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TECHNOLOGICAL INFORMATION

Proposal of a method for routing school buses in a small-sized county

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ABSTRACT. The School Bus Routing Problem (SBRP) is widely discussed in the operations research literature and can be solved by several exact methods and heuristics. This problem seeks to designate the most efficient routes for a fleet of school buses, minimizing the total distance covered and considering variables such as bus stop locations, number of passengers, and the assigned destination for each of them. This study aims at solving a real case SBRP of a small-sized county located in the state of Paraná. The proposed method is based on the Capacitated Vehicle Routing Problem (CVRP) and Travelling Salesman Problem (TSP) combined with a heuristic correction that guarantees sequence constraints, in which the student has to be collected before visiting their destination school. It was possible to obtain two routes of 30.76 km and 17.42 km respectively and both with the total vehicles' capacity of 24 students, which corresponds to the reduction of about 10% in the daily distance covered by two buses.

Keywords: school bus routing problem; vehicle routing problem; minimum path.

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Introduction

Since its first publication by Dantzig and Ramser (1959) as an adaptation of the Traveling Salesman Problem (TSP), the Vehicle Routing Problem (VRP) has been constantly adapted and improved in such a way that there are different sub-methods capable of encompassing different variables and constraints to find the solution of several routing problems (Pillac, Gendreau, Guéret, & Medaglia, 2013). One of the VRP's research branches focuses on the collective transportation of students to their educational institutions with applications of methods such as Capacitated Vehicle Routing Problem (CVRP), or Dial-a-Ride Problem (DARP), or the Heterogeneous Fleet VRP (HVRP), among others (D'Souza, Omkar, & Senthilnath, 2012; Mohammed et al., 2017; Sales, Melo, Bonates, & Prata, 2018). When the focus is on the school transport, it is called School Bus Routing Problem (SBRP), initially proposed by Newton and Thomas (1969).

According to Park and Kim (2010), the SBRP branches into a few subproblems: data preparation, bus stop selection, bus route generation, school bell time adjustment, and bus route schedule; it is possible to find studies that solve a single subproblem or even a set of them together in the same study. Still, according to the authors, it is possible to classify a SBRP study in the following categories: number of schools, service environment, problem scope, mixed loads, special-education students, fleet mix, objectives, and constraints. Ellegood, Solomon, North, and Campbell (2020) continued the review by Park and Kim (2010) and both studies were able to list numerous papers that fit within these categories, demonstrating the range of varieties and the high number of possibilities in solving routing problems by this method. In addition, Ellegood et al. (2020) highlighted trends in the expansion of studies applied to scenarios with multiple schools and considering several subproblems simultaneously, as well as the use of heuristics to solve complex problems.

This article deals with a real problem of routing school buses in a network with multiple schools in a small city. The students' transportation to schools is done in the mornings with a fleet of buses with the same capacity, it is possible to load students from different schools on the same bus and it does not involve transporting students with special needs. The objective of the study is to minimize the distance traveled and the consequent minimization of costs, taking into account the known capacities of the buses, the bus stops already defined by the city, the demand of students at each of these stops, and their respective schools of destination.

Among the subproblems defined by Park and Kim (2010), this study addresses only the bus routing problem, since bus stops are already defined, there is no possibility to change the school bell times and the focus is not on the route schedule. To solve the bus routing problem, the CVRP method was initially applied to designate vehicles for all of the bus stops, then the TSP method was applied to find the best route for these vehicles, and a correction heuristic was implemented to ensure the correct sequence between points of origin and destination within the routes, all programmed in Python in the commercial optimization solver Gurobi.

The Vehicle Routing Problem (VRP) is seen as one of the most complex mathematical optimization models in the literature (Rashidi, Zokaei-Aashtiani, & Mohammadian, 2009). It has been studied since its first publication by Dantzig and Ramser (1959), in which the authors proposed an adaptation to the Traveling Salesman Problem (TSP), presenting a method to improve the routes of a fleet of gasoline delivery trucks, considering the demand at each point, and the capacity of the trucks in order to minimize the total distance traveled.

TSP is a problem much debated in the literature, it aims at minimizing the travel time or total distance traveled by an individual or vehicle that leaves its origin, visits all points in a network once and returns to the point of origin. When the capacity of the vehicle is considered, the TSP is treated as VRP, so the difference between the two methods is that the VRP produces numerous routes to visit all points, while the TSP produces a single route (Liu, Lin, Chiu, Tsao, & Wang, 2014).

VRP applications deal with the efficient use of a vehicle fleet that needs to travel through a network of nodes to collect and deliver items, which can be products or passengers (Li & Fu, 2002). According to Pillac et al. (2013), the VRP branched out into several other methods that can be found in the literature, such as the Capacitated Vehicle Routing Problem (CVRP), where each point has a demand and vehicles have limited capacity (Letchford & Salazar-González, 2019); the VRP with Multiple Compartments, in which different products need to be transported separately inside the same vehicles, and the VRP with Time Windows, which defines windows of time that each point must be visited (Chen & Shi, 2019); Pick-up and Delivery (PDP), in which each stop has a defined quantity that needs to be collected and delivered (Koç, Laporte, & Tükenmez, 2020); the Heterogeneous Fleet VRP (HVRP), in which each vehicle has a different load capacity (Belfiore & Yoshida Yoshizaki, 2009); the Open Vehicle Routing Problem (OVRP) whose main difference from the traditional VRP is that the vehicles do not return to the origin after traveling through the entire network (Scárdua, Rosa, Sabino, & Vitorugo, 2016); among other different variations.

Likewise, when the VRP involves school buses and student transport, considering a defined travel time, the so-called School Bus Routing Problem (SBPR) is found in the literature (Rashidi et al., 2009). This method has been studied since its first publication by Newton and Thomas (1969) and seeks to optimize the path taken by one or more buses that pass through all the pickup locations of students and deliver them to their respective schools, considering known variables such as total student demand, maximum travel time, and vehicle capacity (Caceres, Batta, & He, 2019).

According to Park and Kim (2010), the SBRP can be subdivided into five sub-problems or steps: data preparation, in which information about schools, bus stops, and points of origin are collected; selection of bus stops, that determines where the pickup of students will be located within the network; routes generation, which defines the path traveled by buses; school bell time adjustment, usually necessary in scenarios with more than one school in which the entry times can be modified to better suit the vehicle routing and students delivery; and bus route schedule, where school time windows, travel time between points and time to pick up students are considered to build an optimized travel schedule. According to the authors, it is possible to find studies in the literature that contemplate the resolution of all these subproblems and treat them as sequential activities, however, it is also possible to find studies considering only some of the subproblems, since other steps are already solved. The study of Bertsimas, Delarue, and Martin (2019), for example, brings the solution of three subproblems, starting with the definition of the bus stops, then the bus route is created and the bus route schedule is defined. The study of Schittekat et al. (2013) focuses on two subproblems, the selection of bus stops and the generation of routes for them, while the study by Banerjee and Smilowitz (2019) focuses on optimization of school entrance times and route schedules, and the study by Shafahi, Wang, and Haghani (2017) solves the problem of routing and scheduling routes.

Also, according to Park and Kim (2010), it is possible to classify SBRP applications according to the problem characteristics, namely: number of schools, service environment, problem scope, mixed loads, special-education students, fleet mix, objectives and constraints. The first classification concerns the number of schools, it is possible to find studies on routes with a single school (Babaei & Rajabi-Bahaabadi, 2019) or studies on multiple schools within a network (Yao et al., 2016). The second classification considers the environment in which the bus service takes place, which may be in an urban environment or, as in the study of Lima, Pereira, Conceição, and Nunes (2016), attending students from the rural area. The third classification takes into account the time of the day that the bus ride happens, which can be in the morning, in the afternoon or both, such as the paper of Rashidi et al. (2009) where the bus ride only happens in the morning, or the study of Li and Fu (2002) who considered the delivery of students to schools during the morning period and the pick-up of students from schools and delivery to their bus stops during the afternoon.

The fourth classification concerns the possibility or not of a bus to transport students that need to go to different destinations, as the study of Park et al. (2012) that demonstrated that the use of mixed loads has benefits such as reduction in the total fleet and total distance traveled, and a maximization of the total use of buses. The fifth classification considers the inclusive transport of students with special needs, such as the study of Caceres et al. (2019) which stated that the transport of special students has additional requirements, such as the use of special equipment, defined travel times, schools and programs located off the mainstream route, and a small number of students who need to reach these locations. The sixth classification presents two possibilities: homogeneous fleets and heterogeneous fleets, the first concerns a bus fleet of vehicles with the same capacity, as presented by Shafahi et al. (2017), and the second, as Sales et al. (2018) which considers different capacities for each of the buses in a fleet.

For the objective category, Park and Kim (2010) pointed out that it is possible to find studies that focus on optimizing the number of buses used, the total distance or time traveled by the vehicle, the distance or total time traveled by a student, the maximum length of a route, or the balance of load or time travel. Still on this category, the study of Ellegood et al. (2020) expanded the studies found by Park and Kim (2010) and also presented new studies focusing on the use of shared bus stops, capacity utilization, total cost optimization, trip compatibility, safety factor, number of transfers or even number of bus stops. Similarly, in the category of constraints, Park and Kim (2010) found in their review constraints related to vehicle capacity, maximum travel time, school time windows, maximum walking time or distance, earliest pick-up time, or minimum number of students to create a route, while Ellegood et al. (2020) expanded presenting new studies constraints about transfer time, stop time windows, maximum stops per route, the chance of overcrowding and chance of being late.

In addition, there are different applications that go beyond the use of methods known to the SBRP, integrating it with other methodologies and heuristics, such as the study of Park, Tae, and Kim (2012), who developed a SBRP formulated in a similar way to Pick-up and Delivery with Time Windows (PDPTW); or like using CVRP for university buses and applying a genetic algorithm to find the best route and cost minimization solution (Mohammed et al., 2017); the use of VRPTW considering the time windows of each school (Kim, Kim, & Park, 2012); or even problems involving a heterogeneous fleet of school buses solved by the application of a memetic algorithm (Sales et al., 2018). The problem of collecting people by buses can also be addressed by 'Dial-a-Ride Problem' (DARP), a demand-dependent system in which the passenger makes a travel requirement between two known points, with the objective of finding a route with the lowest possible cost where vehicles have the ability to accommodate all orders, considering that the origin takes precedence over the destination point (Pillac et al., 2013; Molenbruch, Braekers, & Caris, 2017; Drakoulis et al., 2018).

According to Park and Kim (2010), the use of metaheuristics in solving traditional VRP problems has increased in the past decade and was a successful application, however its use for SBRP was still limited and lacking, the same was true for the use of exact methods. According to the review by Ellegood et al. (2020), currently, there is a tendency to solve more complex school bus routing problems capable of considering real-life scenarios from multiple schools and several joint subproblems, and the use of metaheuristics has been considered as the natural solution to higher complexity problem. The authors also suggest that only half of the SBRP publications combine exact methods with their heuristics to increase the confidence of the proposed models, which represent a gap in the literature that can be explored.

Material and methods

The problem evaluated in this article considers the data of a small city in the state of Paraná, Brazil, which on every workday needs to collect 48 primary students in 18 different bus stops and deliver them to 10 destinations called Municipal Child Education Centers. Each available bus leaves the warehouse with a limited capacity of 24 places and it is assigned to one route at most. Only the trip of collecting students and delivering them at their respective schools is considered, not the other way around, also, each pickup location of the students must be visited by only one bus in order to guarantee that all students are collected and that there are not partial collections.

Thus, the objective is to assign the best route to each one of the vehicles in order to minimize the total traveled distance. The following steps were defined to solve the problem:

Step 1: Formulate and solve the Capacitated Vehicle Routing Problem (CVRP) in order to assign vehicles to the pickup locations;

Step 2: Formulate and solve the Traveling Salesman Problem (TSP) for each vehicle resulting from the previous stage in order to find the best route for these vehicles;

Step 3: Formulate and implement a heuristic correction in order to guarantee that all students are collected before the vehicle visits their respective destinations.

Figure 1 shows a scheme of the proposed method.

In Figure 1 each tier represents a stage of the method that will be described individually in sections 3.1, 3.2 and 3.3 respectively.

Capacitated Vehicle Routing Problem (CVRP)

Branched out from the set of problems of the VRP, the CVRP determines a set of routes for a set of vehicles with known capacity, that aims to serve a set of customers with respective demands, thus minimizing the transport cost. Normally, the vehicles start and finish at the same point, called a warehouse, defined in this study as node 1 (Amous, Toumi, Jarboui, & Eddaly, 2017). The Mathematical Model 1, adapted from Toth and Vigo (2002), is associated with the CVRP used in Stage 1 of the study, shown in Table 1.

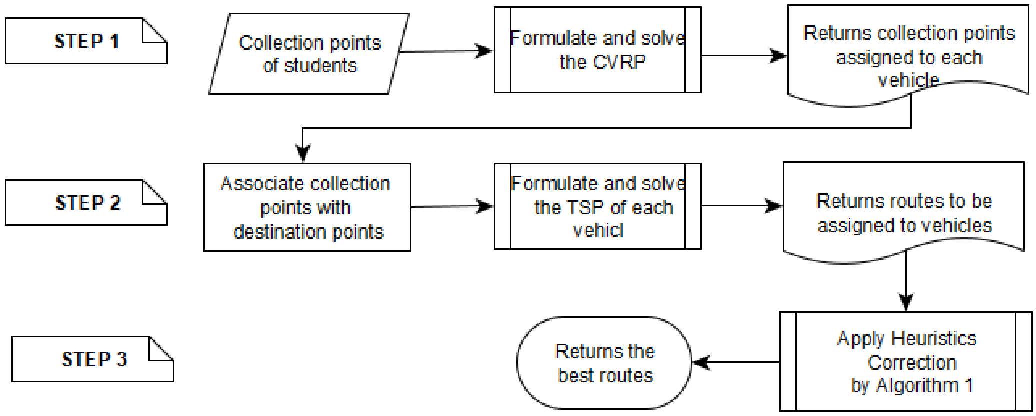

Figure 1. Stages of the Method.

Table 1. Data for Mathematical Model 1.

	Indexes
i	$= originnodeofthearc(i,j);$
j	$= destinynodeofthearc(i,j);$
k	$= vehicle;$
	Set
V	$= \{1,\dots,n\} representsthestops;$
A	$\{(i,j): i,j \in V; i \neq j\} thesetofarcs;$
K	$= \{1,\dots.,k\} representsthenumberofvehicles;$
	Parameters
$G(V,A)$	$= Completenetworkbesolved;$
c_k	$= Capacityofbusk;$
q_i	$Quantityofstudentsonpick-uppointi, i \in V;$
d_{ij}	$Traveldistanceonarc(i,j);$
u_{ik}	$Loadofthevehiclekaftervisitingnodei, i \in V;$

Decision variables:

$x_{ijk} = \{1, ifbusktravelsij, 0else$

$y_{ik} = \{1, ifbuskpick-upstudentsatnodei, 0else$

Objective function:

$$Min \sum_{i=1}^{n} \sum_{j=1, j \neq i}^{n} x_{ijk} d_{ij} \qquad (1)$$

Constraints:

$$\sum_{k \in K} y_{ik} = 1 \qquad \forall i \in V\{1| \quad \} \qquad (2)$$

$$\sum_{k \in K} y_{1k} = K \qquad \forall k \in K \qquad (3)$$

$$\sum_{i \in V} y_{ik} q_i \leq c_k \qquad \forall k \in K \qquad (4)$$

$$\sum_{j \in V, i \neq j} x_{ijk} = y_{ik} \qquad \forall i \in V, \forall k \in K \qquad (5)$$

$$\sum_{i \in V, i \neq j} x_{ijk} = y_{jk} \qquad \forall j \in V, \forall k \in K \qquad (6)$$

$$u_{ik} - u_{jk} + c_k x_{ijk} \leq c_k - q_j \qquad \forall i, j \in V\{1| \quad \}, i \neq j, \forall k \in K, so q_i + q_j \leq c_k \qquad (7)$$

$$q_i \leq u_{ik} \leq c_k \qquad \forall i \in V1\}, \forall k \in K \qquad (8)$$

$$y_{ik} \in \{0; 1\} \qquad \forall i \in V, \forall k \in K \qquad (9)$$

$$x_{ijk} \in \{0; 1\} \qquad \forall i, j \in V, \forall k \in K \qquad (10)$$

In the CVRP problem, the Objective Function (1) aims to minimize the total cost; the constraint (2) guarantees that all points are assigned to exactly one vehicle; the constraint (3) defines that all vehicles must leave the warehouse; the constraint (4) guarantees that the collections made by the vehicles do not exceed its capacity; the constraints (5) and (6) define that if a point is assigned to a vehicle, then this point must be an origin and a destination in the route, respectively; the constraints (7) and (8) are for the possible elimination of sub-tours, where $u_{i,j}$ represents the load of the vehicle after visiting node i, j. Finally, constraints (9) and (10) define the type of variable as binary.

The described mathematical model above was used to define which would be the pickup locations visited for each bus, those being two homogeneous vehicles. After assigning each vehicle to the bus stops, the TSP was implemented in order to find the minimum path for each route defined in this stage.

Traveling Salesman Problem (TSP)

The TSP involves a traveler who leaves the origin and visits a group or subgroup of points only once and returns to the same origin, in order to minimize one or more objectives. The most classic problem and responsible for a great number of studies in the literature is the minimization of the covered distance in the route, the exact same objective treated in this study (Sun, Karwan, & Diaby, 2018).

The great difference between the TSP and the CVRP is that the first one does not consider the capacity of the vehicle that visits the stops and does not allow repetition of covered points. In Table 2 the Mathematical Model 2 is presented, adapted from Orman and Williams (2007), is associated with the TSP.

Table 2. Data for Mathematical Model 2.

	Indexes
i	$= \{1, \dots, n\} originnodesofarc(i, j);$
j	$= \{1, \dots, n\} destinynodesofarc(i, j);$
	Sets
V	$= \{1, \dots, n\} representsthestops;$
A	$\{(i, j): i, j \in V; i \neq j\} setofarcs;$
	Parameters
$G(V, A)$	$= Completenetworkbesolved;$
d_{ij}	$Traveldistanceonarc(i, j);$
u_i	$Sequence \in whichnodeiisvisited(i \neq j);$

Decision variable:

$x_{ij} = \{1 if busktravelsij, 0 else$

Objective function:

$$Min \sum_{i=1}^{n} \sum_{j=1}^{n} x_{ij} d_{ij} \quad (11)$$

Constraints:

$$\sum_{j=1, i \neq j}^{n} x_{ij} = 1 \qquad \forall i \in V \quad (12)$$

$$\sum_{i=1, j \neq i}^{n} x_{ij} = 1 \qquad \forall j \in V \quad (13)$$

$$u_i - u_j + n x_{ij} \leq n - 1 \qquad 2 \leq i \neq j \leq n \quad (14)$$

$$u_i \leq n - 1 \qquad \forall i = 2, \dots, n \quad (15)$$

$$x_{ij} \in \{0; 1\} \qquad \forall i, j \in V \quad (16)$$

In the model previously described, the Objective Function (11) expresses the minimization of the total distance covered in the route; constraints (12) and (13) correspond to the assignments; constraints (14) and (15) refer to the elimination of sub-tours, where n represents the number of stops; and the constraint (16) expresses the type of variable as binary.

This model was implemented to find the minimum path to be covered by each bus, passing through all of its respective assigned pickup locations already defined by the CVRP.

Correction heuristic

When applying the CVRP and the TSP to solve the problem as expressed in the Mathematical Models (1) and (2), respectively, it is not guaranteed that the vehicles are going to visit the bus stops before the schools of its respective picked-up students, therefore, a school could be visited before a bus stop which has students assigned to that school. To correct this problem and to guarantee that this does not happen, Correted Heuristic was created. The flowchart that represents the Algorithm (1) attributed to the heuristic is presented by Figure 2.

As shown by Figure 2, the heuristic algorithm is initiated having as base the route generated for the TSP (1), thus, the points of schools that appear before its respective pickup locations are identified and removed (2 and 3), resulting in a route that will be used in the remain of the code (4). With the points of schools removed, their respective pickup locations and the association among them are identified (5 and 6). From this, step (7) identifies in which position the removed schools can be inserted in a way that its insertion is subsequent to its respective pickup locations. (8) is needed to guarantee that the position is correct, relating to the amount of schools that had been inserted before the point currently analyzed.

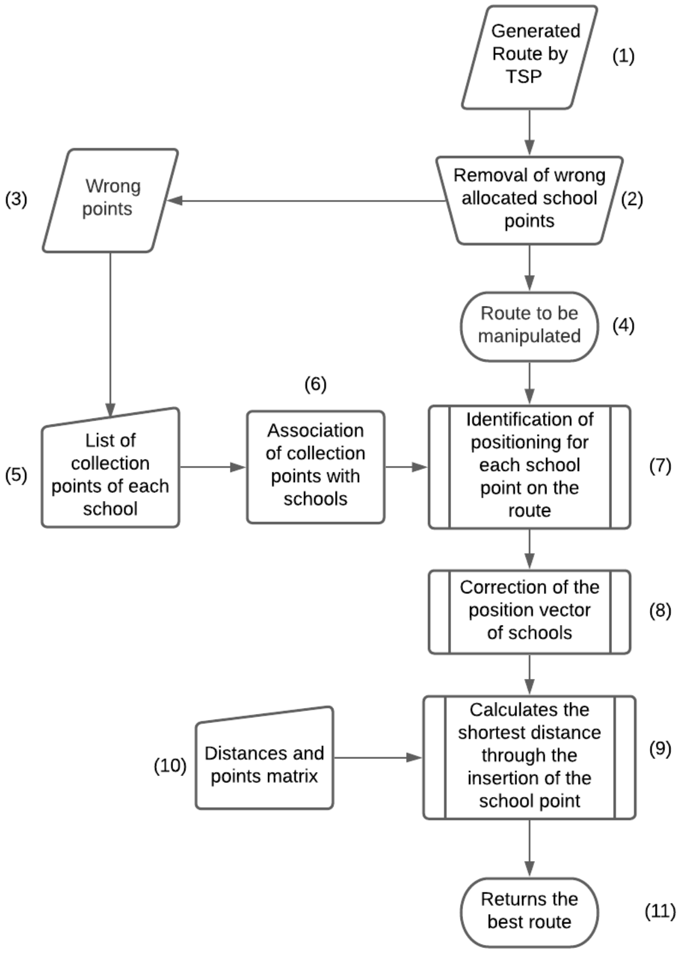

Figure 2. Flowchart of the correction heuristic's algorithm.

Finally, knowing the distances of each point (10), it was inserted a point of school one at a time in an ascending order (respecting the position of which this school must leave) and verified (9) what is the best position for this school in order to minimize the total distance covered for the bus. After making this process for each school point inserted, it is returned the best route, along with the total distance of it.

Finally, to verify if the insertion order of the school points in the route impacts the total distance of the best route defined for the heuristic, the process is made again three times, in a way that the schools are placed in a random sequence in the route.

Results and discussion

To model this problem, a complete network $G(V, A)$ was formulated to be solved, where V represents the set of vertices or nodes and A the set of arcs or edges.

Using Google Maps (Google Maps, 2020), a d_{ij} asymmetric distance matrix, that is $d_{ij} \neq d_{ji}$, of dimension 29 x 29, was built, where each d_{ij} element of the matrix represents the distance between the nodes i and j. In this case, d_{ij} represents the cost of the route assigned to vehicle k.

It was also built a vector of $q_i$demands for each pickup location i and a des_{ij} dropping matrix that provides the information of how many students picked up in node i need to be dropped on node j.

The network with all the nodes is represented in Figure 3.

In Figure 3, the point 1 represents the warehouse from which all vehicles depart and return, points 2 through 19 represent the student pickup points, and points 20 through 29 represent destination schools.

All the steps defined in the methodology were programmed in Python, in the Gurobi commercial optimization solver.

In order to designate each pickup location, from 1 to 19, to one of the two available vehicles, the CVRP was implemented with an execution time limited to 3600 seconds and resulted in two designation vectors, as shown in Table 3.

Table 3 shows that the two vehicles were assigned to pick up locations and filled their entire occupation of 24 seats each. In Figure 4, the network map shows the distribution of these points between buses, with 1 being the starting point and each color the designation of bus 1 or 2.

From the results of the CVRP model, the destination points were added to the route of the vehicles so that every school was on the route of its respective picked up students.

Thus, a TSP was implemented in order to establish a route to the points designated to each bus, minimizing the total distance to be traveled. The resulting routes 1 and 2 are shown in Table 4.

In Table 4, the routes created by the TSP found an optimal solution of 23.97 km to be traveled by bus 1 and 17.33 km to be traveled by bus 2, adding together approximately 41.30 km, but both routes do not consider that the delivery points, that is, schools, should be after the pickup locations with students assigned to them. Thus, in order to ensure that every student is collected before the vehicle visits their respective school, the correction heuristic was applied according to Algorithm 1, based on the results of the TSP. The results are shown in Table 5.

Figure 3. Network of nodes to be visited.

Table 3. Results of CVRP.

Vehicle	Assigned stops	Occupied capacity
Bus 1	1, 16, 8, 2, 18, 11, 12, 6, 14, 13, 19.	24 seats
Bus 2	1, 4, 15, 3, 9, 17, 5, 7, 10.	24 seats

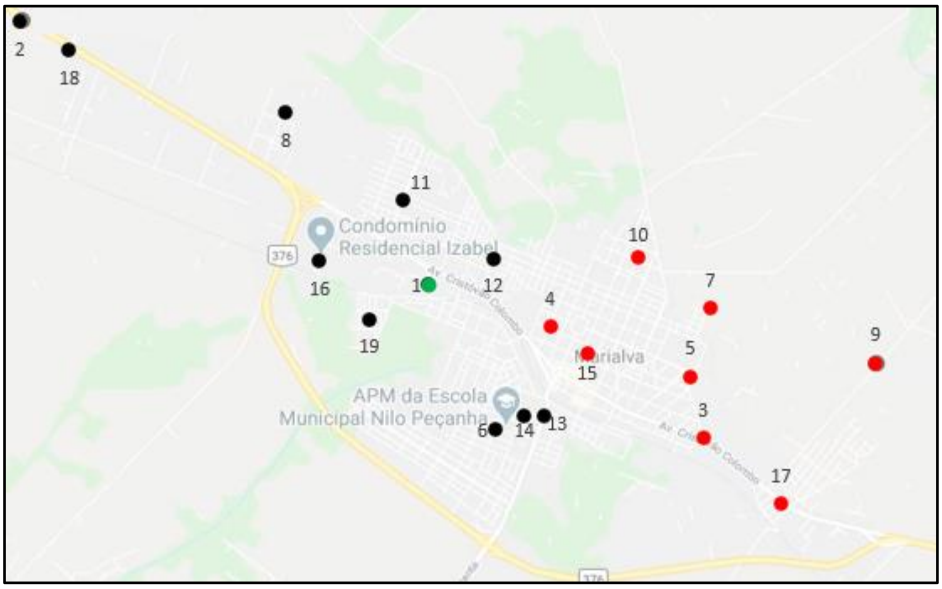

Figure 4. Results from the CVRP model.

Table 4. Results from the TSP model.

Route	Assigned stops	Distance (km)
Route 1	1, 16, 8, 29, 2, 18, 11, 12, 22, 20, 21, 13,14, 6, 24, 27, 19, 1.	23.97
Route 2	1, 26, 25, 15, 3, 9, 28, 17, 5, 7, 10, 22, 23, 4, 24, 27, 1.	17.33

Table 5. Results of Algorithm 1.

Route	Assigned stops	Distance (km)
Route 1 corrected. It should be traveled by bus 1	1, 16, 8, 2, 18, 11, 12, 13, 14, 6, 24, 21, 20, 22, 27, 19, 29, 1.	30.76
Route 2 corrected. It should be traveled by bus 2	1, 15, 3, 9, 17, 28, 5, 7, 10, 22, 23, 25, 4, 24, 27, 26, 1.	18.42

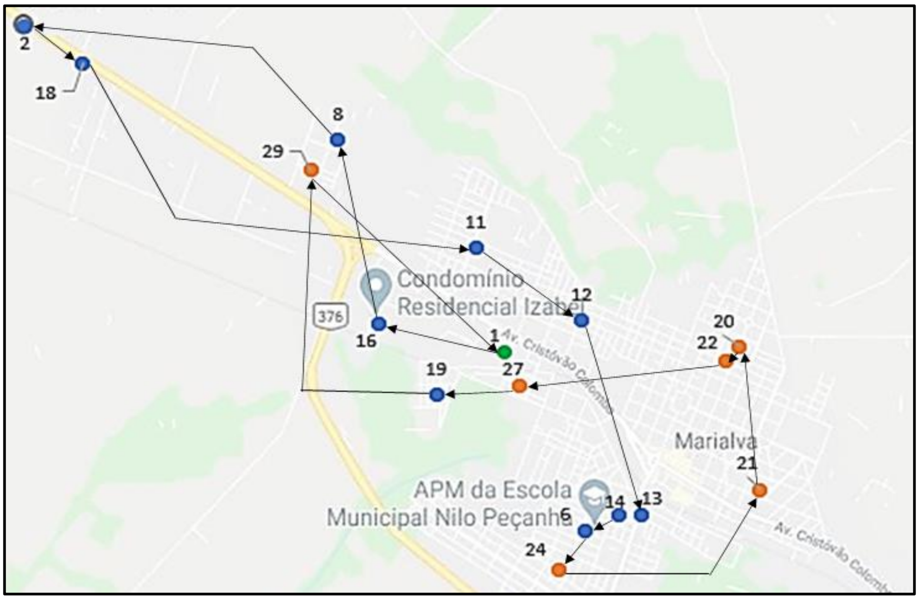

Figure 5. Route 1.

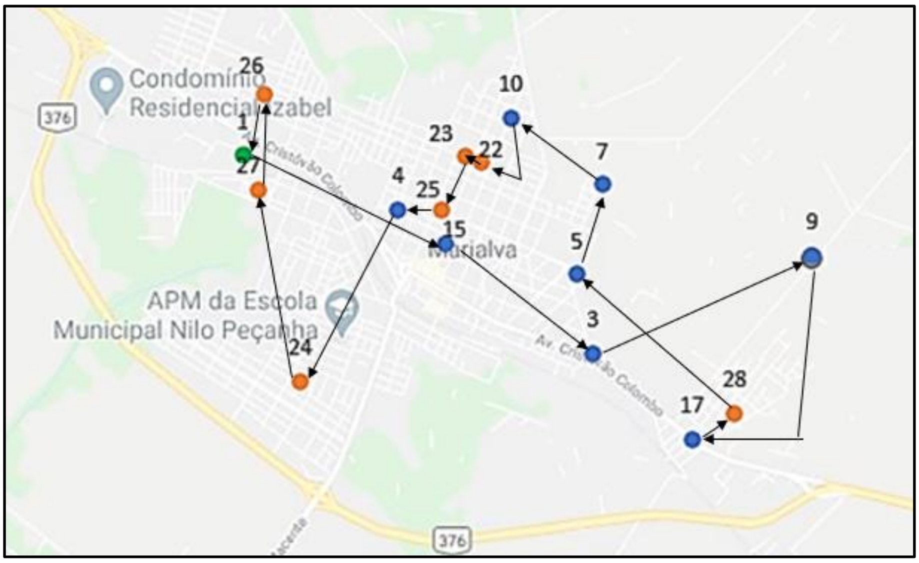

Figure 6. Route 2.

Thus, Table 5 represents the final solution of the problem, with the routes to be traveled, in parallel, daily by the two vehicles, with a total distance of 49.18 km. Figure 5 and 6 present the network map assigned to each of the routes.

Figure 5 shows the circuit of Route 1 to be covered, with a total distance of 30,76 km, starting at point 1, the warehouse, and returning to it after visiting all points.

Figure 6 shows the circuit of route 2 with a total distance of 18.42 km, starting from point 1, warehouse, and returning to it after visiting all points.

With the application of the heuristics from Algorithm 1, the optimal TSP result was lost, shown in Table 2, but a viable solution remained, increasing the total distance of the final routes by 16% compared to the TSP, which is equivalent to approximately 8.2 km. See Table 6.

In Table 6, it is possible to compare the optimal but unfeasible solution given by the TSP concerning the viable solution provided by the use of heuristics. Thus, the distance to be covered by route 1 and route 2 increased by 22.07 and 5.92%, respectively, reflecting in total an increase of 16.02%.

Comparing the routes already taken by the two vehicles in the real scenario - which together travel a total of 54.66 km - with the proposed route of this study, the daily distance could be reduced by 5.48 km, which corresponds to about 10% reduction. Considering that this route is done on every school day during the year (minimum of 200 days), it's possible to decrease a total of 1096 km per year. This value can still increase proportionally with the number of buses and periods in which these services are needed, further reducing the costs involved in school transport.

The use of CVRP allowed the designation of points in two distinct routes; later, with the application of the TSP, these routes were ordered aiming to minimize the total distance to be traveled by the buses. Finally, the heuristics ensured that the solution was feasible, so that the vehicles necessarily visited the pickup locations before visiting the respective schools of the students. Thus, the method met the requirements of the proposed problem and proved to be a viable solution when applied in a small-sized county, but its solution may not be the overall optimal because the use of a correction heuristic can lead to a loss of optimality.

In addition, this resolution tool can still be used and adapted for Pickup-and-Delivery Problems (PDP) because in cases where the pickup locations could also be delivery points, they could be dismembered into two separate points, one for pickup and the other for delivery with a zero distance between the two.

Table 6. Comparison of Results.

Route	O.F. of TSP (km)	O.F. of Heuristic (km)	Difference (%)
Route 1	23.97	30.76	22.07
Route 2	17.33	18.42	5.92
Total	41.30	49.18	16.02

Conclusion

This study contributes to the literature expansion on solutions to the vehicle routing problem by presenting a method that combines two known models, the CVRP and TSP, along with a new correction heuristic to solve a real-life problem. The main limitation is the use of heuristics to ensure that pickup locations are visited before their respective points of delivery, which can compromise its optimization. Possibilities of future studies permeate the construction of a model that eliminates the need for a correction heuristic, and an improved bus stop selection to find better pickup locations according to the students' home addresses.

References

Amous, M., Toumi, S., Jarboui, B., & Eddaly, M. (2017). A variable neighborhood search algorithm for the capacitated vehicle routing problem. *Electronic Notes in Discrete Mathematics*, 58, 231–238. DOI: https://doi.org/10.1016/j.endm.2017.03.030

Babaei, M., & Rajabi-Bahaabadi, M. (2019). School bus routing and scheduling with stochastic time-dependent travel times considering on-time arrival reliability. *Computers and Industrial Engineering, 138*, 106125. DOI: https://doi.org/10.1016/j.cie.2019.106125

Banerjee, D., & Smilowitz, K. (2019). Incorporating equity into the school bus scheduling problem. *Transportation Research Part E: Logistics and Transportation Review, 131*, 228-246. DOI: https://doi.org/10.1016/j.tre.2019.08.006

Belfiore, P., & Yoshida Yoshizaki, H. T. (2009). Scatter search for a real-life heterogeneous fleet vehicle routing problem with time windows and split deliveries in Brazil. *European Journal of Operational Research*, 199(3), 750–758. DOI: https://doi.org/10.1016/j.ejor.2008.08.003

Bertsimas, D., Delarue, A., & Martin, S. (2019). Optimizing schools' start time and bus routes. *Proceedings of the National Academy of Sciences, 116*(13), 5943-5948. DOI: https://doi.org/10.1073/pnas.1811462116

Caceres, H., Batta, R., & He, Q. (2019). Special need students school bus routing: Consideration for mixed load and heterogeneous fleet. *Socio-Economic Planning Sciences, 65*, 10-19. DOI: https://doi.org/10.1016/j.seps.2018.02.008

Chen, J., & Shi, J. (2019). A multi-compartment vehicle routing problem with time windows for urban distribution - A comparison study on particle swarm optimization algorithms. *Computers & Industrial Engineering, 133*, 95-106. DOI: https://doi.org/10.1016/j.cie.2019.05.008

Dantzig, G. B., & Ramser, J. H. (1959). The truck dispatching problem. *Management Science, 6*(1), 80-91. DOI: https://doi.org/10.1287/mnsc.6.1.80

Drakoulis, R., Bellotti, F., Bakas, I., Berta, R., Paranthaman, P. K., Dange, G. R., ... Amditis, A. (2018). A gamified flexible transportation service for on-demand public transport. *IEEE Transactions on Intelligent Transportation Systems, 19*(3), 921-933. DOI: https://doi.org/10.1109/TITS.2018.2791643

Ellegood, W. A., Solomon, S., North, J., & Campbell, J. F. (2020). School bus routing problem: contemporary trends and research directions. *Omega, 95*, 102056. DOI: https://doi.org/10.1016/j.omega.2019.03.014

Google. (2020). *Google Maps of Marialva city in the state of Paraná, Brazil*. Retrieved August 10, 2020, from https://goo.gl/maps/5bPJatwu47zwq1jR7

Kim, B. I., Kim, S., & Park, J. (2012). A school bus scheduling problem. *European Journal of Operational Research, 218*(2), 577-585. DOI: https://doi.org/10.1016/j.ejor.2011.11.035

Koç, Ç., Laporte, G., & Tükenmez, İ. (2020). A review of vehicle routing with simultaneous pickup and delivery. *Computers & Operations Research, 122*, 104987. DOI: https://doi.org/10.1016/j.cor.2020.104987

Letchford, A. N., & Salazar-González, J.-J. (2019). The capacitated vehicle routing problem: stronger bounds in pseudo-polynomial time. *European Journal of Operational Research, 272*(1), 24-31. DOI: https://doi.org/10.1016/j.ejor.2018.06.002

Li, L. Y. O., & Fu, Z. (2002). The school bus routing problem: a case study. *Journal of the Operational Research Society, 53*(5), 552-558. DOI: https://doi.org/10.1057/palgrave.jors.2601341

Lima, F. M. S., Pereira, D. S., Conceição, S. V., & Nunes, N. T. R. (2016). A mixed load capacitated rural school bus routing problem with heterogeneous fleet: algorithms for the Brazilian context. *Expert Systems with Applications, 56*, 320-334. DOI: https://doi.org/10.1016/j.eswa.2016.03.005

Liu, W.-Y., Lin, C.-C., Chiu, C.-R., Tsao, Y.-S., & Wang, Q. (2014). Minimizing the carbon footprint for the time-dependent heterogeneous-fleet vehicle routing problem with alternative paths. *Sustainability, 6*(7), 4658-4684. DOI: https://doi.org/10.3390/su6074658

Mohammed, M. A., Ghani, M. K. A., Hamed, R. I., Mostafa, S. A., Ahmad, M. S., & Ibrahim, D. A. (2017). Solving vehicle routing problem by using improved genetic algorithm for optimal solution. *Journal of Computational Science, 21*, 255-262. DOI: https://doi.org/10.1016/j.jocs.2017.04.003

Molenbruch, Y., Braekers, K., & Caris, A. (2017). Typology and literature review for dial-a-ride problems. *Annals of Operations Research, 259*(1), 295-325. DOI: https://doi.org/10.1007/s10479-017-2525-0

Newton, R. M., & Thomas, W. H. (1969). Design of school bus routes by computer. *Socio-Economic Planning Sciences, 3*(1), 75-85. DOI: https://doi.org/10.1016/0038-0121(69)90051-2

Orman, A. J., & Williams, H. P. (2007). A survey of different integer programming formulations of the travelling salesman problem. *Optimisation, Econometric and Financial Analysis, 9*, 91-104. DOI: https://doi.org/10.1007/3-540-36626-1_5

Park, J., & Kim, B.-I. (2010). The school bus routing problem: a review. *European Journal of Operational Research, 202*(2), 311-319. DOI: https://doi.org/10.1016/j.ejor.2009.05.017

Park, J., Tae, H., & Kim, B.-I. (2012). A post-improvement procedure for the mixed load school bus routing problem. *European Journal of Operational Research, 217*(1), 204-213. DOI: https://doi.org/10.1016/j.ejor.2011.08.022

Pillac, V., Gendreau, M., Guéret, C., & Medaglia, A. L. (2013). A review of dynamic vehicle routing problems. *European Journal of Operational Research, 225*(1), 1-11. DOI: https://doi.org/10.1016/j.ejor.2012.08.015

Rashidi, T. H., Zokaei-Aashtiani, H., & Mohammadian, A. (2009). School bus routing problem in large-scale networks: new approach utilizing tabu search on a case study in developing countries. *Transportation Research Record, 2137*(1), 140-147. DOI: https://doi.org/10.3141/2137-15

Sales, L. P. A., Melo, C. S., Bonates, T. O., & Prata, B. A. (2018). Memetic algorithm for the heterogeneous fleet school bus routing problem. *Journal of Urban Planning and Development, 144*(2). DOI: https://doi.org/10.1061/(ASCE)UP.1943-5444.0000454

Scárdua, R. F., Rosa, R. A., Sabino, J. A., & Vitorugo, L. R. (2016). Planejamento do transporte de empregados por uma frota de ônibus fretada por meio de um modelo matemático baseado no Open Vehicle Routing Problem (OVRP). *Transportes, 24*(3), 20-28. DOI: https://doi.org/10.14295/transportes.v24i3.1089

Schittekat, P., Kinable, J., Sörensen, K., Sevaux, M., Spieksma, F., & Springael, J. (2013). A metaheuristic for the school bus routing problem with bus stop selection. *European Journal of Operational Research, 229*(2), 518-528. DOI: https://doi.org/10.1016/j.ejor.2013.02.025

Shafahi, A., Wang, Z., & Haghani, A. (2017). Solving the school bus routing problem by maximizing trip compatibility. *Transportation Research Record: Journal of the Transportation Research Board, 2667*(1), 17-27. DOI: https://doi.org/10.3141/2667-03

Souza, C., Omkar, S. N., & Senthilnath, J. (2012). Pickup and delivery problem using metaheuristics techniques. *Expert Systems with Applications, 39*(1), 328-334. DOI: https://doi.org/10.1016/j.eswa.2011.07.022

Sun, L., Karwan, M. H., & Diaby, M. (2018). The indefinite period traveling salesman problem. *European Journal of Operational Research, 270*(3), 1171-1181. DOI: https://doi.org/10.1016/j.ejor.2018.04.028

Toth, P., & Vigo, D. (2002). *The vehicle routing problem*. Philadelphia, PA: SIAM.

Yao, B., Cao, Q., Wang, Z., Hu, P., Zhang, M., & Yu, B. (2016). A two-stage heuristic algorithm for the school bus routing problem with mixed load plan. *Transportation Letters, 8*(4), 205-219. DOI: https://doi.org/10.1080/19427867.2015.1110953