Acta Scientiarum

http://periodicos.uem.br/ojs/acta
ISSN on-line: 1807-8664
Doi: 10.4025/actascitechnol.v42i1.41950

Estimation of lacunarity using gamma regression model

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ABSTRACT. In this study we evaluate estimated lacunarity in three different databases using the power (standard) and gamma model. Results showed that estimates of lacunarity using gamma regression model was superior to those with the power regression model. The gamma regression model had a higher coefficient of model determination than the power regression model for all three used databases and, additionally, had smaller sums of residuals squared. The Gamma model was chosen the most appropriate model for lacunarity estimates.

Keywords: lacunarity; gamma model; fractal.

Received on March 7, 2018.
Accepted on June 12, 2018

Introduction

The concept of lacunarity, a word derived from the Latin 'gap', and meaning 'emptiness', was initially developed for the detection of texture patterns of fractal objects. However, it can be generalized to any spatial pattern, including those with fractal and multifractal properties, be used with both quantitative and binary data, and in 1D, 2D and 3D dimensions (Plotnick, Gardner, Hargrove, Prestegaard, & Perlmutter, 1996).

Several methods for calculating lacunarity have been developed as a result of computational advancement (Gefen, Meir, Mandelbrot, & Aharony, 1983; Lin & Yang, 1986; Allain & Cloitre, 1991; Voss, 1991; Dong, 2009), of which the gliding box algorithm is prominent (Allain & Cloitre, 1991).

The concept of lacunarity has been widely used in medical studies, analysis of human retinal patterns (Cheng & Huang, 2003), assessment of epidermal tissues (Karperien & Jelinek, 2015), as well as in dental radiographs (Yasar & Akgünlü, 2005), and computer tomography images of trabecular bones (Dougherty & Henebry, 2001).

For ecological studies, Plotnick, Gardner, and O'Neill (1993) have used ~~a~~ lacunarity to develop a forest texture index, while Malhi and Román-Cuesta (2008) evaluated IKONOS images of the Amazon rainforests. In geology, Roy, Perfect, Dunne, Odling, and Kim (2010) used lacunarity analysis to study fracture networks and test scale-dependent clustering.

Du and Yeo (2002) created a new technique of estimation of lacunarity applying in the segmentation of SAR images. In the area of food technology, Velazquez-Camilo, Bolaños-Reynoso, Rodriguez, and Alvarez-Ramirez (2010) detected patterns in sugarcane crystallisation using fractal images with the aid of 2D lacunarity analysis. In the area of urban planning, Myint & Lam (2005) were able to distinguish urban patterns in texture images in Oklahoma City using 2D lacunarity analysis.

The analysis of lacunarity is also well developed in climatology. Uses have included the characterization of daily pluviometric regimes in the Iberian Peninsula from 1950 to 1990 (Martínez, Lana, Burgueño, & Serra, 2007); studies of the complex behavior of pluviometric regimes in Europe between 1950 to 2000 (Lana, Martínez, Serra, & Burgueño, 2010); investigating the dynamics of monthly North Atlantic oscillation intensity (Martínez, Lana, Burgueño, & Serra, 2010), temperatures in northeastern Spain between 1917 and 2005 (Lana, Burgueño, Serra, & Martínez, 2015).

In recent studies in Brazil, lacunarity analysis has been used to study precipitation behaviour (Lucena & Campos, 2014; Lucena, Stosic, & Filho, 2015; Lucena, 2015; Lucena, Stosic, Filho, & Santos, 2016); wind directions (Lucena, 2016; Lucena, Souto, Stosic, & Filho, 2017a); water flow within the Piracicaba river basin (Lucena, Stosic, & Filho, 2014), temperatures in the northeast (Lucena et al., 2016), spatial variability of forest fires detected in the Legal Amazon (Lucena, Santos, Stosic, & Filho, 2017b).

In all above-mentioned studies calculations for lacunarity estimation were made via a power regression model. Thus, to find a new form that best estimates the value of lacunarity, we proposed for the estimation of lacunarity the gamma regression model with link function one over the squared mean.

Material and methods

Data

Three databases were used, two of which were obtained from studies of Lucena et al. (2014) and Lucena et al. (2017b). The third refers to an edge cube 100, where for each its elements of cube the values 0 and 1 were simulated, where 0 is the non-occupation of the element of cube and 1 is the occupation of the element of cube. The other two database refer to (i) a historical series (1930-1974) of 16436 daily flow records for the Atibaia River hydrological station 3D-006 (Lucena et al., 2014), and (ii) records of fires occurring during the year of 2000 within the legal Amazon region. The latter database was obtained from the INPE website (www.inpe.br), always considering the reference satellite indicated in the referred site NOAA-12 satellite, in this database were used information on detection time and spatial location (longitude and latitude) of fires in the Legal Amazon, is composed of a matrix of dimension 3500X2450 where each element of this matrix is constituted of the number of records of fires in a determined latitude and longitude of the evaluated area throughout the year (Lucena et al., 2017b).

Lacunarity analysis

Procedures for the calculation of lacunarity-1D:

I - A box of size *r* is placed at the origin of the database, and the number of occupied sites above a pre-determined threshold is counted.

II - The box is moved along the whole set of observations and its mass is calculated.

III - This process is repeated for the whole set of observations, obtaining the frequency distribution of the mass of the box *n(s, r),* that is number of boxes of size *r* with mass *s* and *s* is the number of occupied sites of box, and a corresponding distribution probability $P(s,r) = \frac{n(s,r)}{N(r)}$, where $N(r) = N - r + 1$ is the total number of boxes of size *r*, and *N* is the total of observations. The associated random variable P (s, r) is the number of occupied sites of size r for the vector.

IV - Lacunarity for box size r is defined by Equation 1:

$$L(r) = \frac{M_2}{{M_1}^2} \qquad (1)$$

where:

$M_1 = \sum_{s=1}^{r} s\, P(s,r)$ and $M_1 = \sum_{s=1}^{r} s^2\, P(s,r)$ are the first and the second moments of the distribution of $P(s,r)$, respectively.

Procedures for the calculation of lacunarity-2D:

I - A box of size *r x r* is placed at the origin of an information matrix *L x C* and its mass representing the number '*s*' of occupied sites is counted.

II - The box is moved along the whole set of observations and its mass is calculated.

III- This process is repeated for the whole set of observations, obtaining the frequency distribution of the mass for the box *n(s, r)*, that is, the number of boxes of size *r* with mass *s* where *s* is the number of occupied sites of box, and a corresponding distribution probability $(s,r) = \frac{n(s,r)}{N(r)}$, where $N(r) = (L - r + 1)(C - r + 1)$ is the total number of size boxes *r x r* slid in the information matrix. The associated random variable P (s, r) is the number of occupied sites of size r in the matrix.

IV - Lacunarity for box size *r x r* is defined by Equation 2:

$$L(r) = \frac{M_2}{{M_1}^2} \qquad (2)$$

where:

$M_1 = \sum_{s=1}^{r^2} s\, P(s,r)$ and $M_2 = \sum_{s=1}^{r^2} s^2\, P(s,r)$ are the first and the second moments of the distribution of P (s, r), respectively.

Procedures for calculating lacunarity-3D:

I - A cube of size *r x r x r* is placed at the origin of a 3D information object *LX x LY x LZ* and the number *s* of occupied sites other than a pre-defined threshold is counted, in *LX, LY* and *LZ* are the dimensions of the cube.

II - The cube is moved along the entire 3D object, and its mass is calculated.

III- This process is repeated for the entire 3D object, obtaining the frequency distribution of the cube mass *n(s, r)*, that is number of boxes of size *r* with mass *s* and *s* is the number of occupied sites of box, and a corresponding distribution probability $P(s,r) = \frac{n(s,r)}{N(r)}$, where $N(r) = (LX - r + 1)(LY - r + 1)(LZ - r + 1)$ is the total number of cubes of size *r x r x r* and *N* is the edge of the 3D object. The associated random variable P (s, r) is the number of occupied sites of size r within the cube.

IV - Lacunarity for cube size *r x r x r* is defined by Equation 3:

$$L(r) = \frac{M_2}{{M_1}^2} \tag{3}$$

where:

$M_1 = \sum_{s=1}^{r^3} s\, P(s,r)$ and $M_2 = \sum_{s=1}^{r^3} s^2\, P(s,r)$ are the first and the second moments of the distribution of $P(s,r)$, respectively (Martínez et al., 2007).

Regression model

Let X be a random variable with Gaussian distribution with mean 0 and constant variance σ^2, we define its probability density function by Equation 4:

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp^{\frac{-x^2}{2\sigma^2}} \tag{4}$$

The power regression model (Nascimento, Ribeiro, Rocha, & Lucena, 2017) is defined by Equation 5:

$$Y_i = \beta_0 {r_i}^{\beta_1} \varepsilon_i \tag{5}$$

where:

Y_i is the ith observed lacunarity; r_i is the size of the ith box and ε_i is the ith error associated with lacunarity, where ε_i presents normal distribution of mean 0 and constant variance σ^2. The unknowns β_0 and β_1 are the parameters associated with the model, and the parameters of model were estimated by least squares method (Nascimento et al., 2017).

Let X be a random variable with gamma distribution with parameters α (shape) and β (scale), then define its probability density function by Equation 6:

$$f(x;\alpha;\beta) = \frac{1}{\Gamma(\alpha)\beta^{\alpha}} x^{\alpha-1} \exp^{\frac{-x}{\beta}} \tag{6}$$

where:

$\alpha > 0$ and $\beta > 0$.

The gamma regression model (Nascimento et al., 2017), with link function $g(\mu_i) = \frac{1}{(\beta_0+\beta_1 r_i)^2}$ is defined by Equation 7:

$$Y_i = (1/\beta_0 + \beta_1 r_i + \varepsilon_i)^{1/2} \tag{7}$$

where:

Y_i is the ith observed lacunarity; r_i is the size of the ith box, and ε_i is the ith error associated with lacunarity, where ε_i presents gamma distribution of parameters α and β. The unknowns β_0 and β_1 are the parameters associated with the model, and the parameters of model were estimated by least squares method.

Selection criteria of model

Models were evaluated by the following criteria: coefficient of determination of the model of McFadden (R^2_{MF}), Akaike information criterion (AIC), and the sum of the squares of the residuals (SQR).

Let $\widehat{Y}_i$ be the ith lacunarity value after fitting the model; then we define the sum of the squares of the residues, for this study, by the following Equation 8:

$$SQR = \sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2 \quad (8)$$

The coefficient of determination of the model of McFadden (R_{MF}^2) is expressed by the ratio between of the log-likelihood of the final model with the model composed only of the intercept (null model), that is Equation 9:

$$R_{MF}^2 = 1 - \frac{L_1}{L_0} \quad (9)$$

where:

L_0, L_1 are log-likelihood of null and final model, respectively.

The Akaike information criterion (AIC) defined by Akaike (1974) is given by Equation 10:

$$AIC = -2L(x \backslash \hat{\theta}) + 2p \quad (10)$$

where:

p is number of parameters of model and L (x\ θ̂) is the maximum likelihood function, defined as the density function product (Nascimento et al., 2017), and given by Equation 11:

$$L(x \backslash \hat{\theta}) = \prod_{i=1}^{n} f(x_i) \quad (11)$$

Results and discussion

Using the lacunarity results for a historical series (1930-1974) of daily flow records from Atibaia river station 3D-006, obtained by (Lucena et al., 2014); In Figure 1 we showed that the gamma model gave the best estimates in comparison to the power model when explaining lacunarity as a function of box size, because the values adjusted by gamma model are closer to observed lacunarity values than those estimated by the power model.

Table 1 shows that the gamma model had AIC = 3.422, SQR = 0.43 and R_{MF}^2 = 99.45%, while the power model had AIC = -15.5, SQR = 1.8 and R_{MF}^2 = 99.15%. Gamma and power models adjusted for lacunarity estimation in relation to box size were defined in Table 1.

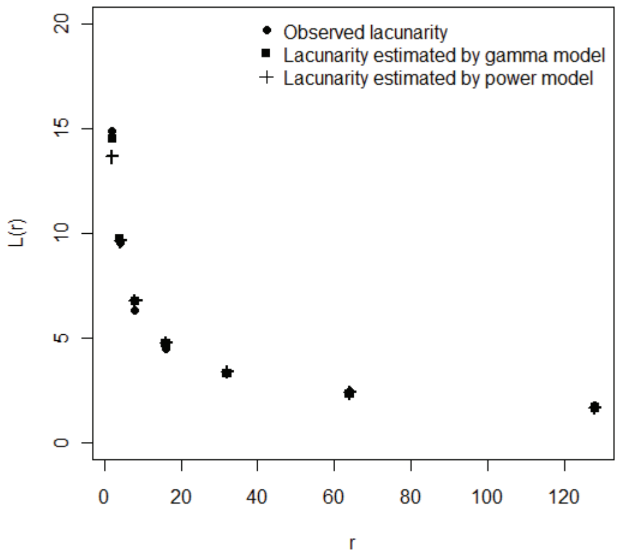

Figure 1. Lacunarity values and lacunarity estimates using gamma and power models in relation to the size of the box, using a historical flow series database.

Table 1. Regression model and adequacy criteria of model using as a database the historical flow series.

Distribution	Equation of model	R_{MF}^2 (%)	AIC	SQR
Power	$L(r) = 19.375\ r^{-0.505}$	99.15	-15.50	1.80
Gamma	$L(r) = (1/-0.001 + 0.00288r)^{1/2}$	99.65	3.42	0.43

Figure 2 graphically displays the data on the number of fires occurring in the legal Amazon in 2000 (Lucena et al., 2017b). It can be seen that results obtained via estimation with the gamma model are much more accurate than the conventional approach using the power model. For all box sizes, gamma model-derived estimates are more accurate than those from the power model, because the values adjusted by gamma model are closer to observed lacunarity values than those estimated by the power model.

The power model had Akaike (AIC = -0.626), sum of squares of residuals (SQR = 887.54), and determination coefficient (R^2_{MF} = 97.08%), while the gamma model had AIC = 32.929, SQR = 19.59 and R^2_{MF} = 99.57%, Table 2.

Figure 3 shows the use of a simulated edge cube 100. It can be seen that the gamma model had estimates close to the observed lacunarity values, the same can be seen for the power model.

Although the models show optimal estimates for the observed lacunarity values, the gamma model is better suited to explain this phenomenon, since it had a determination coefficient R^2_{MF} = 94.99% higher than the power model R^2_{MF} = 93.11%, as well as a smaller sum of squares of residual (SQR = 20.288 gamma model and SQR = 52.946 model power), Table 3.

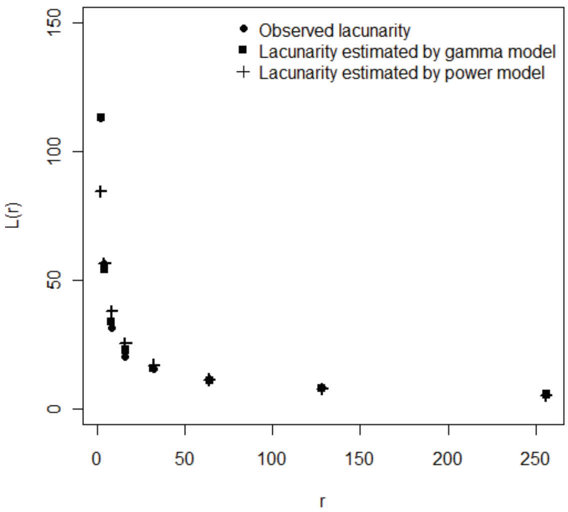

Figure 2. Lacunarity values and estimates of lacunarity using the gamma and power models for the size of the box, using a database of legal Amazon forest fires in 2000.

Table 2. Regression model and adequacy criteria of model using a database of legal Amazon forest fires in 2000.

Distribution	Equation of model	R^2_{MF}(%)	AIC	SQR
Power	$L(r) = 126.469\ r^{-0.581}$	97.08	-0.626	887.54
Gamma	$L(r) = (1/-0.0001876 + 0.0001329r)^{1/2}$	99.57	32.929	19.59

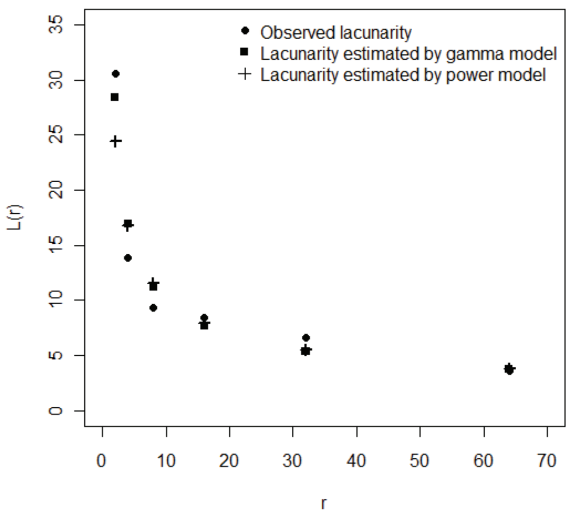

Figure 3. Lacunarity values and lacunarity estimates using the gamma and power models in relation to box size with a simulated cube of edge 100 as the database.

Table 3. Regression model and adequacy criteria of model using a simulated cube of edge 100 as the database.

Distribution	Equation of model	R^2_{MF}(%)	AIC	SQR
Power	$L(r) = 35.517\ r^{-0.539}$	93.11	1.9265	52.946
Gamma	$L(r) = (1/ -0.00099 + 0.00112r)^{1/2}$	94.99	27.526	20.288

Conclusion

For estimating lacunarity values, the gamma regression model, using a one-way squaring function, was the most appropriate model when compared to the power model, since in all three databases it showed a higher determination coefficient and a smaller sum of squares of residues.

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