (WEB + Tie)	kN
$\epsilon_{c2} = 2,0\text{‰}$	Struts plastic level	-	-	1.150,00
$\epsilon_{cu} = 3,5\text{‰}$	struts rupture strain	-	-	1.347,10
-	-	-	$\epsilon_{c2} = 2,0\text{‰}$ Struts plastic level	1.400,00
-	-	-	$\epsilon_{aco} = 2,38\text{‰}$ Steel yielding	1.459,70
-	-	$\epsilon_{c2} = 2,0\text{‰}$ Struts plastic level	-	1.600,00
-	-	-	$\epsilon_{cu} = 3,5\text{‰}$ struts rupture strain	1.660,00
-	-	$\epsilon_{aco} = 2,38\text{‰}$ Steel yielding	-	1.900,00
-	-	$\epsilon_{cu} = 3,5\text{‰}$ struts rupture strain	-	2.000,00
-	$Q_y/Q_R = -$	$Q_y/Q_R = 0,95 \rightarrow \gamma = 1,053$	$Q_y/Q_R = 0,879 \rightarrow \gamma = 1,138$	

$$Q_y/Q_R \leq \frac{1}{\gamma} \tag{6}$$

The ratios of Q_y/Q_R from the studied models are shown in Table 3. Therefore, for the presented deep beam designed only with the main ties according to the STM, this ratio could not even be calculated. The concrete has reached the ultimate compressive strength before the steel started to yield, consequently, that value would be higher than one. However, for the same method, but now considering the minimum web reinforcement, the γ factor is equal to 1,053. For the model designed by the NL-EPSF, the value is equal to 1,138. Then, it is possible to conclude that the last model has proven to be safer in terms of ductility than the others.

The last statement can also be checked through the load versus displacement behavior shown in Figure 13. The curve used to represent the model designed by the STM only with the main tie does not show a yielding baseline. On the other hand, the curve of the model which was designed by the STM with the minimum web reinforcement and the main tie is better in terms of vertical displacement and ultimate load.

The curve that represents the model designed by the NL-EPSF approach (SFM - web + tie) has similar behavior to that one from the deep beam reinforced only with the main tie (STM - Tie). The compressive limit strength for struts and nodal regions are presented in chapter 22.3.2 of the ABNT (2014). The curves presented in Figure 14 compare the recommended values of the Brazilian structural code with the values of Eta2 (NL-EPSF) obtained from the numerical analysis using Jcon.

Analyzing the results of Figure 14 it is possible to identify that the values of Eta2 taking into account the failure loads are very close to the limit strength ($f_{cd1}/f_{cd} = 0.748$) recommended by the Brazilian code. However, the strength value recommended by the ABNT (2014) for struts crossed by one tie is $f_{cd3}/f_{cd} = 0.633$. In summary, the values, which penalize the concrete strength in struts, obtained through NL-EPSF, is less conservative than the values recommended by the Brazilian structural code.

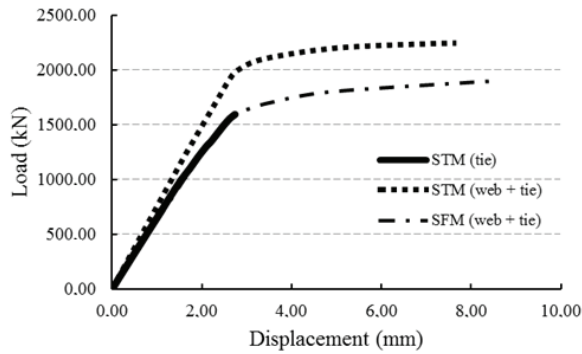


Figure 13. Diagram load versus displacement for the investigated models.

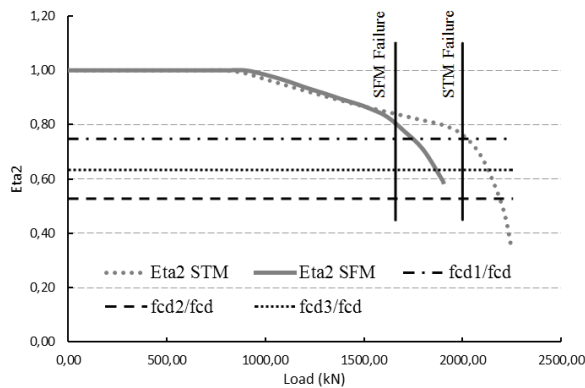


Figure 14. Diagram eta2 versus point load for the investigated models.

The purpose of Table 4 is to compare the amount of steel used in each investigated model, assuming that the model designed by STM only with the main tie is not relevant for the comparisons. Therefore, only the results according to the STM with the minimum web reinforcement and the main tie, the simple beam model and the NL-EPSF model will be compared. The values obtained by the STM modeled with the main tie and the minimum web reinforcement are considered as the reference. Thus, the positive values refer to a larger use of reinforcement while negative values

mean an economy of reinforcement when compared to the reference model.

The amount of tie in the simple beam model is identical to this value on the STM model. For the horizontal reinforcement, the simple beam model used 60.61% more steel than the reinforcement provided in the STM model. For the vertical reinforcement, the relationship of steel amount between simple beam model and STM was of 17.78%. In general, the simple beam model demanded 20.20% more reinforcement than the STM model.

The value -26,67% refers to the difference between the STM model and the NL-EPSF model, comparing just the quantity of steel used on the main tie. Considering this fact, the NL-EPSF method were efficient in the design of the main ties, by the material economy point of view. The -12,15% value refers to the relative difference of all reinforcement used to build the global deep beam, i.e., the NL-EPSF model for this case is more economical than the STM model.

Conclusion

It is possible to conclude that the SFM combined with a nonlinear software package (Jconc) provides conditions for taking into account the minimum web reinforcement (usually mandatory in the structural codes) during the design stage of deep beams.

Considering that the minimum web reinforcement is not considered in the design process using the STM model, i.e., the minimum web reinforcement is normally included in the final design in order to just satisfy the structural codes, important distortions may arise in the idealized truss model. In this way, the approach considering the minimum reinforcement and numerical analysis using the SFM may be considered more realistic and provides a more economical design of the main tie when compared to the STM approach.

Table 4. Reinforcement comparisons for the investigated deep beam.

Steel Type	Position	Diameter (mm)	Amount (bars)	Length (cm)	Total Length (cm)	Total (kg)	Relative Difference
Strut-and-Tie Model (STM)							
CA60	Hor. Dist.	5	15	634	9510,0	1450,70	0,00%
	Vert. Dist.	5	30	386	11580,0	1766,47	0,00%
CA50	Tie	10	15	294	4410,0	2691,62	0,00%
	Total					5908,80	0,00%
Simple Beam Model							
CA60	Hor. Dist.	6,3	15	634	9510,0	2329,95	60,61%
	Vert. Dist.	6,3	22	386	8492,0	2080,54	17,78%
CA50	Tie	10	15	294	4410,0	2691,62	0,00%
	Total					7102,11	20,20%
Non-Linear Elastic-Plastic Stress Fields Model							
CA60	Hor. Dist.	5	15	634	9510,0	1450,70	0,00%
	Vert. Dist.	5	30	386	11580,0	1766,47	0,00%
CA50	Tie	10	11	294	3234,0	1973,86	-26,67%
	Total					5191,03	-12,15%

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