



Optimization of sizing of annual water storage reservoirs considering return period association

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ABSTRACT. An increasing concern among water resources managers is the search for ways to improve reservoir sizing techniques. A simple technical and scientifically based technique is the sequent peak method, which has been widely disseminated, but presents limitations with respect to the length of the available data series. The objective of this study was to propose modifications to the sequent peak method to overcome the limitations related to the impossibility of associating the reservoir storage capacity with return periods different from the length of the data series. Storage capacities were estimated for every year, considering gauge stations located in the Urucuia River Basin, based on the sequent peak method. In order to associate such capacities with a frequency factor (return period), the Gumbel distribution was applied to the estimated storage values. This association with return periods which differ from the number of years in the data series showed great applicability.

Keywords: streamflow regularization; water storage; return period.

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Introduction

The main purpose of a reservoir is to regulate natural streamflow by storing excess water in the rainy season and releasing the stored water in the dry season. In general, most of the annual streamflow is available during a few months of the rainy season, but water demands takes place all year round, thus storage is necessary to attend the demands in periods when the natural streamflow is not sufficient to do so (Adeloye, Soundharajan, & Mohammed, 2017; Xu et al., 2017).

In this sense, according to Marino and Loaiciga (1985) a successful sizing method must be developed considering the characteristics of the system, in order to obtain simple mathematical formulations and overcome the usual dimensionality problems. There are simplified methods and more complex models of simulation and optimization. The simplified methods have some limitations that make it difficult to be applied to the dimensioning of complex systems. Simulation and optimization models, on the other hand, describe the behavior of the system over time and space depending on a given operating scenario. They can take into account the stochastic nature of rain and streamflow, and still preserve the characteristics of the natural hydrological regime through the use of extensive series (observed or generated).

Several classifications of reservoirs are possible depending on their purpose, size and storage capacity. Based on the capacity, a reservoir may be classified as an annual (seasonal) or pluriannual (over-year) storage. An annual reservoir is designed to attend the requirements in months of water deficit within a year. These reservoirs are usually built on small tributaries to serve relatively small areas. A pluriannual reservoir is designed to attend water demands for periods longer than a year, where the storage accumulated at the end of a water year is carried over to the following year (Jain & Singh, 2003).

Among the methods used to compute storage capacities for annual regularization reservoirs, those which are based on the critical period concept are highlighted. The critical period is defined as the period of convergence between the highest demand and lowest water supply (Adeloye et al., 2017). Thus, many researchers have used the sequent peak algorithm (SPA) in a variety of hydrological studies for water resources planning and management (Silva, Sánchez-Román, Teixeira, Franzotti, & Folegatti, 2013; Patskoski & Sankarasubramanian, 2015; Kuria & Vogel, 2015; Turner & Galelli, 2016; Adeloye et al., 2017; Al-Zakar, Sarlak, & Agha, 2017). Soundharajan, Adeloye, and Remesan (2016) and Adeloye, Soundharajan, Ojha, and

Remesan (2016), for instance, reported that the SPA is one of the methods used to calculate the capacity of the reservoir under conditions of climate change.

One limitation of sizing techniques, including the sequent peak algorithm, is the fact that they do not allow the association of the reservoir capacity to a return period. Their extreme dependence upon the available database frequently limits their application (Nunes & Pruski, 2015; Patskoski & Sankarasubramanian, 2018). In this sense, some initiatives have been proposed. Sawatpru and Konyai (2016), for example, performed frequency analyses to learn the severity of streamflow droughts along the Yom River and quantified the amounts of storage needed along the river with acceptable risks. A deficit volume was used to characterize a drought event and the Weibull distribution model was chosen for analysis after comparison of log normal and Pareto models with an empirical distribution.

Nunes and Pruski (2015) proposed potential modifications to the Reservoir Operation Study to overcome the dependence upon the first year of the time series and inability of associating the reservoir storage capacity with a frequency (return period). To make the reservoir capacity independent from the first year of the time series, they created $(N - 1)$ synthetic series of streamflow ($N =$ the number of years in the time series) and applied the Reservoir Operation Study method to each one; and to associate the reservoir capacity with a frequency factor (return period), they applied a Gumbel distribution to the reservoir capacity estimated from each one of the synthetic series. In this case, they worked with pluriannual reservoirs.

In view of the foregoing, the present study was based on the hypothesis that the technical and scientific basis of the sequent peak algorithm combined with a minor dependence on the characteristics of the database can increase the use of this method, being still a research line little explored. The paper proposes modifications to the method in order to overcome the aforementioned limitations.

Material and methods

Sequent peak method

The regulation volume computed by the sequent peak method is equal to the largest amplitude of the net accumulated volume (output volume subtracted from the input volume) estimated for the streamflow data series (Patra, 2008; Silva et al., 2013). The algorithm steps are:

- Calculate $Q_i - D_i$ (input volume minus the regulated volume – 40% of regularization) for $i = 1, 2, \dots, N$. The net accumulated volume is then calculated $V_i = \sum_{i=1}^N (Q_i - D_i)$;
- Locate the first peak P_1 (local maxima, equal to the value of V_i higher than the previous V_{i-1} and higher than the posterior V_{i+1}), in the column of the net accumulated volumes V_i ;
- Locate the following peak P_2 , which is the second largest peak, i.e., $P_2 \geq P_1$;
- Between the pair of peaks P_1 and P_2 , find the lowest value T_1 of the net accumulated volume V_i and then calculate $P_1 - T_1$;
- Starting with P_2 , find the next sequent peak P_3 , which must be higher than P_2 ;
- Find the lowest value T_2 of V_i , between P_2 and P_3 , and calculate $P_2 - T_2$;
- Starting with P_3 , find P_4 and T_3 , and calculate $P_3 - T_3$;
- Continue for all sequent peaks of the N data series; and
- Calculate the required reservoir capacity, given by: $C = \max (P_k - T_k)$, as shown in Figure 1.

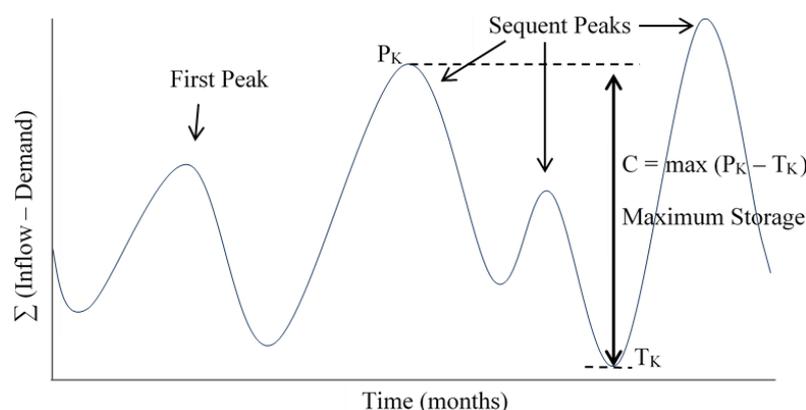


Figure 1. Representation of the critical period in the sequent peak method.

However, as cited in the introduction, there are limitations associated to this method, as the fact that it does not allow the association of the reservoir capacity to a return period. The extreme dependence upon the available database frequently limits its application.

Modifications on the sequent peak method

Given the limitations of the sequent peak method, we propose the application of one statistical distribution in order to associate the reservoir capacity to return periods different from the number of years in the data series and, consequently, minimize its dependence upon the length of the available time series data.

Considering the characteristics of the data and according to Kuria and Vogel (2015), Youn, Chung, Kang, and Sung (2012) and Sawatpru and Konyai (2016) Gamma, Generalized Extreme Value (GEV), Gumbel, Lognormal 2P, Lognormal 3P and Pearson type III distributions were considered appropriated to fit the probability distribution. Kolmogorov–Smirnov (K–S) test was used to examine the goodness-of-fit of these distribution fittings with a significance level of 5%. ALEA software was used for data processing.

Generally, the more parameters a distribution has, the better it will fit to the data. However, difficulty in the parameter estimation arises, and the distribution maybe too rigid to accurately extrapolate beyond the range of the available data (Catalunha, Sediya, Leal, Soares, & Ribeiro, 2002). In this work, all the distribution fittings significantly passed the goodness-of-fit test and Gumbel distribution was applied to the 'N' reservoir capacity values calculated by the sequent peak method for every year.

According to Naghettini and Pinto (2007), the magnitude of the event is given by

$$x(T) = \beta - \alpha \ln \left(- \ln \left(1 - \frac{1}{T} \right) \right) \quad (1)$$

where:

$x(T)$ = reservoir capacity with a return period T , m^3 ;

β = position parameter;

α = scale parameter;

T = return period, years.

Estimating the distribution parameters by the Moments Method (MOM) are obtained:

$$\beta = \bar{X} - 0,45 s_x \quad (2)$$

where:

\bar{X} = sample mean of reservoir capacities, m^3 ;

s_x = sample standard deviation of reservoir capacities, m^3 .

and

$$\alpha = \frac{s_x}{1,283} \quad (3)$$

Case study

The study was carried out using consistent time series data from 3 gauge stations located in the Urucua River Basin (Figure 2). The stations belong to the hydro-meteorological network operated by the Brazilian National Water Agency (ANA).

After the obtention of the time series for each station, we analyzed the availability of data for each year and selected those which did not present missing or inconsistent values.

Results and discussion

Table 1 shows the reservoir capacities obtained for each gauge station, for each year, using the provided time series data and the sequent peak method. The highest capacity values found for each station are highlighted in grey.

The highlighted values are representative of a specific dry year, which will change only if an even drier year than the critical one is incorporated into the series. The consideration of this specific value does not allow the association of the reservoir capacity with the variations in the period of analysis. Such association occurs when using one statistical distribution, as proposed in this work.

Table 2 shows the fitted probability distributions under the different stations. A ranking scheme was developed to judge the overall goodness-of-fit of each distribution by comparing the test statistic and p-value

of K-S test. A distribution with the lowest test statistic or highest p-value would be given a rank of 1. Table 2 summarizes the overall ranking results.

Examination of the goodness-of-fit test results reveals that in many cases there was little difference between the various distributions for each station. The best-fitting probability distributions of Station 43429998 were Gumbel and GEV distributions, respectively. In Station 43670000, Pearson type III and GEV. GEV and Lognormal 3P provided the best fit for Station 43880000. The Lognormal 2P distributions was rank consistently poorly compared to the other stations. However, the fact that a distribution has a low ranking does not necessarily mean that it performed poorly, since the differences of good fit between different distributions may or may not be statistically significant. Thus, all fitted probability distributions were concluded to be appropriate for all stations because the K-S Test results.

After assessing how well each distribution fit to the overall data sets, Table 3 shows the reservoir capacity values obtained by the Gumbel distribution for return periods of 10, 20, 30, 40, 50 and 100 years, as well as the highest capacity values found for each station using the sequent peak method.

For the station 43429998, the capacities ranged from 418.6 to 710.1 hm^3 for return periods between 10 and 100 years, and the highest capacity found for the station using the sequent peak method was 523.2 hm^3 . If we consider the capacity estimated for the return period equal to N years of the time series, then for $T = 20$, the estimated value was 507.9 hm^3 , i.e., 3% lower than the highest value found for the series containing 18 years of data (523.2 hm^3). This difference was expected considering that errors in the fitting process of using a probability distribution function instead of using empirical values are expected to occur.

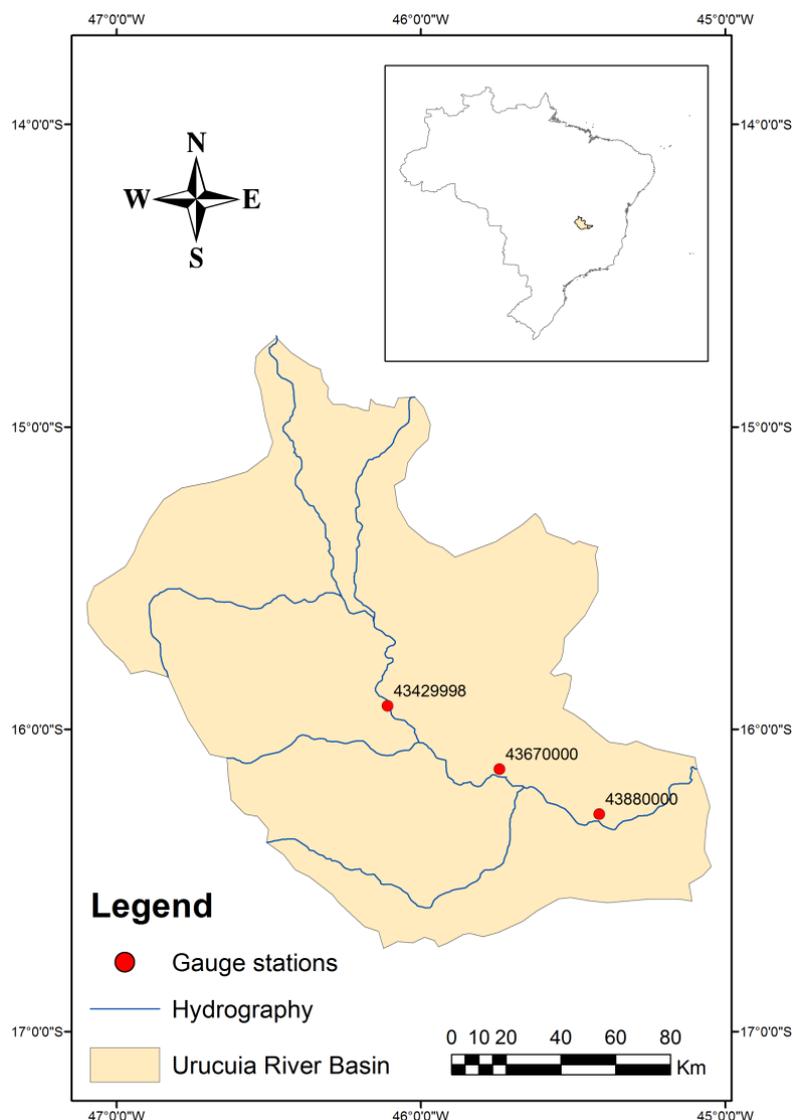


Figure 2. Location of the gauge stations used in the study.

Table 1. Reservoir capacities (hm³) for each year of the time series data obtained from the 3 gauge stations used in the study.

Time series	Gauge stations		
	43429998	43670000	43880000
1	120.3266	128.0377	282.0893
2	26.7466	492.9772	196.0743
3	30.8126	258.0749	547.9255
4	100.6093	759.0358	238.2758
5	27.4717	550.2261	847.1246
6	59.3723	415.328	590.231
7	71.2807	158.9672	452.2674
8	195.5576	548.1334	153.6418
9	211.6412	845.6145	582.4335
10	341.2273	799.5169	932.298
11	144.7304	215.6076	834.9728
12	152.6533	35.0263	162.0272
13	256.189	68.8858	60.7616
14	409.672	125.26	127.6616
15	258.202	74.6221	44.5741
16	468.552	115.6499	105.6295
17	523.217	143.0886	88.3623
18	398.243	311.0642	327.8253
19		357.7396	298.2856
20		594.6413	611.8619
21		355.3714	371.1377
22		574.142	241.4594
23		256.1985	362.2388
24		279.9636	253.5866
25		147.2421	314.7832
26		497.1003	535.337
27		405.8016	877.162
28		630.4472	495.5092
29		900.5687	975.5069
30		579.3605	1120.147
31		994.995	848.9773
32		1074.339	1131.537
33		834.0279	111.4959
34			946.7663
35			383.409
36			593.0431
37			145.1838

Table 2. Fitted probability distribution under different gauge stations.

Probability distributions	43429998			43670000			43880000		
	K-S Test			K-S Test			K-S Test		
	Test Statistic (Critical Value = 0.309)	p-value	Ranking	Test Statistic (Critical Value = 0.231)	p-value	Ranking	Test Statistic (Critical Value = 0.218)	p-value	Ranking
Gamma	0.118	0.950	5°	0.130	0.606	5°	0.119	0.642	4°
GEV	0.108	0.978	2°	0.105	0.842	2°	0.114	0.694	1°
Gumbel	0.106	0.983	1°	0.125	0.648	4°	0.122	0.610	5°
Lognormal 2P	0.178	0.578	6°	0.188	0.173	6°	0.157	0.296	6°
Lognormal 3P	0.112	0.969	4°	0.106	0.833	3°	0.116	0.671	2°
Pearson type III	0.110	0.973	3°	0.104	0.846	1°	0.117	0.664	3°

Table 3. Reservoir capacities (hm³) obtained using the sequent peak method (SPM) and the Gumbel distribution for return periods of 10, 20, 30, 40, 50 and 100 years.

Stations	Return period (T)						SPM
	10	20	30	40	50	100	
43429998	418.6	507.9	559.3	595.5	623.5	710.1	523.2
43670000	823.8	988.8	1,083.7	1,150.6	1,202.4	1,362.4	1,074.3
43880000	885.9	1,067.1	1,171.4	1,244.9	1,301.7	1,477.5	1,131.5

When comparing the other capacities obtained by the Gumbel distribution with the highest value obtained by the sequent peak method (last column of Table 3), for the station 43429998, it was possible to observe that for T < N years of the series (T = 10), the estimated capacity was 20% lower than the highest value obtained

by the method. For $T > N$ years of the series ($T = 30, 40, 50, 100$) the capacities exceeded in 7% (for $T = 30$) and 36% (for $T = 100$) the value obtained using the sequent peak method.

The behavior observed for the station 43429998 is quite similar to that observed for the stations 43670000 and 43880000, as shown in Figure 3. The highest capacities estimated by the sequent peak method are represented by SPM lines and those estimated by the Gumbel distribution for different return periods were plotted in the MSPM (modified sequent peak method).

Figure 3 shows that when considering the effect of associating capacities with different return periods (MSPM), for $T = N$ years of the series, the estimated value differed by 1% from the highest value estimated by the sequent peak method for the station 43670000, and by 10% for the station 43880000.

Considering the capacities associated with return periods shorter than the number of years (N) of the series, $T = 10$ and 20 years, it was possible to observe that they presented coherent behaviors. For $T = 10$ years, the capacities estimated by the Gumbel distribution were 23 and 22% lower than the highest value estimated by the sequent peak method, for the stations 43670000 and 43880000, respectively. For $T = 20$ years, the capacities estimated by Gumbel were 8 and 6% lower than the highest value estimated by the sequent peak method.

For the capacities associated with return periods longer than the number of years (N) of the series, for the station 43670000, the differences with respect to the highest value estimated by the sequent peak method and those estimated by the Gumbel distribution ranged from 7%, for $T = 40$ to 27%, for $T = 100$. For the station 43880000, such differences ranged from 15%, for $T = 50$ to 31%, for $T = 100$.

Similar results were obtained by Nunes and Pruski (2015), which proposed potential modifications to the ROS to overcome the dependence upon the first year of the time series and inability of associating the reservoir storage capacity with a frequency (return period). Applying Gumbel distribution, in this case working with pluriannual reservoirs, when the return period was greater than the number of years in the time series, the differences ranged from 10 to 62 % for $T = 40$ and from 20 to 78% for $T = 100$.

Still in this context, Sawatpru and Konyai (2016) performed frequency analyses to learn the severity of streamflow droughts along the Yom River and quantified the amounts of storage needed along the river with acceptable risks. Weibull distribution model was chosen for analysis after comparison of log normal, and Pareto models with an empirical distribution. The authors concluded that the method presented can help to quantify the severity of hydrological drought along any river so that drought management measures can be undertaken.

Considering, then, future works, probability functions different to Gumbel distribution can be used, according to data profile, in order to better associate the reservoir capacity with a frequency factor (return period). It is also important to mention that it was calculated annual storage for high return periods as 50-100 years, considering 40% of the potential of regularization (regulated volume). For higher percentages of regularization, shorter return periods should be evaluated, considering that the assumption of intra-annual regulation and independence between years can be compromised.

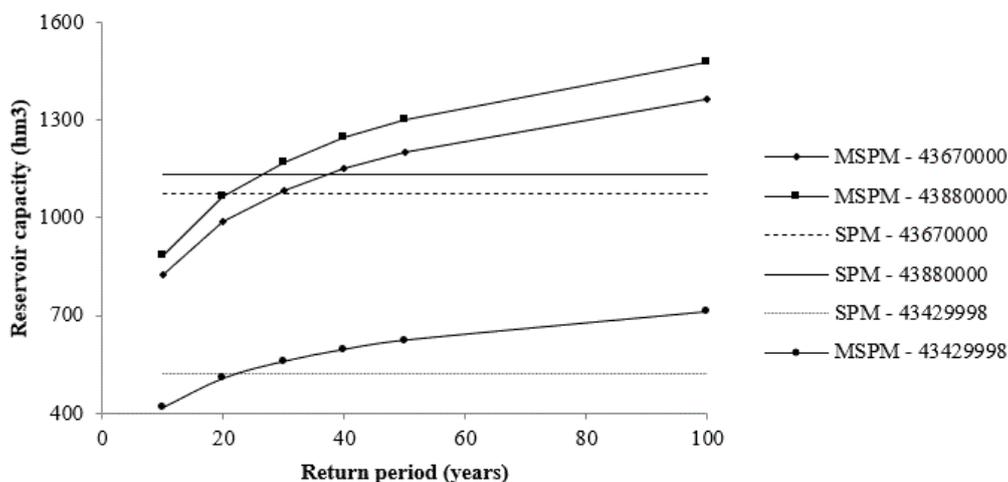


Figure 3. Reservoir capacities estimated by the sequent peak method (SPM) and Gumbel distribution (MSPM) for different return periods, and for stations 43429998, 43670000 and 43880000.

Conclusion

The modifications proposed to the sequent peak method showed great applicability to overcome the limitations related to the dependence upon the length of the time series, i.e., they allowed the association of the reservoir capacity with return periods different from the number of years in the data series.

The resulting approach suggests that the technique is a tool that is more reliable than the classical empirical and more practical than the synthetic hydrology approaches in reservoir storage studies. However, it is important to mention, this is a simplified method with limitations to be applied in the dimensioning of complex systems.

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