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MATHEMATICS

On intuitionistic fuzzy hilbert ideal convergent sequence spaces

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ABSTRACT. In this paper, by using the triangle Hilbert matrix H and the notion of ideal convergence for the sequences in intuitionistic fuzzy normed spaces, we introduce some new intuitionistic fuzzy normed sequence spaces as a domain of Hilbert matrix H, that is, $c^I_{\square_{0(\mu,\nu)}}(H)$ and $c^I_{\square_{(\mu,\nu)}}(H)$. Here, $c^I_{\square_{0(\mu,\nu)}}(H)$ denotes the Hilbert ideal null convergent sequence space with respect to the intuitionistic fuzzy norm and $c^I_{\square_{(\mu,\nu)}}(H)$ denotes the Hilbert ideal convergent sequence space with respect to the intuitionistic fuzzy norm. We also define an open ball with respect to defined sequence space and prove that these open balls are the open sets of these spaces. Further, we study some of its topological and algebraic properties. We prove that these sequence spaces are linear spaces of . In addition, we define a topology with respect to these sequence spaces and obtain that the defined topology is first countable and these topological sequence spaces are Hausdorff spaces. We also obtain if and only if results that give an idea about when a sequence belonging to these spaces is classical convergent with respect to the intuitionistic fuzzy norm and when a sequence belonging to these spaces is ideal convergent with respect to the intuitionistic fuzzy norm.

Keywords: Hilbert matrix; ideal convergence; intuitionistic fuzzy normed space.

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Introduction

Let R and N denote the sets of real and natural numbers respectively. By ω we denote linear space of sequence of real or complex numbers. Any vector subspace of ω is called a sequence space. We use the notations l_∞, c and c_0 to denote the sequence space of bounded, convergent and null sequences, respectively. A family of sets $I \subset P(X)$ (where $P(X)$ is the power set of X) of subsets of X is said to be ideal in X if and only if (i) $\emptyset \in I$, (ii) for each $A, B \in I$ we have $A \cup B \in I$, (iii) for each $A \in I$ and $B \subset A$ we have $B \in I$ and I is called an admissible in X if and only if $I \neq X$ and it contains all singletons. A filter on X is a non-empty family of sets $F \subset P(X)$ satisfying (i) $\emptyset \notin F$, (ii) for each $A, B \in F$ we have $A \cap B \in F$, (iii) for each $A \in F$ and $B \supset A$ we have $B \in F$. For each ideal I there is a filter $F(I)$ corresponding to I, that is, $F(I) = \{K \subseteq X: K^c \in I\}$, where $K^c = X$. Depending on the structure of ideals of subsets of N, (Kostyrko, Macaj, & Šalát, 1999) defined the notion of I-convergence as a generalization of statistical convergence introduced by (Fast, 1951) and (Steinhaus, 1951). Where the notion of ideal convergence is defined as: A sequence $(x_k) \in \omega$ is said to be I-convergent to a number $\xi \in R$ if for every $\epsilon > 0$, $\{k \in N: \vee x_k - \xi \vee \geq \epsilon\} \in I$, and write $I - \lim_{\square} x_k = \xi$. Later, the notion of I-convergence was further investigated from the sequence space point of view and linked with the summability theory by (Šalát, Tripathy, & Ziman, 2004; 2005; Khan, Khan, & Khan, 2016; Khan, Rababah, Alshlool, Abdullah, & Ahmad, 2018c; Filipów & Tryba, 2018) and many other authors. On the other hand, (Kumar & Kumar, 2009) defined ideal analogue of convergence sequences on intuitionistic fuzzy normed space (IFNS) as: Let X be an IFNS, a sequence $x = (x_k) \in X$ is said to be I-convergent to $\xi \in R$ with respect to the intuitionistic norm (μ, ν), if for every $\epsilon > 0$ and $t > 0$, $\{k \in N: \mu(x_k - \xi, t) \leq 1 - \epsilon \vee \nu(x_k - \xi, t) \geq \epsilon\} \in I$ and write $I_{(\mu,\nu)} - \lim_{\square} x = \xi$.

For further details on ideal convergence and, we refer to (Tripathy & Hazarika, 2008; Esi & Hazarika, 2013; Subramanian, Esi, & Özdemir, 2017; Khan, Yasmeen, Esi, & Fatima, 2017; Khan, Alshlool, & Abdullah, 2018;

Khan, Makharesh, Alshlool, Abdullah, & Fatima, 2018b; Khan, Abdullah, Esi, & Alshlool, 2019; Kemal Ozdemir, Esi, & Subramanian, 2019).

Material and methods

In the classical summability theory the idea of the generalization of the convergence of sequences of real or complex numbers is to assign a limit of some sort to divergent sequences by considering a matrix transform of a sequence rather than the original sequence. Let λ be a sequence spaces and $A = (a_{nk})$ be an infinite matrix of real or complex numbers a_{nk}, where $n, k \in N$. Then, the matrix domain of an infinite matrix A in a sequence space λ is a sequence space defined by

$$\lambda_A := \{x = (x_k) \in \omega : Ax \in \lambda\}.$$

The study of such matrices attracted the attention of many researchers to dig deeper in this area, for instance (Mursaleen, 1983; Djolović & Malkowsky, 2008; Mursaleen & Noman, 2012; Nergiz & Başar, 2013; Candan, 2014), and the references therein. Recall in (Hilbert, 1894) that, the Hilbert matrix is an infinite matrix $H = (a_{nk})$ which is defined as $a_{nk} =$ for n, k ∈ N. The Hilbert matrix was used as a bounded linear operator on the spaces of all p-summable sequences l_p with the norm $\vee H \vee \square_p = \frac{\pi}{\sin(\frac{\pi}{p})}$ for $1 < p < \infty$ (see, Hardy, Littlewood, & Pólya, 1934). Recently, (Polat, 2016) introduced the sequence spaces h_∞, h_c and h_0 as the sets of all sequences whose H-transforms are in the spaces l_∞, c and c_0, respectively, i.e.,

$$\lambda_H = \left\{x = (x_k) \in \omega : \left(\sum_{k=1}^{m} \frac{x_k}{n+k-1}\right) \in \lambda\right\} for\ \lambda \in \{c_0, c, l_\infty\}.$$

Quite recently, by using the triangle Hilbert matrix $H = (a_{nk})$ defined by

$$a_{nk} = \begin{cases} \frac{1}{n+k-1}, & if \quad 1 \le k \le n \\ 0, & if \quad k > n, \end{cases}$$

and the notion of I-convergence, (Khan, Alshlool, & Alam, 2020) introduced some new Hilbert (Hardy et al., 1934) I-convergent sequence spaces $c^I_{\square_0}(H)$, $c^I(H)$, $l^I_{\square_\infty}(H)$ and $l_\infty(H)$ as the set of all sequences whose H-transform are in the spaces $c^I_{\square_0}$, c^I, $l^I_{\square_\infty}$ and l_∞, respectively. Further, they defined the sequence $H_k(x)$ which is frequently used as H-transform of the sequence $x = (x_k) \in \omega$, defined by:

$$H_k(x) = \sum_{k=1}^{n} \frac{x_k}{n+k-1}\ for\ n, k \in N. \tag{1}$$

In this paper, by using the triangle Hilbert matrix H and the notion of ideal convergence of sequences in intuitionistic fuzzy normed space, we introduce some new spaces of Hilbert ideal convergent sequences with respect to intuitionistic fuzzy norm (μ, ν), that is, $c^I_{\square_{(\mu,\nu)}}(H)$ and $c^I_{\square_{0(\mu,\nu)}}(H)$. Further, we study some inclusion relations and some of its topological and algebraic properties.

We recall the definition of intuitionistic fuzzy normed space defined by (Saadati & Park, 2006), and remarks for the sequel of this paper.

The five-tuple $(X, \mu, \nu, *, \diamond)$ is said to be an intuitionistic fuzzy normed space (for short, IFNS) if X is a vector space, is a continuous t-norm, $\diamond$ is a continuous t-conorm and μ, ν are fuzzy sets on $X \times (0, \infty)$ satisfying the following conditions for every $x, y \in X$, $c \in R$ and for all $s, t > 0$:

$\mu(x, t) + \nu(x, t) \le 1$,

$\mu(x, t) > 0$,

$\mu(x, t) = 1$ if and only if $x = 0$,

$\mu(cx, t) = \mu(x, \frac{t}{|c|})$,

$\mu(x, t) * \mu(y, s) \le \mu(x + y, t + s)$,

$\mu(x, \cdot) : (0, \infty) \longrightarrow [0,1]$ is continuous,

$\lim_{\square} \square_{t\to\infty} \mu(x, t) = 1$ and $\lim_{\square} \square_{t\to 0} \mu(x, t) = 0$.

$\nu(x, t) < 1$,

$\nu(x,t)=0$ and if and only if $x=0$,

$\nu(cx,t)=\nu(x,\frac{t}{|c|})$,

$\nu(x,t)\diamond\nu(y,s)\geq\nu(x+y,t+s)$,

$\nu(x,\cdot):(0,\infty)\longrightarrow[0,1]$ is continuous,

$\lim_{t\to\infty}\nu(x,t)=0$ and $\lim_{t\to 0}\nu(x,t)=1$.

In this case (μ,ν) is called intuitionistic fuzzy norm on X.

Remark.1 (Park, 2004)
For any $r_1,r_2\in(0,1)$ with $r_1>r_2$ there exist $r_3,r_4\in(0,1)$ such that $r_1*r_3\geq r_2$ and $r_1\geq r_4\diamond r_2$.
For any $r_5\in(0,1)$ there exist $r_6,r_7\in(0,1)$ such that $r_6*r_6\geq r_5$ and $r_7\diamond r_7\leq r_5$.

Results and discussion

In this fragment, we define different type of sequence spaces first one is $c^I_{(\mu,\nu)}(H)$, contains those sequences whose H-transform is convergent to some $L\in R$ with respect to intuitionistic fuzzy norm (μ,ν), and second one $c^I_{0(\mu,\nu)}(H)$, contains those sequences whose H-transform I-convergent to 0 with respect to intuitionistic fuzzy norm (μ,ν), i.e.,

$$c^I_{0(\mu,\nu)}(H)=\{x=(x_k)\in l_\infty:\{k\in N:\mu(H_k(x),t)\leq 1-\epsilon\vee\nu(H_k(x),t)\geq\epsilon\}\in I\}, \quad (2)$$

$$c^I_{(\mu,\nu)}(H)=\{x=(x_k)\in l_\infty:\{k\in N:\mu(H_k(x)-l,t)\leq 1-\epsilon\vee\nu(H_k(x)-l,t)\geq\epsilon\}\in I\}. \quad (3)$$

Further, we define an open ball with center x and radius r with respect to t as follows:

$$B_x(r,t)(H)=\{y=(y_k)\in l_\infty:\{k\in N:\mu(H_k(x)-H_k(y),t)>1-r\wedge\nu(H_k(x)-H_k(y),t)<r\}\}. \quad (4)$$

Theorem. 1. The spaces $c^I_{0(\mu,\nu)}(H)$ and $c^I_{(\mu,\nu)}(H)$ are vector spaces over R.

Proof. Let us show the result for $c^I_{(\mu,\nu)}(H)$ and the proof for other space will follow on the similar lines. Let $x=(x_k)$ and $y=(y_k)\in c^I_{(\mu,\nu)}(H)$. Let $0<\epsilon<1$ so we may choose a s in (0,1) such that $(1-s)*(1-s)>1-\epsilon$ and $s\diamond s<\epsilon$. Then by definition there exists ξ_1 and ξ_2 and for $t>0$, we have

$$A=\left\{k\in\mathbb{N}:\mu\left(H_k(x)-\xi_1,\frac{t}{2|\alpha|}\right)\leq 1-s\ \vee\ \nu\left(H_k(x)-\xi_1,\frac{t}{2|\alpha|}\right)\geq s\right\}\in I,$$

$$B=\{k\in\mathbb{N}:\mu(H_k(y)-\xi_2,\frac{t}{2|\beta|})\leq 1-s\ \vee\ \nu(H_k(y)-\xi_2,\frac{t}{2|\beta|})\geq s\}\in I,$$

where:
α and β are scalars.

Define $E=A\cup B$ so that $E\in I$. Thus $E^c\in F(I)$ and therefore is non-empty. We will show

$$E^c\subset\{k\in N:\mu(\alpha H_k(x)+\beta H_k(y)-(\alpha\xi_1+\beta\xi_2),t)>1-\epsilon\wedge\nu(\alpha H_k(x)+\beta H_k(y)-(\alpha\xi_1+\beta\xi_2),t)<\epsilon\}.$$

Let $k\in E^c$. Then,

$$\mu(H_k(x)-\xi_1,\frac{t}{2|\alpha|})>1-s\ \wedge\ \nu(H_k(x)-\xi_1,\frac{t}{2|\alpha|})<s,$$

$$\mu(H_k(y)-\xi_2,\frac{t}{2|\beta|})>1-s\ \wedge\ \nu(H_k(y)-\xi_2,\frac{t}{2|\beta|})<s.$$

Consider,

$$\begin{aligned}\mu(\alpha H_k(x)+\beta H_k(x)-(\alpha\,\xi_1+\beta\,\xi_2),t)\ &\geq\mu\left(\alpha H_k(x)-\alpha\,\xi_1,\frac{t}{2}\right)*\mu\left(\beta H_k(y)-\beta\,\xi_2,\frac{t}{2}\right)\\&=\mu\left(H_k(x)-\xi_1,\frac{t}{2|\alpha|}\right)*\mu\left(H_k(y)-\xi_2,\frac{t}{|\beta|}\right)\\&>(1-s)*(1-s)>1-\epsilon\end{aligned}$$

and

$$\begin{aligned}\nu\big(\alpha H_k(y)+\beta H_k(y)-(\alpha\,\xi_1+\beta\,\xi_2)\big) &\le \nu\left(\alpha H_k(x)-\alpha\,\xi_1,\frac{t}{2}\right)\diamond\nu\left(\beta H_k(y)-\beta\,\xi_2,\frac{t}{2}\right)\\ &= \nu\left(H_k(x)-\xi_1,\frac{t}{2|\alpha|}\right)\diamond\nu\left(H_k(y)-\xi_2,\frac{t}{2|\beta|}\right)\\ &< s\diamond s<\epsilon.\end{aligned}$$

Thus

$$E^c\subset\{k\in N:\mu(\alpha H_k(x)+\beta H_k(y)-(\alpha\xi_1+\beta\xi_2),t)>1-\epsilon\wedge\nu(\alpha H_k(x)+\beta H_k(y)-(\alpha\xi_1+\beta\xi_2),t)<\epsilon\}.$$

$E^c\in F(I)$, therefore by definition of filter, the set on the right side of the above equation belongs to $F(I)$ so that its complement belongs to I. This implies $(\alpha x+\beta y)\in c^I_{(\mu,\nu)}(H)$. Hence $c^I_{(\mu,\nu)}(H)$ is a vector space over R.

Theorem 2. Every open ball $B_x(r,t)(H)$ is an open set in $c^I_{(\mu,\nu)}(H)$.

Proof. We have defined open ball as follows:

$$B_x(r,t)(H)=\{y=(y_k)\in l_\infty:\{k\in N:\mu(H_k(x)-H_k(y),t)>1-r\wedge\nu(H_k(x)-H_k(y),t)<r\}\}.$$

Let $y=(y_k)\in B_x(r,t)(H)$ so that $\mu(H_k(x)-H_k(y),t)>1-r$ and $\nu(H_k(x)-H_k(y),t)<r$. Then there exists $t_0\in(0,t)$ with $\mu(H_k(x)-H_k(y),t_0)>1-r$ and $\nu(H_k(x)-H_k(y),t_0)<r$. Put $p_0=\mu(H_k(x)-H_k(y),t_0)$ so we have $p_0>1-r$, there exists $s\in(0,1)$ such that $p_0>1-s>1-r$. Using Remark 1, given $p_0>1-s$ we can find $p_1,p_2\in(0,1)$ with $p_0*p_1>1-s$ and $(1-p_0)\diamond(1-p_2)<s$. Put $p_3=max\{p_1,p_2\}$. We will prove $B_y(1-p_3,t-t_0)(H)\subset B_x(r,t)(H)$. Let $z=(z_k)\in B_y(1-p_3,t-t_0)(H)$.

Hence

$$\mu(H_k(x)-H_k(z),t)\ge\mu(H_k(x)-H_k(y),t_0)*\mu(H_k(y)-H_k(z),t-t_0)>p_0*p_3\ge p_0*p_1>1-s>1-r,$$

and

$$\begin{aligned}\nu(H_k(x)-H_k(z),t)&\le\nu(H_k(x)-H_k(y),t_0)\diamond\nu(H_k(y)-H_k(z),t-t_0)\le(1-p_0)\diamond(1-p_3)\\&\le(1-p_0)\diamond(1-p_2)<r.\end{aligned}$$

Hence $z\in B_x(r,t)(H)$ and therefore $B_y(1-p_3,t-t_0)(H)\subset B_x(r,t)(H)$.

Remark. 2. Let $c^I_{(\mu,\nu)}(H)$ be IFNS. Define $\tau^I_{(\mu,\nu)}(H)=\{A\subset c^I_{(\mu,\nu)}(H)$: for given $x\in A$, we can find $t>0$ and $0<r<1$ such that $B_x(r,t)(H)\subset A\}$. Then $\tau^I_{(\mu,\nu)}(H)$ is a topology on $c^I_{(\mu,\nu)}(H)$.

Remark. 3. Since $\{B_x(\frac{1}{k},\frac{1}{k})(H):k\in N\}$ is a local base at x. The topology $\tau^I_{(\mu,\nu)}(H)$ is first countable.

Theorem 3. The spaces $c^I_{(\mu,\nu)}(H)$ and $c^I_{0(\mu,\nu)}(H)$ are Hausdorff.

Proof. Let $x,y\in c^I_{(\mu,\nu)}(H)$ with x and y to be different. Then $0<\mu(H_k(x)-H(y),t)<1$ and $0<\nu(H_k(x)-H_k(y),t)<1$. Put $\mu(H_k(x)-H(y),t)=p_1$ and $\nu(H_k(x)-H_k(y),t)=p_2$ and $r=max\{p_1,1-p_2\}$. Using Remark 1 for $p_0\in(r,1)$ we can find $p_3,p_4\in(0,1)$ such that $p_3*p_3\ge p_0$ and $(1-p_4)\diamond(1-p_4)\le 1-p_0$. Put $p_5=max\{p_3,p_4\}$. Clearly $B_x(1-p_5,\frac{t}{2})(H)\cap B_y(1-p_5,\frac{t}{2})(H)=\emptyset$. Let on contrary $z\in B_x(1-p_5,\frac{t}{2})(H)\cap B_y(1-p_5,\frac{t}{2})(H)$. Then we have

$$\begin{aligned}p_1=\mu(H_k(x)-H_k(y),t)&\ge\mu\left(H_k(x)-H_k(z),\frac{t}{2}\right)*\mu\left(H_k(z)-H_k(y),\frac{t}{2}\right)\\&\ge p_5*p_5\ge p_3*p_3>p_0>p_1,\end{aligned}$$

and

$$\begin{aligned}p_2=\nu(H_k(x)-H_k(y),t)&\le\nu\left(H_k(x)-H_k(z),\frac{t}{2}\right)\diamond\nu\left(H_k(z)-H_k(y),\frac{t}{2}\right)\\&\le(1-p_5)\diamond(1-p_5)\le(1-p_4)\diamond(1-p_4)\le 1-p_0<p_2.\end{aligned}$$

which is a contradiction. Therefore $c^I_{(\mu,\nu)}(H)$ is a Hausdorff space. The proof for $c^I_{0(\mu,\nu)}(H)$ follows similarly.

Theorem. 4. Let $c^I_{(\mu,\nu)}(H)$ be IFNS and $\tau^I_{(\mu,\nu)}(H)$ is a topology on $c_{\mu,\nu}❑^I(H)$. A sequence $(x_k)\in c^I_{(\mu,\nu)}(H)$ convergent to ξ iff $\mu(H_k(x)-H_k(\xi),t)\to 1$ and $\nu(H_k(x)-H_k(\xi),t)\to 0$ as $k\to\infty$.

Proof. Suppose $x_k\to\xi$, then given $0<r<1$ there exists $n_0\in N$ such that $(x_k)\in B_x(r,t)(H)$ for all $k\ge n_0$. Given $t>0$. Hence, we have $1-\mu(H_k(x)-H_k(\xi),t)<r$ and $\nu(H_k(x)-H_k(\xi),t)<r$. Therefore $\mu(H_k(x)-H_k(\xi),t)\to 1$ and $\nu(H_k(x)-H_k(\xi),t)\to 0$ as $n\to\infty$.

Conversely, if $\mu(H_k(x)-H_k(\xi),t)\to 1$ and $\nu(H_k(x)-H_k(\xi),t)\to 0$ as $k\to\infty$ holds for each $t>0$. For $0<r<1$, there exists $n_0\in N$ such that $1-\mu(H_k(x)-H_k(\xi),t)<r$ and $\nu(H_k(x)-H_k(\xi),t)<r$ for all $n\ge n_0$ which implies $\mu(H_k(x)-H_k(\xi),t)>1-r$ and $\nu(H_k(x)-H_k(\xi),t)<r$. Thus $x_k\in B_x(r,t)(H)$ for all $k\ge n_0$ and hence $x_k\to\xi$.

Theorem. 5. A sequence $x=(x_k)\in c^I_{❑(\mu,\nu)}(H)$ is ideal convergent if and only if for every $\epsilon>0$ and $t>0$ there exists number $N=N(x,\epsilon,t)$ such that

$\{k \in N: \mu(H_k(x) - \xi, \frac{t}{2}) > 1 - \epsilon \wedge \nu(H_k(x) - \xi, \frac{t}{2}) < \epsilon\} \in F(I)$

Proof. Suppose that $I_{(\mu,\nu)}\text{-}\lim_{\square} x = \xi$ and $\epsilon > 0$ and $t > 0$. For a given $\epsilon > 0$, choose $s > 0$ such that $\epsilon * \epsilon > 1 - s$ and $\epsilon \diamond \epsilon < s$. Then for each $x \in c^I_{\square_{(\mu,\nu)}}(H)$,

$A = \{k \in N: \mu(H_k(x) - \xi, \frac{t}{2}) \leq 1 - \epsilon \vee \nu(H_k(x) - \xi, \frac{t}{2}) \geq \epsilon\} \in I,$

which implies that

$A_c = \{k \in N: \mu(H_k(x) - \xi, \frac{t}{2}) > 1 - \epsilon \wedge \nu(H_k(x) - \xi, \frac{t}{2}) < \epsilon\} \in F(I).$

Conversely, let us choose $N \in A$, then

$\mu(H_k(x) - \xi, \frac{t}{2}) > 1 - \epsilon \wedge \nu(H_k(x) - \xi, \frac{t}{2}) < \epsilon$

Now we want to show that there exists a number $N = N(x, \epsilon, t)$ such that

$\{k \in N: \mu(H_k(x) - H_N(x), t) \leq 1 - s \vee \nu(H_k(x) - H_N(x), t) \geq s\} \in I.$

For this define for each $x = (x_k) \in c^I_{\square_{(\mu,\nu)}}(H)$

$B = \{k \in N: \mu(H_k(x) - H_N(x), t) \leq 1 - s \vee \nu(H_k(x) - H_N(x), t) \geq s\} \in I.$

Now we have to show that $B \subset A$. Suppose $B \not\subseteq A$ then there exists $k \in B$ and $n \notin A$. Therefore, we have

$\mu(H_k(x) - H_N(x), t) \leq 1 - s \vee \mu(H_k(x) - \xi, \frac{t}{2}) > 1 - \epsilon$

In Particular $\mu(H_k(x) - \xi, \frac{t}{2}) > 1 - \epsilon$. Therefore, we have,

$1 - s \geq \mu(H_k(x) - H_N(x), t) \geq \mu(H_k(x) - \xi, \frac{t}{2}) * \mu(H_N(x) - \xi, \frac{t}{2}) \geq (1 - \epsilon) * (1 - \epsilon) > 1 - s$

which is not possible. On the other hand

$\nu(H_k(x) - H_N(x), t) \geq s \vee \nu(H_k(x) - \xi, \frac{t}{2}) < \epsilon$

In particular $\nu(H_k(x) - \xi, \frac{t}{2}) < \epsilon$. Therefore, we have

$s \leq \nu(H_k(x) - H_N(x), t) \leq \nu(H_k(x) - \xi, \frac{t}{2}) \diamond \nu(H_N(x) - \xi, \frac{t}{2}) \leq \epsilon \diamond \epsilon < s$

which is not possible. Hence, $B \subset A$. $A \in I$ implies $B \in I$.

Conclusion

In this paper, we have formally defined the notions of Hilbert ideal convergence for the sequences in intuitionistic fuzzy normed spaces. Further, we defined the open ball $B_x(r, t)(H)$ with respect to the topology that induced by intuitionistic fuzzy normed space using Hilbert matrix H and show that this open ball is an open set in the space of all Hilbert ideal convergent sequences $c^I_{(\mu,\nu)}(H)$ with respect to an intuitionistic fuzzy norm (μ, ν). In addition, we proved some theorems that would support the results. These new results will further help the researchers expand their work in the area of sequence spaces in view of fuzzy theory.

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