



## A Brief multifractal analysis of rainfall dynamics in Piracicaba, São Paulo, Brazil

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**ABSTRACT.** The aim of this study was to evaluate the statistical properties of a daily rainfall time series recorded in Piracicaba river basin, state of Sao Paulo, Brazil in the period 1917-2016. We apply Multifractal Detrended Fluctuation Analysis (MF-DFA) on deseasonalized data and calculate generalized scaling exponents,  $h(q)$ . We find that  $h(q)$  is a decreasing function of  $q$ , indicating multifractal behavior of precipitation dynamics. We also perform calculations on two subseries corresponding to the periods before and after 1970, where the second period is characterised by a significant increase of positive trend in rainfall intensity. We find no significant change in behaviour between these two subseries and the entire series, indicating that the dynamics of the underlying process did not change. Our results agree with previous studies on multifractal properties of rainfall time series in other regions of the world supporting the hypotheses of the existence of multiplicative cascade processes in the atmosphere.

**Keywords:** rainfall; power law; MF-DFA.

## Uma breve análise multifractal da dinâmica da chuva em Piracicaba, São Paulo, Brasil

**RESUMO.** O objetivo deste trabalho foi examinar as propriedades estatísticas de uma série temporal de chuvas diárias registrada na bacia do rio Piracicaba, Estado de São Paulo, Brasil, no período de 1917 a 2016. A metodologia *Multifractal Detrended Fluctuation Analysis* (MF-DFA) foi aplicada em dados dessazonalizados e foi calculado o expoente de escala generalizada,  $h(q)$ . Os resultados mostraram que  $h(q)$  é uma função decrescente de  $q$ , indicando o comportamento multifractal da dinâmica de precipitação. Foram realizados cálculos em duas subséries correspondentes aos períodos anteriores e posteriores a 1970, em que o segundo período é caracterizado com aumento significativo da tendência positiva na intensidade da precipitação. Não foram encontradas mudanças significativas de comportamento entre essas duas subséries e toda a série, indicando que a dinâmica do processo subjacente não mudou. Os resultados obtidos estão em concordância com estudos anteriores sobre as propriedades multifractais das séries temporais de chuvas em outras regiões do mundo, apoiando as hipóteses da existência de processos cascata multiplicativos na atmosfera.

**Palavras-chave:** precipitação; lei de potência; MF-DFA.

### Introduction

Rainfall is one of the most remarkable hydro-meteorological variables studied because of its non-homogeneous behaviour in event and intensity, leading to water runoff, drought with adverse environmental and social consequences (Walther et al., 2002). Rainfall time series often exhibit strong variability in time and space (Yu et al., 2014).

Over the last assorted decades, the multifractal theory (Feder, 1988), initially developed to model velocity fluctuations in turbulent flow (Benzi, Paladin, & Vulpiani, 1984), has been widely used to outline and model temporal and spatial distribution of rainfall, runoff and stream flow (Garcia-Marin, Jiménez-

Hornero, & Ayuso, 2008; Gupta & Waymire, 1993; Kantelhardt et al., 2006; Labat, Mangin, & Ababou, 2002; Schertzer & Lovejoy, 1987; Svensson, Olsson, & Berndtsson, 1996; Yu et al., 2014).

By analysing the data correlations on different scales, the multifractal technique thus makes it possible to evaluate the full range of precipitation dynamics; which is essential for understanding and modeling several hydrological phenomena such as droughts, floods, runoff, soil erosion, pollution transport, water infiltration etc. (Berne, Delrieu, Creutin, & Obled, 2004; Moriasi & Starks, 2010; Zhang, Ni, Li, & Zhou, 2016).

Piracicaba river basin is situated in the Southeastern region of Brazil. This region is experiencing an accelerated urban and agro-

industrial development is leading to quantitative and qualitative degradation of hydric resources (Lara et al., 2001; Martinelli, Gat, De Camargo, Lara, & Ometto, 2004; Coltri, Ferreira, Freitas, & Demetrio, 2009). In the early 80's a series of reservoirs were constructed to export approximately  $31 \text{ m}^3 \text{ s}^{-1}$  from the Piracicaba river basin to the metropolitan region of São Paulo city. Earlier analysis performed on records of precipitation and streamflow showed a positive trend for precipitation for the entire basin and a negative trend for stream flow for some locations within basin. While a decrease in water discharge can be explained by anthropogenic intervention, the increase in precipitation is due to the result of complex interactions between human and natural factors, and it is not yet completely understood (Moraes et al., 1998; Oki & Kanae, 2006; Haddeland et al., 2014).

Global historical rainfall series obtained by traditional rain gauge measurements are mostly available at coarse time scales (daily or higher resolution). This facilitates the manipulation, storage and dissemination of data, but can lead to the lack of information. The lack of high-resolution rainfall data, due to their costly and time-consuming acquisition, can be overcome by i) developing efficient downscaling models that produce synthetic data at a higher temporal and spatial resolution based on low-resolution observations, or ii) by downscaling the output of the regional climatic models and global circulation models.

Our objective in this paper focuses on the examination of the multifractal features of a long-term daily rainfall time series recorded in the Piracicaba river basin. To contribute to a better understanding of hydrological processes in this basin and possible relation with natural and anthropogenic factors, we apply multifractal analysis on daily rainfall time series. We compare the results of multifractal analysis for the periods before and after 1970, when the positive trend in precipitation was found to be much stronger (Liebmann et al., 2004; Moraes et al., 1998).

In the following section, we describe the data and present the multifractal analysis. The subsequent section deals with the results of our analysis, and finally, the conclusions are drawn.

## Material and methods

### Rainfall data

The data used in this work are provided by the Agrometeorological Sector of the Department of Exact Sciences of the College of Agriculture Luiz de Queiroz, University of São Paulo (Escola Superior

de Agricultura Luiz de Queiroz-ESALQ, USP), Brazil. The data are collected at an agrometeorological station located in the city of Piracicaba (latitude  $22^{\circ}42' \text{ S}$ , longitude  $47^{\circ}38' \text{ W}$  and altitude 546m), state of São Paulo, southeast Brazil, and they are available at <http://www.lce.esalq.usp.br/base.html>. The climate in the region is sub-tropical C with a wet season from October to March and a dry season from April to September. Eighty percent of the total annual rainfall of 1200-1500 mm occurs in the rainy season with an average monthly precipitation of more than 300 mm, while average monthly precipitation in the dry season is less than 29 mm. The average temperature in Piracicaba ranges from  $19^{\circ}\text{C}$  in the winter to  $24^{\circ}\text{C}$  in the summer (Lara et al., 2001). We analyze daily rainfall records collected during the period 01-jan-1917 to 31-dec-2016, with a total of 36864 observations.

### Multifractal analysis

Multifractal time series are characterized by a hierarchy of scaling exponents that describe different scaling behavior of many subsets of the series (Feder, 1988). The standard multifractal analysis based upon partition function multifractal formalism (Feder, 1988) is not appropriate for non-stationary time series, for which the improved methods were developed, as wavelet transform modulus maxima (WTMM) method (Muzy, Bacry, & Arneodo, 1991) and multifractal detrended fluctuation analysis (MF-DFA) method (Kantelhardt et al., 2002). In this work, we use MF-DFA method which produces slightly more reliable results than WTMM method (Kantelhardt et al., 2003; Oswiecimka, Kwapień, & Drożdż, 2006), and is characterized by simplicity of the implementation algorithm. This approach can systematically detect non-stationarities and overcome trends at all timescales, and was successfully applied to various phenomena such as physiological signals (Figliola, Serrano, & Rosso, 2007; Dutta, Ghosh, Samanta, & Dey, 2014), neurological diseases (Dutta, Ghosh, & Chatterjee, 2013), geophysical and soil properties data (Yu, Anh, & Eastes, 2009; Guadagnini, Martínez, & Pachepsky 2013; Subhakar & Chandrasekhar, 2016), hydrological processes (Kantelhardt et al., 2006; Zhang, Xu, Chen, & Yu, 2008; Serinaldi, 2010; Gires, Tchiguirinskaia, Schertzer, & Lovejoy, 2013), and financial time series and stock market (Matia, Ashkenazy, & Stanley, 2003; Zunino et al., 2008; Zunino et al., 2009, Oh et al., 2012).

The MF-DFA procedure is briefly described as follows. The original temporal series  $x(i), i =$

$1, \dots, N$  is integrated to produce  $y(k) = \sum [x(i) - \langle x \rangle], k = 1, \dots, N$ , where  $\langle x \rangle = \frac{1}{N} \sum x(i)$  is the average. Next, the integrated series  $y(k)$  is divided into  $N_n$  non-overlapping segments of length  $n$  and in each segment, the linear (or higher order polynomial) least square fit (representing local trend) is estimated. The integrated series  $y(k)$  is then detrended by subtracting the local trend  $y_i(k)$  (ordinates of straight line or higher order polynomial segment) from the data in each segment and a  $q$ -th order fluctuation function is calculated as

$$F_q(n) = \left\{ \frac{1}{N_n} \sum_{i=1}^{N_n} \left[ \frac{1}{n} \sum_{k=(i-1)n+1}^{in} [y(k) - y_i(k)]^2 \right]^{q/2} \right\}^{1/q} \quad (1)$$

where, in general,  $q$  can take any real value except zero. Repeating this calculation for all box sizes provides the relationship between fluctuation function  $F_q(n)$  and box size  $n$ , where typically  $F_q(n)$  increases with  $n$  as power law  $F_q(n) \sim n^{h(q)}$ . The scaling exponent  $h(q)$  is obtained as the slope of the regression (least square line fitting) of  $\log F_q(n)$  versus  $\log n$ . For stationary time series,  $h(2)$  is identical to well-known Hurst exponent  $H$  (Feder, 1988), and therefore  $h(q)$  is called the generalised Hurst exponent. For monofractal time series  $h(q)$  is independent of  $q$  (a constant), while for multifractal time series, for which small and large fluctuations scale differently,  $h(q)$  is a decreasing function of  $q$  (Kantelhardt et al., 2002).

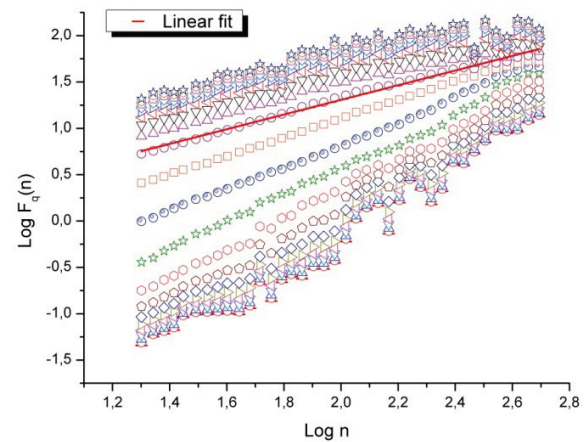
## Results and discussion

In order to make sure that seasonal periodicity does not affect power law behaviour we apply MF-DFA on the deseasonalized (normalised) series

$$X(t) = \frac{x(t) - \langle x(t) \rangle}{\sigma}, \quad (2)$$

where  $\langle x(t) \rangle$  is the mean daily precipitation calculated for each calendar date by averaging over all years in the record, and  $\sigma$  is the standard deviation of  $x(t)$  for each calendar date (Kantelhardt et al., 2006). To verify if the multifractality is due to broad probability density function for the values of precipitation time series, or due to different long-range correlations for small and large fluctuations, we also apply the MF-DFA on the shuffled series. In order to apply the MF-DFA technique, we wrote a C programming language using the Microsoft Visual Studio Compiler 2008.

The results of MF-DFA analysis are presented on Figure 1 and Figure 2. Figure 1 shows  $\log F_q(n)$  versus  $\log n$  plots for  $-10 \leq q \leq 10$  we can see linearity within scaling region  $10 < n < 100$  days, with decreasing slopes (representing generalized Hurst exponents  $h(q)$ ) from negative to positive  $q$  values, indicating multifractal behaviour. Figure 1 also displays the linear fit (red line) for  $q = 2$ , using the Ordinary Least Squares Method (OLSM). We obtained an adjusted value of  $R^2 = 0.999$ , indicating an optimal fit.

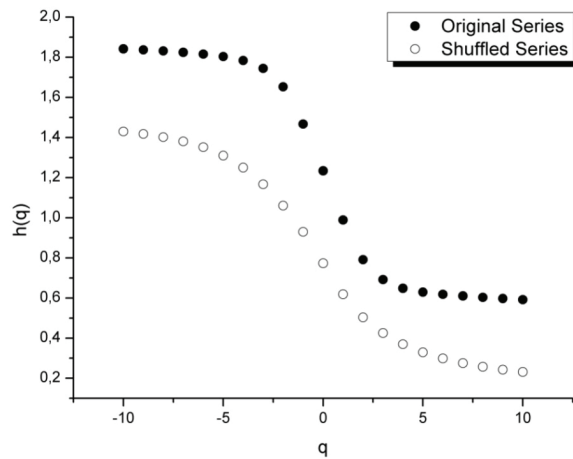


**Figure 1.** Fluctuation function  $F_q(n)$  versus box size  $n$  on double logarithmic scale, for different values of  $q$  from -10 to 10 with a step of 1 (from bottom to top).

Figure 2 shows scaling exponents  $h(q)$  for original and shuffled series. After shuffling the range of  $h(q)$  decreases but does not become constant, meaning that both large probability density function and different long-term correlations for small and large fluctuations contribute to observed multifractality (Kantelhardt et al., 2002). It is seen from Figure 2 that  $h(q)$  is a decreasing function of  $q$ , indicating multifractal behavior of precipitation dynamics, which is in agreement with the results obtained for other locations (Kantelhardt et al., 2006), supporting the hypotheses of the existence of multiplicative cascade processes in the atmosphere (Lovejoy & Schertzer, 2010; De Souto Araújo, Stosic, & Stosic, 2014). These empirical findings provide the base for evaluation of the applicability of universal multifractal model (Schertzer & Lovejoy, 1987) for generating synthetic rainfall data of higher resolution in this location, which may be used to improve the estimation of extreme values (Douglas & Barros, 2003).

Motivated by the results of Moraes et al. (1998) that reveal a strong positive trend in precipitation for the entire basin after 1970, we repeat MF-DFA analysis for two subseries, corresponding to the periods before and after 1970, and obtained an

almost identical  $h(q)$  spectrum. These findings indicate that although the intensity of the rainfall increased, the dynamics of the underlying process did not change.



**Figure 2.** Generalised Hurst exponent  $h(q)$  before and after shuffling of the data.

## Conclusion

In this work, the dynamics of a long-term daily rainfall time series for one station located in Piracicaba River basin were analyzed via Multifractal Detrended Fluctuation Analysis (MF-DFA). Numerical results show that these time series have multifractal behavior which stems from both the broad probability density function and from different long-term correlations for small and large fluctuations. The current findings should be taken into account when formulating and/or validating new simulation models, in the sense that any future "microscopic" statistical models should reflect the nature of long-term correlations observed in real data, thus leading to more reliable predictions.

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