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The influence of physical properties in bubble collapse and pressure field in liquids

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ABSTRACT. Numerical simulations of the collapse of bubbles (or cavities) are shown, using the finite difference method. Simulations were made using: water, benzene, glycerin, and mercury. Pressure fields values are calculated in an area of 800 x 800 mm, first for the case of one bubble under the hypothesis of spherical symmetry, and then for the case of four bubbles, where the spherical symmetry no longer exists. Results are shown as pressure curves in the plane. The existing method doesn't take into account the physical properties of the fluid and it is available for one bubble only.

Key words: bubble, cavity, cavitation.

RESUMO. Influência de propriedades físicas no colapso de bolhas e no campo de pressão em líquidos. Simulações numéricas do colapso de cavidades em líquidos, utilizando o método das diferenças finitas foram feitas para os seguintes fluidos: água, benzeno, glicerina, e mercúrio. O campo de pressões foi calculado em uma área de 800 x 800 mm, primeiro para uma única cavidade, sob a hipótese de simetria esférica, depois para quatro cavidades, onde a simetria esférica é quebrada. Os resultados são apresentados na forma de curvas de pressão no plano. O método existente até então não leva em conta as propriedades físicas do fluido e serve apenas para uma cavidade.

Palavras-chave: bolha, cavidade, cavitação.

In the process of formation of bubbles in liquids, air and vapour are always trapped inside the bubbles, since the nucleation begins in a micro-bubble of air (Hammitt, 1980), and the bubble is filled by vapour as it grows. Such process begins when the pressure in the liquid reaches its vapour pressure. So the presence of vapour must be taken into account as well as air. Although Poritsky (1952), studying the collapse of a bubble, considered a pressure inside the bubble, he didn't take into account the compression of vapour and air during the collapse.

When the bubble is submitted to high pressure, collapse will occur. Some photographic studies of collapsing bubbles on a flow over ogives were carried out (Plesset, 1949). The pressure field due to the collapse of bubbles in liquids may be calculated as a function of the radius of the bubble and the physical properties of the fluid, for special values of initial and some boundary conditions.

Recent studies about sonoluminescence, as result of cavitation, (Barber *et al.,* 1997; Moss *et al.,* 1994), have been creating an increasing interest in cavitation research again, since it is believed by some researchers that it could be a possible start-up for the cold fusion.

Pressure field shall be calculated for several fluids under the hypothesis of adiabatic collapse, since there is no time for heat transfer to occur. Better results for adiabatic hypothesis, when compared to isothermal hypothesis, can be seen in Bazanini *et al.* (1998). The choice of the fluids was based on their physical properties. One selected fluid has a high value of one physical property, when compared to water, as follows: benzene has high value of vapour pressure; glycerin has high value of viscosity and mercury has high value of density. So the influence of physical properties in the pressure field of a collapsing bubble in an incompressible liquid may be investigated.

Method

The basic equations for the motion of the bubble wall during the collapse of a spherical bubble in an incompressible liquid is obtained from the Navier-Stokes equation in the vector form below (Swansom, 1970):

$$\rho_L \vec{B} - \nabla P = \rho_L \frac{D\vec{v}}{Dt} - \mu \nabla^2 \vec{v}\,, \qquad (1)$$

where μ is the viscosity and ρ_L is the liquid density. $\vec{B}$ represents the body forces, P is the pressure and $\vec{v}$ is the velocity.

For a spherical bubble in an incompressible liquid, the motion will be in the radial direction, and the velocity shall be:

$$\vec{v} = \vec{v}_r\,, \qquad (2)$$

$$\nabla^2 \vec{v}_r = 0. \qquad (3)$$

Neglecting body forces, we set for the Navier-Stokes equation the following result:

$$-\frac{\nabla P}{\rho_L} d\vec{r} = \frac{D\vec{v}_r}{Dt} d\vec{r}\,. \qquad (4)$$

As the motion is in the radial direction, equation (4) shall be written in the scalar form. For the first member we set

$$-\frac{\nabla P}{\rho_L} d\vec{r} = -\frac{\partial P}{\partial r}\frac{dr}{\rho_L} = -\frac{dP}{\rho_L} \qquad (5)$$

And the second member may be written as

$$\frac{D\vec{V}_r}{Dt} d\vec{r} = \frac{DV_r}{Dt} dr = \left(\frac{\partial V_r}{\partial t} + V_r \frac{\partial V_r}{\partial r}\right) dr \qquad (6)$$

Or

$$\frac{D\vec{V}_r}{Dt} d\vec{r} = \frac{\partial V_r}{\partial t} dr + V_r\, dV_r \qquad (7)$$

Substituting equations (5) and (7) in equation (4) leads to the following result

$$-\frac{dP}{\rho_L} = \frac{\partial v_r}{\partial t} dr + v_r dv_r\,. \qquad (8)$$

Integrating equation (8) between a radial position in the liquid r and a position far enough in the liquid (where no effects of the collapse are felt, named ∞), we obtain

$$\frac{P_\infty - P_r}{\rho_L} - \frac{{v_r}^2}{2} + \int_r^\infty \frac{\partial v_r}{\partial t} dr = 0\,. \qquad (9)$$

To evaluate the last term in equation (9) above, one more equation is necessary, for the radial velocity v_r as a function of time t. The continuity equation in the vector form (Swanson, 1970) shall be used for this purpose and applied in a variable spherical control volume situated between the radius r and $R(t)$. Then, we have

$$0 = \frac{\partial}{\partial t}\int_{VC} \rho_L\, dV + \int_{SC} \rho_L\, \vec{v}\, d\vec{A}. \qquad (10)$$

For an incompressible fluid, the continuity equation becomes

$$v_r = \frac{[R(t)]^2}{r^2}\frac{dR(t)}{dt}\,. \qquad (11)$$

Now it is possible to evaluate the desired term, which results in

$$\int_r^\infty \frac{\partial v_r}{\partial t} dr = \frac{[R(t)]^2}{r}\frac{d^2R(t)}{dt^2} + \frac{2R(t)}{r}\left[\frac{dR(t)}{dt}\right]^2. \qquad (12)$$

Substituting equation (12) in equation (9), we have that

$$\frac{{v_r}^2}{2} - \frac{[R(t)]^2}{r}\frac{d^2R(t)}{dt^2} - \frac{2R(t)}{r}\left[\frac{dR(t)}{dt}\right]^2 = \frac{P_\infty - P_r}{\rho_L}\,. \qquad (13)$$

In the bubble wall

$$r = R\,(t) \qquad (14)$$

$$v_r = \frac{dR(t)}{dt} \qquad (15)$$

$$P_r = P, \qquad (16)$$

and then equation (13) becomes

$$R(t)\frac{d^2R(t)}{dt^2} + \frac{3}{2}\left[\frac{dR(t)}{dt}\right]^2 = \frac{1}{\rho_L}[P - P_\infty]. \qquad (17)$$

Vapor and air trapped inside the bubble will be assumed as ideal gases. Since the collapse is very fast (about 0,7 ms for water (Knapp and Hollander, 1948)), the process will be assumed as adiabatic because there is no time for heat transfer to occur.

Equations considering surface tension S and viscosity effects may be written as

$$P = P_e - 2\mu \frac{\partial v_r}{\partial r}. \qquad (18)$$

For the pressure external to the bubble P_e, we can use the following equation

$$P_i - P_e = \frac{2S}{R(t)}, \qquad (19)$$

where the internal pressure P_i is due to the presence of vapor and air inside the bubble, and is such that

$$P_i = P_g + P_v. \qquad (20)$$

Substituting equations (18), (19) and (20) into equation (17), and taking $R(t) =$

R, $dR(t)/dt = R'$ and $d^2R(t)/dt^2 = R''$, we set the following equation for the collapse of a bubble

$$RR'' + \frac{3}{2}R'^2 = \frac{1}{\rho_L}\left[P_g + P_v - P_\infty - \frac{2S}{R} - 2\mu\frac{\partial v_r}{\partial r}R\right]. \qquad (21)$$

Under the hypothesis of adiabatic collapse and substituting equation (11), it results in

$$RR'' + \frac{3}{2}R'^2 = \frac{1}{\rho_L}\left[\frac{P_{g_0} R_0^{3K_g}}{R^{3K_g}} + \frac{P_{v_0} R_0^{3K_v}}{R^{3K_v}} - P_\infty - \frac{2S}{R} - \frac{4(\mu_g + \mu_L)}{R}R'\right]. \qquad (22)$$

Air and vapor initial pressure inside the bubble, P_{g0} and P_{v0} respectively, shall be considered as well as air and vapor adiabatic constants, K_g and K_v.

Available method for pressure field calculation during the collapse of a bubble is appropriate for one empty bubble only, and disregards physical properties of the fluid (Rayleigh, 1917).

To calculate the pressure field taking into account the physical properties of the fluid, bubbles shall be assumed as sinks in the potential flow theory. Since equation (22) is in the difference form, manipulations of the Navier-Stokes and continuity equations in the difference form is necessary in order to find an equation to calculate pressure field. The pressure field will be calculated in an area of 800 x 800 mm. For the two-dimensional case, Navier-Stokes and continuity equations are (Welty *et al.*, 1984)

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho_L}\frac{dP}{dx} + \nu_L\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \qquad (23)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho_L}\frac{dP}{dy} + \nu_L\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \qquad (24)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (25)$$

where u and v are the velocities in the x and y direction respectively, and ν_L is the kinematic viscosity of the liquid.

Differentiating equation (23) with respect to y and equation (24) with respect to x, subtracting the latter from the former equation and simplifying this result using equation (25), we set

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} = -2\rho_L\frac{\partial v}{\partial x}\frac{\partial u}{\partial y} - \rho_L\left(\frac{\partial u}{\partial x}\right)^2 - \rho_L\left(\frac{\partial v}{\partial y}\right)^2 \qquad (26)$$

The flow function for the two-dimensional case (Swansom, 1970) is

$$v = -\frac{\partial \psi}{\partial x} \qquad (27)$$

$$u = \frac{\partial \psi}{\partial y} \qquad (28)$$

Using the flow function definition above, equation (26) becomes:

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} = 2\rho_L\left[\frac{\partial^2 \psi}{\partial x^2}\frac{\partial^2 \psi}{\partial y^2} - \left(\frac{\partial^2 \psi}{\partial x \partial y}\right)^2\right] \qquad (29)$$

This equation may be used for any number of bubbles. Once the bubbles are treated as sinks, the flow function field may be obtained simply by adding the flow function for every bubble. Flow function field may be calculated for every sink using the following equation

$$\psi = -R R' C, \qquad (30)$$

where C is the position of the calculated point in relation to the sink.

Equations (22) and (29) above were solved using the finite difference method, as described in the following section.

Results

To calculate the pressure field it is necessary to solve equations (22) and (29), whose analytical solutions are difficult to find. Therefore let us use numerical methods to solve these equations. Equations (22) and (29) were solved using the finite difference method, for the following conditions: R_0 = 3.56 mm, P_{g0} = 40 Pa (initial conditions) as measured by Knapp and Hollander (1948); P_∞ = 50,000 Pa, boundary condition; step = 10^{-5} s, enough for such calculations. Calculations are finished when the bubble radius is 1 mm, since it is not possible to assure the existence of the bubble beyond this value.

Equations (22), (29) and (30) take into account the physical properties of the fluid and may be used for several bubbles.

In Table 1 below, values of initial vapour and air pressure are shown. Vapour pressure is expected to have a great influence on the pressure field, once, as the vapour and air are compressed as the bubble collapses, vapour and air pressures inside the bubble raise, and initial vapour pressure is much higher than initial air pressure, as may be seen in Table 1.

Table 1. Initial air and vapour pressure

Fluid (vapour)	P_0 (Pa)
Benzene	10,000
Glycerin	0.014
Mercury	0.17
Water	2,340
Air	40

First calculations were made in a square area of 800 x 800 mm, for one collapsing bubble located in the centre of the area. Results are shown in Figures 1 to 4. Since the calculation method presented here is adequate to the presence of several bubbles, it shall be used to four bubbles randomly disposed in the same area of 800 x 800 mm, under the initial and boundary conditions:P_{g0} = 40 Pa; R_0 = 3.56 mm and P_∞ = 50,000 Pa. Results are shown in Figures 5 to 8 below for each fluid.

It can be seen from Figures 1 to 8 that pressure values are higher for benzene and water respectively, because these fluids have the highest values of initial vapour pressure.

Discussion

Initial vapour pressure was of major influence in the pressure field of collapsing spherical bubbles in an incompressible liquid as may be seen in Figures 1 and 2. This fact is due to the compressing process during the collapse of the bubble. As the bubble collapses, vapour and air are compressed and pressure inside the bubble raises. Since water was used as a parameter, it may be conclude from Figures 3 and 4 that properties such as viscosity (represented by glycerin), surface tension and mercury (represented by mercury) are not of major influence in the pressure field.

According to Hammitt (1980), the effect of surface tension is negligible during the collapse of cavities. But it may be seen in Figures 2, 3 and 4 that the effects of viscosity, surface tension, and density are negligible in the pressure field during the collapse, when compared to vapour pressure effects.

Calculations presented here are adequate to any fluid of known physical properties, but there is an infinite number of possibilities for calculations when varying parameters such as number and positions of bubbles, initial and boundary conditions, and physical properties are present.

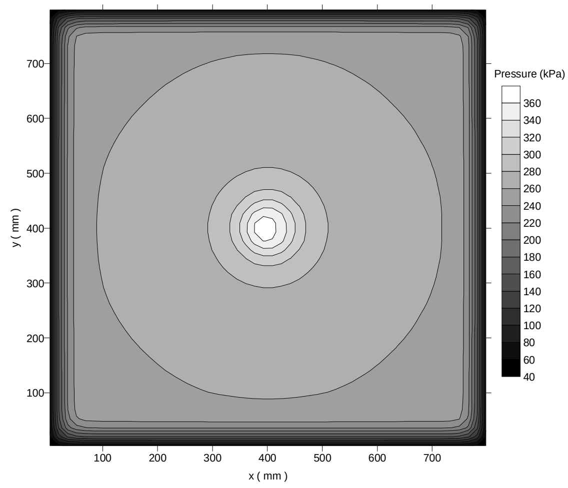

Figure 1. Pressure field for water (01 bubble)

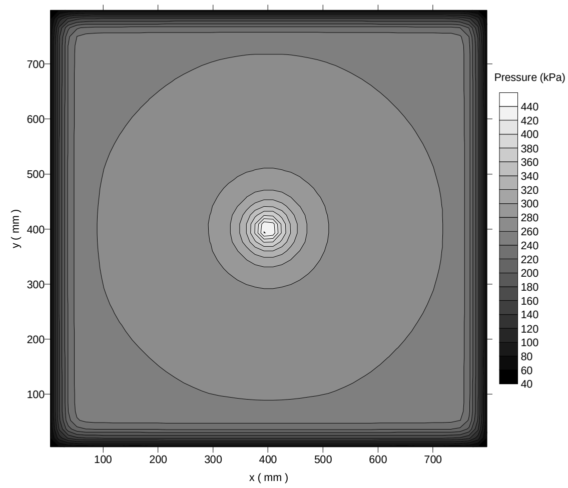

Figure 2. Pressure field for benzene (01 bubble)

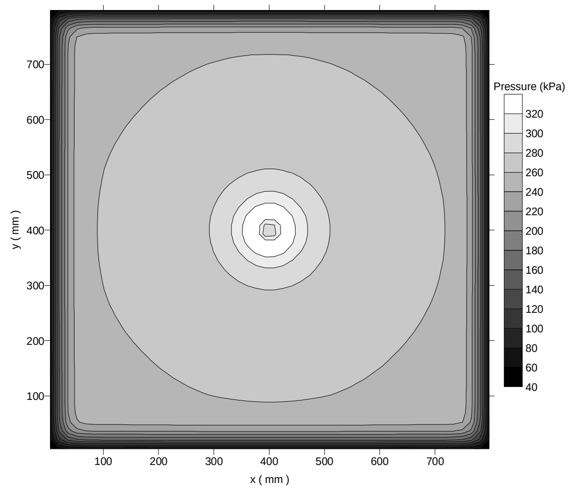

Figure 3. Pressure field for glycerin (01 bubble)

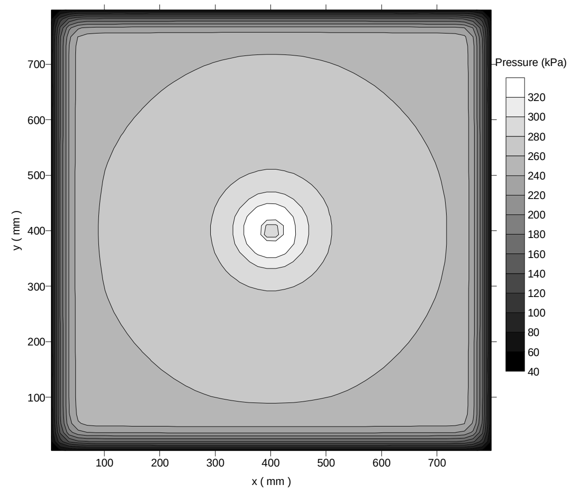

Figure 4. Pressure field for mercury (01 bubble)

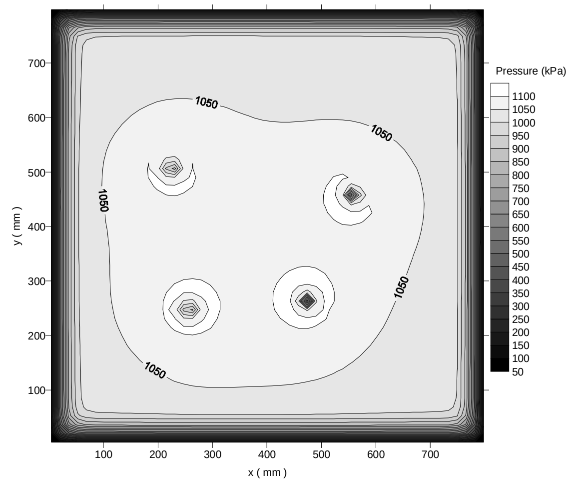

Figure 5. Pressure field for water (04 bubbles)

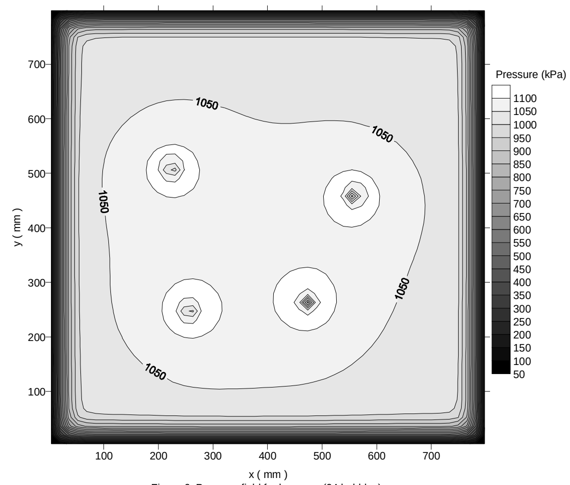

Figure 6. Pressure field for benzene (04 bubbles).

Figure 6. Pressure field for benzene (04 bubbles)

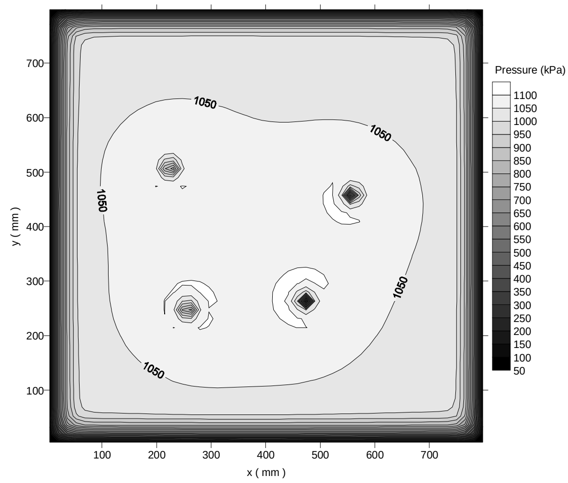

Figure 7. Pressure field for glycerin (04 bubbles)

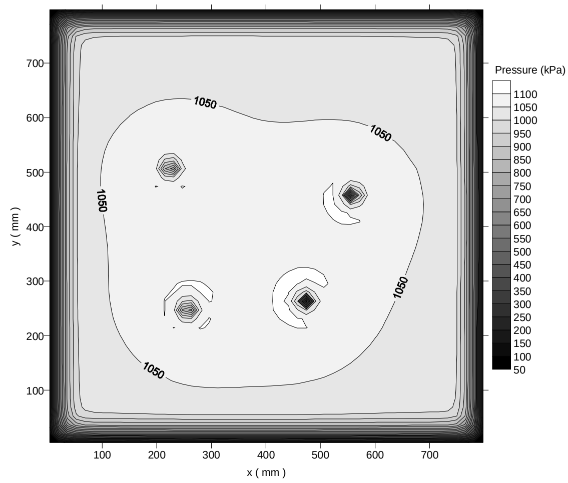

Figure 8. Pressure field for mercury (04 bubbles)

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