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Hands-on experiments in Fourier's optics

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ABSTRACT. Simple experiments obtained with a laser beam are proposed in this article. Diffraction phenomena and their interpretation in terms of Fourier's transforms in the spatial domain are discussed in order to introduce some interesting application of the spatial filtering in optics. A straightforward introduction of convolution theorem is also suggested. A stimulus is given to the discussion concerning the utility of Fourier transforms in high school and undergraduate physics teaching.

Key words: diffraction, Fourier's optics, spatial filtering.

RESUMO. Uma introdução à ótica de Fourier. Alguns experimentos simples com um feixe de laser (laser pointer, p.ex.) são propostos no presente artigo. Serão discutidos os fenômenos de difração e suas interpretações em termos das transformadas de Fourier no domínio espacial, com o intuito de introduzir algumas aplicações interessantes da filtragem espacial em ótica. Será mencionado brevemente o teorema de convolução, além de discutir a utilidade das transformadas de Fourier na Física da Escola Média e de cursos de graduação.

Palavras-chave: difração, ótica de Fourier, filtragem espacial.

Light is an electromagnetic field, varying in space and in time: in wave optics we are well accustomed to studying how this field can be represented by a sum of harmonic components of different time frequency.

If we use a particular non-monocromatic laser beam entering a dispersing prism, we can see the emerging rays having deflected their original direction by angles which are different for each colour, that is, for each component of different time frequency.

Obviously if we use a beam of white light (obtained by the use of an overhead projector) we obtain the empirical evidence of the existence in white light of a continuum spectrum of time frequencies.

Method

Diffraction of the same white light beam through a grating demonstrates the different behaviour of the different spectral components: various coloured images of the source corresponding to slightly different diffraction angles spread out in series of beautiful continuous spectra to the left and to the right of the white zero order maximum.

But what if we use a monochromatic laser beam through the same diffraction grating? In this case the monochromatic beam spreads out on the image plane determining a periodic spatial distribution of maxima whose intensity may be described by a cos^2x function. A similar effect can be obtained using now a single slit, but the diffraction pattern is slightly different from the previous one since its intensity I is proportional to $(sinx/x)^2$. In Figure 1, we can see its plot against $sin\theta$(where θ is the angle between the undeflected ray and the diffracted one). The position of the maxima is given by the well-known formula:

$$b \sin \theta = m \lambda,$$

where b is the width of the slit. Thus $sin\ \theta$ is also equal to $(m\lambda/b)$, hence it is proportional to the reciprocal of a unit of length: so that we can call it "spatial frequency". The plot may be now interpreted as the presentation of the intensity of light as a function of these "spatial frequencies".

In Figure 2, the black line is a plot of the square root of the intensity function I, that is, proportional to $sinx/x$.

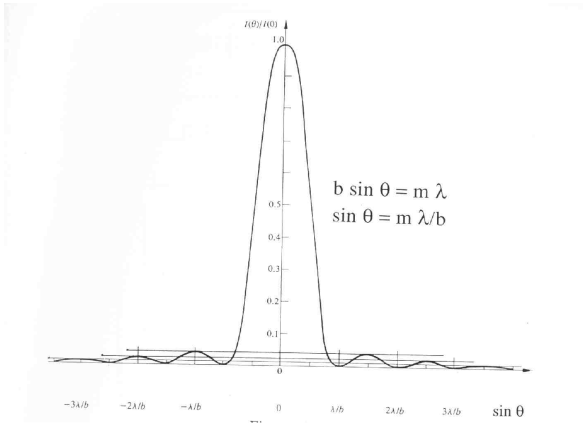

Figure 1

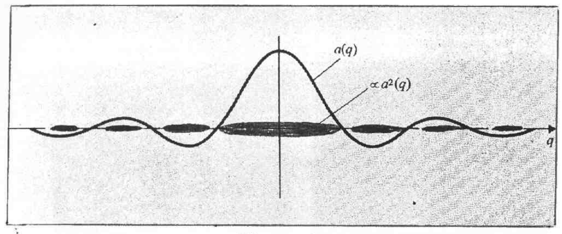

Figure 2

If we represent, as shown in Figure 3, the so-called "aperture function", that is, the electromagnetic field of light distribution across the object illuminated by the source light (the single slit, in this case), we obtain the square pulse profile. Comparing this function with the pattern of the square root of the intensity I of the diffracted light, we ask ourselves if there is a mathematical link between these two functions: the answer is *yes*. Each function can be considered as Fourier's transform of the other.

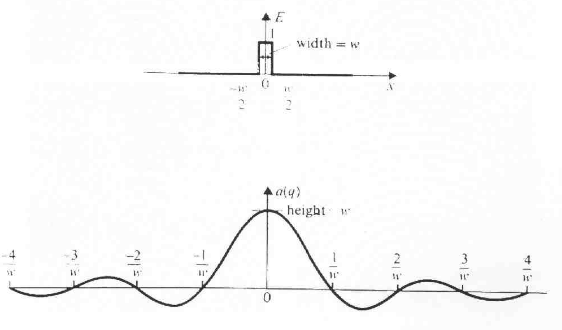

Figure 3

In Figure 4, we can see the aperture function of a grating and, below, its Fourier's transform as a function of spatial frequencies: the square of this spectrum gives the typical diffraction pattern of light. If we choose a grating in which the slits are more separated, we notice that Fourier's transform and, in other words, the diffraction pattern shows maxima that are less spread out. At last, if we let the separation among the slits tend to infinity, we obtain the well-known single slit's diffraction pattern which we discussed before (in the bottom part of the figure).

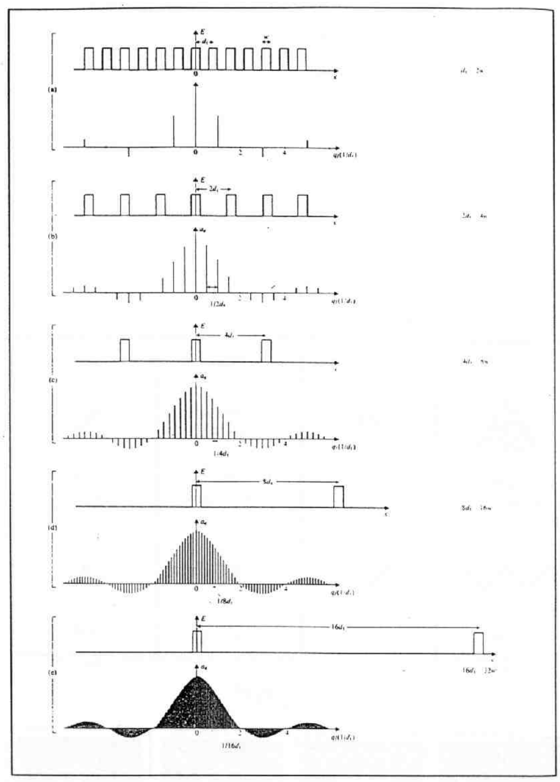

Figure 4

We also see, from a phenomenological point of view, that if the aperture function is periodic (and this is the case of a grating), the diffraction pattern does not change if we move the grating horizontally, because the components of different spatial frequencies remain the same. But, if we now rotate the grating the diffraction pattern rotates, too: we can conclude that the diffraction pattern is constantly perpendicular to the slits of the grating.

It is well known that diffraction phenomena are obtained when light encounters an opening (such as a hole, for example), but also when it encounters an obstacle (such as a disk, for example). In optics, hole and disk are said "complementary diffracting screens", or, in other words, the transparent regions on the one, correspond exactly to the opaque regions on the other, and vice-versa. *Babinet's principle,* valid for any wave phenomena, states that diffraction patterns from complementary screens are identical (see Figure 5). Therefore, the diffraction pattern obtained from a fine thread is almost identical to the one obtained from a single slit.

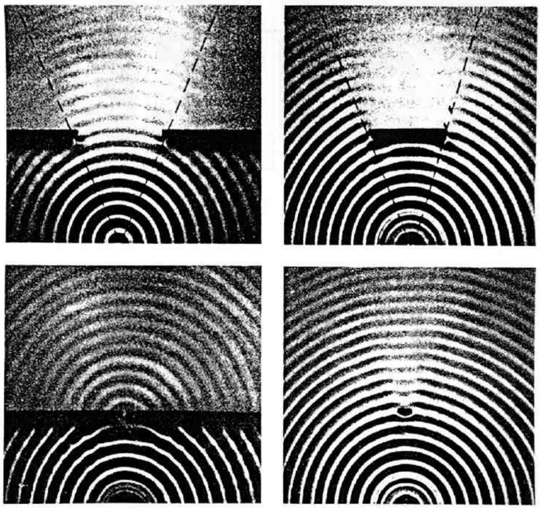

Figure 5

Now, we want to study diffraction patterns produced by the obstacles shown in Figure 6, which are characterized by particular symmetries (Hodkingson, 1992). Obviously, the reasoning does not change if we use complementary apertures for Babinet's principle. As a consequence, we obtain diffraction patterns as shown in the set of Figures 7.

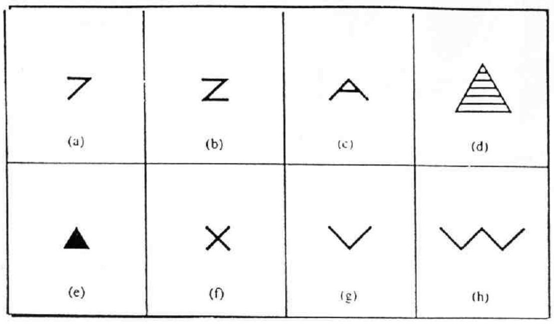

Figure 6

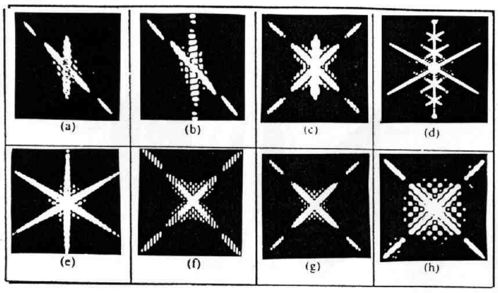

Figure 7

Results

In Figure 8, the sketch of the experimental set is shown to demonstrate the role of a converging lens interposed on the optical path beyond the diffraction pattern: we can observe that it produces on the screen a Fourier antitransform of the diffraction pattern, giving us back on the screen the original obstacle (object - in this case the letter “E”).

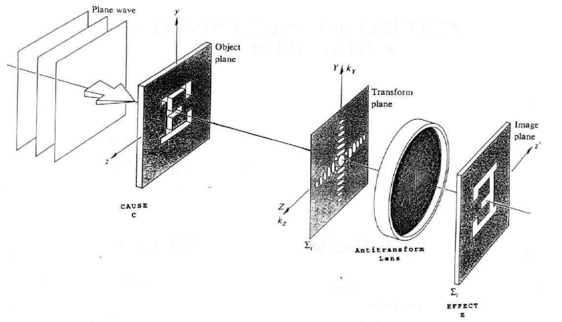

Figure 8

We want now to introduce an important application of Fourier’s optics known as “spatial filtering”. In particular, we want to ideally pose the “spatial filtering” in the mainstream of the so-called “linear response theory”. This formalism, based essentially upon Fourier’s transforms, has been extensively adopted in solid state physics and in the theory of electromagnetic signal processing and, recently, its introduction has been proposed in high school level by some physics teaching researches (Vicentini, 1995; Wanderlingh, 1991).

We can start now focusing on three important parts constituting our ideal spatial filtering set (see Figure 9). First, the role of the source object illuminated by the laser beam which we will call the *cause*. Then, the spatial filter (the single slit in the Figure 9) we will call *response,* and, at last, the image obtained on the screen we will call *effect*.

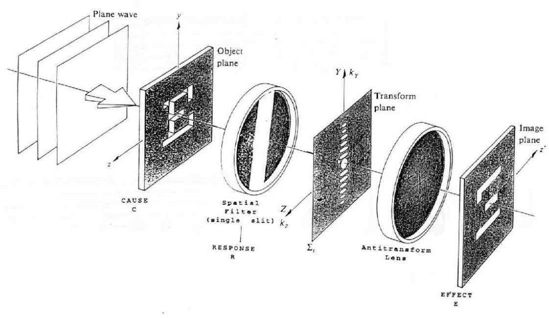

Figure 9

One can demonstrate (see Figure 10) that simple mathematical relations exist between the *cause,* the *response*, and the *effect.* We have a double slit illuminated by a laser beam; mathematically the aperture function, *g(x),* may be represented by two δ-functions. Below, the diffraction pattern, *G(k),* is represented mathematically corresponding to a $(\cos k)^2$. Both *g(x)* and *G(k)* may be thought as *causes* in our experiment. Then, we interpose a spatial filter, which is in this case a single slit. It is mathematically described by a square pulse, *H(k)*. Its diffraction pattern, as we have seen before, is a $(\sin x/x)^2$ function, *h(x)*: both *H(k)* and *h(x)*, may be thought as *responses* in our experiment.

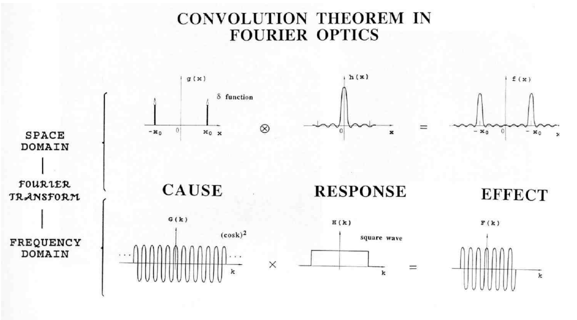

Figure 10

At last, we have a new diffraction pattern: it corresponds mathematically to a symmetrically truncated $(\cos k)^2$ function, *F(k)*. Its Fourier's transform, which experimentally may be obtained using a converging lens, corresponds mathematically to a *f(x)* function, where in the place of the two δ-functions now appear two $(\sin x/x)^2$ functions. We call *effect* both *f(x)* and *F(k)* functions.

The essence of the convolution theorem (a milestone of linear response theory) is this: in the space domain, the *effect* is obtained *convolving* the *response* with the *cause*. In the frequency domain, on the contrary, the *effect* is obtained as a simple algebric multiplication between the *response* and the *cause*.

The final part of this work present's a couple of applications of the spatial filtering technique (Xuan, 1995, personal communication). We produced an image of a musical partiture on slide and illuminated it with a laser beam. Truncating the horizontal maxima in the diffraction pattern by the use of a variable single slit as a spatial filter, we obtained on the screen, after the interposition of a converging lens, a new figure without the vertical lines, as we can see in the Figure 11a. Truncating now the vertical maxima in the diffraction pattern, we obtain a filtered image sketched in Figure 11b.

The same technique (Heicht, 1987) is used in the composite photography of the Moon, taken by Lunar Orbiter (see Figure 12a), to eliminate horizontal lines. In the unfiltered Fourier's transform, we can distinguish (Figure 12b) the vertical dot pattern describing the horizontal lines of the source image. When it is filtered out (Figure 12c), we obtain a version of the photo without horizontal lines (Figure 12d).

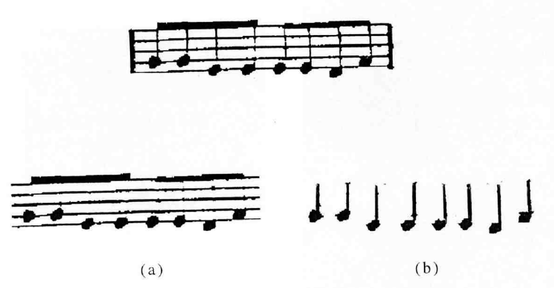

Figure 11

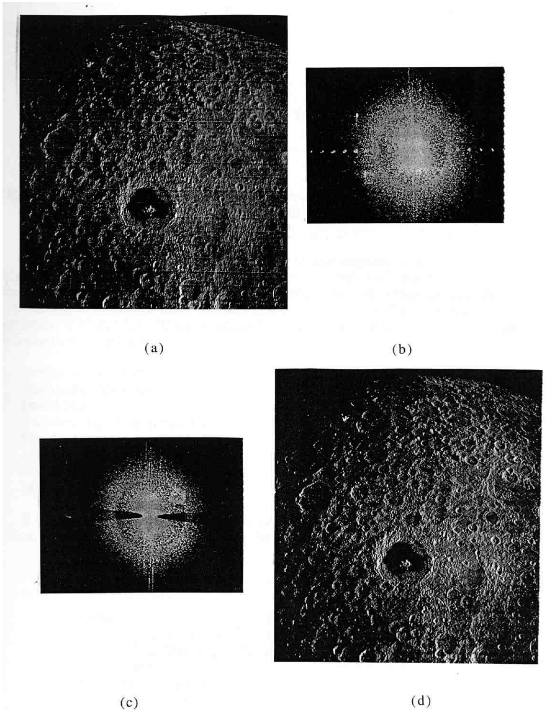

Figure 12

Discussion

The interest in optical processing, raised since the 1960s: in particular, in the areas of photographic image enhancements, radar and sonar signal processing, pattern recognition and so on.

To appreciate these technological applications, it is necessary, as we have seen, to have a good knowledge of this new branch of optics: this is why we presented, in a rather intuitive way, some of the most important ideas upon which Fourier's optics is based.

We also hope to have given a stimulus to the discussion recently raised in teaching research, concerning the proposal to introduce the "linear response approach" in high school level.

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