Acta Scientiarum 20(4):473-476, 1998.
ISSN 1415-6814.

The universe of Ptolemy revisited

Marcos Cesar Danhoni Neves

Departamento de Física, Universidade Estadual de Maringá, Av. Colombo, 5790, 87020-900, Maringá-Paraná, Brazil. phone: 00 55 44 261-4330, e-mail: macedane@fisica.dfi.uem.br

ABSTRACT. In this work we will present an alternative method to teach the first two Kepler's laws, using basic concepts of geometry, but achieving conclusions in the important notions of reference systems and the equivalence between heliocentrism (Copernican-Keplerian system) and geocentrism (Ptolemaic system). We used this method for students of junior (last year), high school and for undergraduate courses (Physics and Mathematics).

Key words: geocentrism, heliocentrism, Ptolemy, Kepler, referential systems

RESUMO. O universo de Ptolomeu revisitado. Será apresentado um método alternativo para se ensinar as duas primeiras leis de Kepler, usando conceitos básicos de geometria, procurando-se chegar à importante noção de sistemas de referência e à equivalência entre o heliocentrismo (sistema Copernicano-Kepleriano) e o geocentrismo (sistema Ptolomaico). O método foi utilizado para estudantes de Ensino Fundamental (último ano), Ensino Médio e em cursos de graduação (Física e Matemática).

Palavras-chave: geocentrismo, heliocentrismo, Ptolomeu, Kepler, sistema de referências.

The historical development of Astronomy, specially in the period between the transition from Ptolemaic to Copernican conception of the universe, found in the person of Johannes Kepler its maximum exponent. In high school teaching and in the first years of undergraduate courses, the description of planetary motions is reduced using simple memorization of the Kepler's laws, without a true phenomenon comprehension. The mathematical relations involved in this kind of teaching are not so simple to encourage the teachers to treat this theme with a different form from that presented in common (and, frequently, very complicated way) astronomical textbooks.

It is a great disappointment for students use the memorization of laws without a real and effective comprehension of the studied phenomenon. Some authors (Brehme, 1976) worked with Kepler's laws using alternative methods, based in science history or in geometrical arguments. The present work will treat this important and forgotten chapter of the Physics using geometry as tool to try to find out an alternative way to teach Mathematics (geometry), Astronomy and History of Science.

Our method consist in using two first of the Kepler's laws in a close form: replacing the ellipsis by non-centered circumferences (Neves and Arguello, 1986), and using, also, the very old equant's point instead the area's velocity to calculate the planetary trajectories as seeing in the Earth's sky. Results were confronted with that presented in current bibliography and, at the end, it was made a change of the reference systems (heliocentrism to geocentrism). This method was tested with junior (last year), high school and college students.

Non-centered circumference's method

Kepler's first law in a simplified form. Kepler's first law (or orbit's law) states that motion of the planets forms an ellipsis with the sun posed in the focus of this ellipsis.

We know that the planetary eccentricities are very small. Nevertheless, we can replace the elliptical shape of a planetary motion by a circular shape. But, this circumference will not be centered, e.g., with the sun occupying the center of this new circular orbit. The sun must be shifted from the center O until the distance $s = R \, . \, \varepsilon$, where R is the circumference radius (Figure 1).

The point occupied by the sun is called *solar focus* F, and the other point, symmetrically located around the center O, *angular focus* F'.

The equivalence between non-centered circumferences and ellipses with very low eccentricities can be shown using the ellipse's equation in its polar form:

$$r = R / [1 - \varepsilon (\cos \theta)] \quad (1)$$

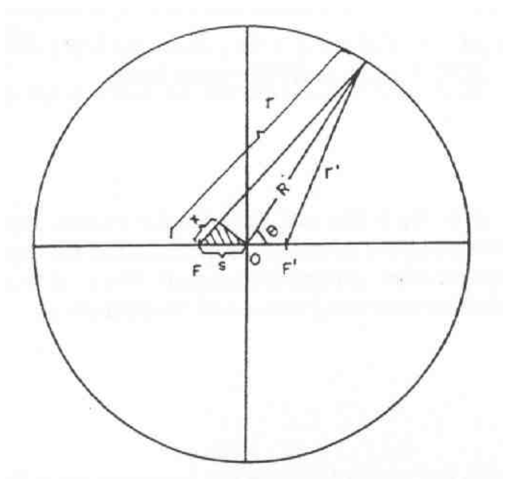

Figure 1

Equation 1, using a very low eccentricity ($\varepsilon^2 <<$ 1), becomes:

$$r = R + R\,\varepsilon\,(\cos\theta) \qquad (2)$$

Observing Figure 1 we can write the following mathematical relation:

$$x = s\,(\cos\theta) \qquad (3)$$

And, finally,

$$r = R + s\,(\cos\theta) \qquad (4)$$
$$r' = R - s\,(\cos\theta) \qquad (5)$$

As we know that $\varepsilon = s / R$, we can finally write (4) in the following form:

$$r = R + R\,\varepsilon\,(\cos\theta)$$

that corresponds to equation 2 valid for an ellipse with low eccentricity and, for our purpose, also valid for a circumference of radius R having two "focuses", both shifted from the center by the distance $s = R\,.\,\varepsilon$.

Kepler's second law in a simplified form. Kepler's second law (or area's law) states that areas swept in equal units of time by a planet orbiting around the sun are always the same.

For our purpose, we will enunciate this law in an alternative way, using the concept of the angular velocity with respect to angular focus F', or, as we can denominate it, *equant point.*

Using equation 1 and putting in it $\varepsilon = s / R$, and making use of the relations 4 and 5, we can find:

$$r = R^2 / r'$$

We know that R is constant (corresponding in planetary motion, to the mean distance between the sun and the planet so we have that:

$$r\,.\,r' = \text{constant} \qquad (6)$$

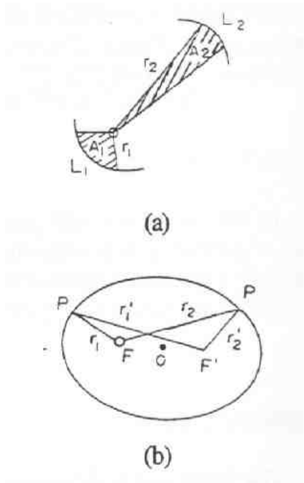

Figure 2

From the Figure 2a e 2b we can find also:

$$L_1\,r_1 = L_2\,r_2 \approx 2\,.\,\text{Area}$$

(where L is the length of the arc traversed by the planet).

Using 6, we can finally arrive to:

$$L1 / r_1' = L2 / r_2' \approx \xi \qquad (7)$$

where ξ is the angle, with respect to equant point (focus F'), that always remains constant in equal time intervals.

Nevertheless, considering the angles described by the planets always constant with respect to F', and linking the points on the circumference until the solar focus F (where the sun is placed), equal areas in equal time intervals are obtained (this fact can be proved using the Hero's formula to calculate the area of a triangle of unequal sides a, b and $c \rightarrow Area = [p\,(p - a)\,(p - b)\,(p - c)]^{1/2}$, where p is the semiperimeter $= (a+b+c)/2)$.

It is very interesting to observe that until now we have worked with three different concepts of velocity: angular velocity, constant with respect to focus F'; area's velocity, constant with respect to focus F; and linear velocity, inversely proportional to the planet-sun distance.

Using the non-centered circumferences method to calculate the orbits of Mercury and Earth. To draw Mercury's orbit (valid also to any other planet) we need know the following orbital elements: longitude ϖ, ascendent node Ω, longitude λ in the epoch (here, epoch = 1980.0) and the period T of a complete revolution of the planet around the sun.

The final drawing of the orbit is possible transferring orbital elements in spatial representation by a plane representation. Figure 3 shows the final result obtained. The most external circumference corresponds to fixed stars, with an infinite radius when it is compared to the radii of the planet orbits.

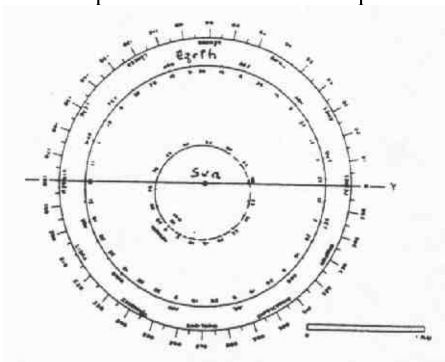

Figure 3

Drawing the apparent trajectory of Mercury. To obtain the apparent behaviour (resulting from the motion composition: Earth and planet) of "Mercury's motion around the Earth", we must work with a specific coordinate system. The system most used is the equatorial or absolute coordinate system (right ascension α and declination δ). However, for this specific method developed here, it ecliptical coordinates will be used: ecliptical longitude λ and ecliptical latitude β.

Taking the longitude λ close to right ascension α, we can calculate the ecliptical latitude β using the following formula:

$$\beta = i\,(DSP / DEP)\ \mathrm{sen}\,(\lambda - \Omega) \quad (8)$$

where i is the inclination of planetary orbit with respect to ecliptic, *DSP* is the sun-planet distance, and *DEP* is the Earth-planet distance.

Knowing λ and, obviously, α, corresponding to a specific epoch, 1980.0, and using precedent results, it is possible to determine the position of the Earth and the planet (in present section, Mercury) to sucessive posterior times (in this example, 1985). The geocentric apparent trajectory is obtained finding the ecliptical longitude, e.g., reducing to a point the radius of the Earth and the planet orbit, considering the radius of the fixed stars circumference as infinite (Menzel, 1964).

Using a graph right ascension α versus declination δ, and drawing the ecliptic "line" (that is our reference frame), we can obtain the geocentric apparent motion made by Mercury in 1985. Figure 4 shows this trajectory. Some of the assumptions made here seems to be not so accurate, but the results obtained are the best validity justification of the method developed here, using non-centered circumferences (Anuário Astronômico, 1985). Using this method we can change the origin of coordinate systems to any other planet. So, in an analogous way, we may suggested as an exercise: what would be the trajectory of the Earth by hypothetic in habitant of Saturn.

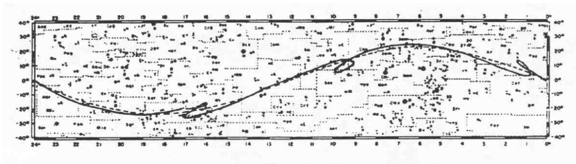

Figure 4

Conclusion: from Kepler to Ptolemy

The present work has been restricted until this point to the presentation of an alternative method, using basic and simple geometry to calculate and make previsions about the apparent trajectories of the planets in the sky. It seems to be a contradiction with the title: *from Kepler to Ptolemy*. However, this contradiction is only apparent, because by using the equant point we are working with a powerful tool characteristic of the Ptolemaic system (geocentric). The use of *equant law* as presented in the previous section is an alternative and analogous way to meet to the Kepler's second law (area's law).

However, observing Figure 3, and linking the points where Mercury and Earth were in 1985 (or, if we wish, to any other year), as illustrated in Figure 5, and having registered the length of Earth-planet (DEP) segments and angles with respect to the ecliptic, we can obtain a geocentric or geostatic system as imagined by Ptolemy. Figure 6 illustrates the final result after this change of reference system.

Observing the model of the universe as conceived by Ptolemy, in a very simplified way, the retrogradation motions were explained using a great deferent circumference, with the Earth placed at point *E* (Figure 7). The retrogradation motions like that presented in Figure 6 were obtained using an epicycle (little circumference on the deferent), with a constant angular velocity with respect to equant point *D*.

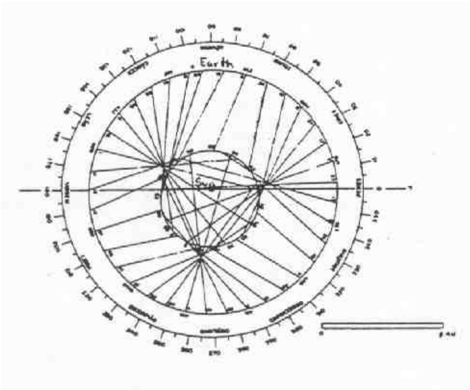

Figure 5

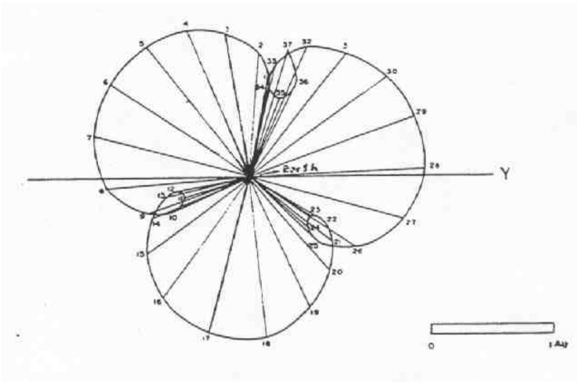

Figure 6

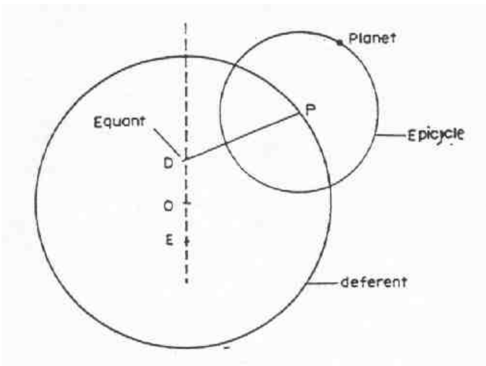

Figure 7

This last figure states correspondence between heliocentrism and geocentrism, using two different reference frames. Kuhn (1974) writes that Ptolemy and Copernicus had chosen equally justifiable processes to describe the position of the planets and the Earth.

Using non-centered circumferences method and observing Figure 6 we can, intuitively, make predictions about the number of the retrograde motions for each planet in the Earth's sky. In the case of external visible (with naked eye) planets (Mars, Jupiter and Saturn), the number of retrogradations corresponds to the number of rotations by the Earth during a complete orbital period around the sun. For example, the orbital period of Jupiter is 12 years. So, the number of the retrogradations is around twelve, for each twelve-year orbital period of the Earth around the sun.

In the case of internal planets (Venus and Mercury), the same scheme is valid. For example: to Mercury, which revolves around the sun in 88 days, at least, three retrogradations, are predictable, because this number of days is within the duration of a complete revolution of the Earth around the sun.

Using this method in schools we can introduce geometrical notions (perspective and spatial views in an applied form), history of science and physical principles of reference frames. The most important thing that we can show using this method is that both reference systems, Ptolemaic and Copernican-Keplerian, are equally valid and correct to describe the phenomena of the observable world.

References

Anuário Astronômico: 1985. São Paulo: Instituto Astronômico e Geofísico da USP, 1985.

Brehme, R.W. New look at the ptolemaic system, *Am. J. Phys.*, *44*(6):506-514, 1976.

Kuhn, T.S. A Função do dogma na investigação científica. In: Deus J.D. (ed.), *A crítica da ciência:* sociologia e ideologia da ciência, Rio de Janeiro: Zahar, 1974.

Menzel, D.H. *A field guide to the stars and planets.* Boston: Houghton Mifflin, 1964.

Neves, M.C.D.; Arguello, C.A. *Astronomia de régua e compasso:* de Kepler a Ptolomeu. Campinas: Papirus, 1986.

Received on November 19, 1998.
Accepted on November 30, 1998.