Acta Scientiarum
http://periodicos.uem.br/ojs
ISSN on-line: 1807-8664
Doi: 10.4025/actascitechnol.v46i1.64563

An assessment of the true Gini coefficient regarding the fulfilment of the basic criteria for inequality measures

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ABSTRACT. The Gini coefficient emerged more than a hundred years ago, but it is still the well-known and most used measure for assessing the degree of inequality in a distribution. Historically, the Gini coefficient has mainly been used to study income or wealth distributions, but, as highlighted by Gini himself, the coefficient's power to measure inequality extends to other contexts. In order to adapt it to the needs and points of view of those who use it, both in its classical and non-traditional applications, the Gini coefficient is frequently modified and extended, which resulted in a multitude of mathematical expressions, interpretations and generalizations of this coefficient. The so-called True Gini coefficient, one of the multiple formulations of the Gini coefficient for discrete distributions that can be found in the literature, is a correction of the Gini coefficient that is directly derived from the mean difference between n quantities, originally proposed by Gini, in 1912, and follows from the exclusion of self-on-self differences in the calculation. References to the main motivation for using the Gini coefficient point to its good properties, however, the fulfilment of the criteria for inequality measures is not common to the different formulations of the Gini coefficient. In this work we assessed the fulfillment of the four basic criteria for inequality measures by the True Gini coefficient, having shown that this formulation of the coefficient fulfills the principle of transfers, symmetry, and scale invariance. However, it does not comply with the principle of population, therefore, it cannot be included in the class of relative inequality indexes.

Keywords: Gini index; dispersion; relative inequality index; variability.

Received on July 31, 2022.
Accepted on January 23, 2023.

Introduction

For many people, inequality rhymes with Gini coefficient. In fact, even though the Gini coefficient emerged more than a hundred years ago, it is still the well-known and popular measure of inequality (Luebker, 2010; Langel & Tilé, 2011). Conrado Gini's contribution to Statistics was quite extensive, in such a way that Gini (1884, Motta di Livenza, Italy – 1965, Rome, Italy) is considered by many to be the greatest Italian statistician (Montanari & Monari, 2008) although Gini's worldwide recognition is due to the coefficient that carries his name.

Gini had a wide range of scientific interests. As evidence of this, we can mention his change of direction to Statistics after studying Law at the University of Bologna (Boldrini, 1966), during his academic career having taught, in addition to Statistics, subjects such as Political Economy, Constitutional Law, Demography and Economic Statistics, and the motivation of many of his contributions to Statistics lay in his desire to apply statistics to practical problems (Giorgi, 2011).

Gini's contribution to Statistics, in an initial phase, focused on developments to the theory of averages, variability, and statistical relations, progressed later to issues related to social and economic problems. This change of focus is framed by the beginning of the 1st World War and Gini's activity as advisor to the Italian Government and expert to the League of Nations. In a third phase, Gini was involved in a critical review of the foundations of Statistics, which ended up giving unity to his method (Montanari & Monari, 2008).

The vastness and diversity of Gini´s scientific work can be framed fundamentally in the areas of the statistical methods, demography and biometrics, sociology, and economics (Boldrini, 1966). Between 1906 and 1915, among the approximately four dozen most important Italian works on statistical methodology, almost two dozen belong to Gini (Prévost, 2016).

In a link between theory and statistical practice, in his 1912 book 'Variabilità e Mutabilità' (Variability and Mutability) Gini introduced his most famous contribution to Statistics, the Gini coefficient, also known as the Gini index or the Gini (concentration) ratio, or rather, introduced a preliminary form of this measure, suggesting different variations reflecting Gini's view that the nature of the characters, that being studied, shapes the purpose of a measure of dispersion. In his 1912 work, Gini also highlights what differentiates his measure from the popular variability measures at the time: the simple, squared, or probabilistic mean deviation of the arithmetic mean; and the simple or probabilistic mean deviation from the median. An extensive analysis of the original version of the Gini coefficient(s) can be found in Ceriani and Verme (2012), where some extracts from the book 'Variabilità e Mutabilità' are presented and commented on.

Historically, the Gini coefficient has mostly been used to study income or wealth distributions, where its adoption by the United Nations development program and the World Bank stands out. However, as Gini himself highlighted, the coefficient´s power to measure inequality extends to other contexts. To name just a few, we refer the applications of the Gini coefficient to topics on the agenda such as the Environment and Ecology, or Covid-19 pandemic, as in Teng, He, Pan, and Zhang (2011), where the Gini coefficient is used in climate change area, to measure inequality of carbon space allocation, in Lexerød and Eid (2006), in which the coefficient is used to discriminate forest structural types, in the work in which Valbuena, Eerikäinen, Packalen, and Maltamo (2016) use the Gini coefficient as an ecological indicator highlighting differences in forest structure driven by human activity, in Rouvinen and Kuuluvainen (2005) who use the Gini coefficient as a concise indicator for describing the variability in tree diameter distributions of forests, and in the work by Sobieszek, Lipniacka, and Lipniacki (2022) on Covid-19 deaths and vaccination. But also, applications in the field of transportation (e.g. Hörcher & Graham, 2021), criminology (e.g. Bernasco & Steenbeek, 2017), epidemiology (Abeles & Conway, 2020), tourism (e.g. Fernández-Morales, Cisneros-Martínez, & McCabe, 2016), logistics (Gutjahr & Fischer, 2018) or education (e.g. Thomas, Wang, & Fan, 2001; Castelló & Doménech, 2002).

Both in the context in which it was initially used, and in its non-traditional applications, the Gini coefficient was frequently modified and extended to suit the needs and points of view of those who use it, which resulted in a multitude of mathematical expressions, interpretations, and generalizations of this coefficient, which can be found in the vast literature where it is addressed. In Yitzhaki and Schechtman (2013) extensive analysis of multiple representations of the Gini coefficient can be found.

Choosing the measure of inequality to be adopted for a given purpose involves, from the outset, the difficulty of selecting from a wide range of measures of inequality.

Faced with such diversity, researchers often "[...] base their choice on convenience, familiarity, or vague, methodological reasons" (Allison, 1978, p. 865). The literature on the various practical applications of inequality measurement in the most diverse fields, and the evidence of transferability of that studies, provide the researcher with important indications on the measure that best suits the object of his study, but, as highlighted by Cowell (2009), in choosing the inequality measure, one should consider the type of work one performs, whether it is adequately sensitive to changes in the distribution pattern and whether it adequately reflects changes in the general (income) scale. Selecting one measure over another, therefore, requires knowing the strengths and weaknesses of each measure, understanding its ability to provide the full picture.

Just as the use of different measures of inequality does not necessarily result in equal ranking of distributions, the use of different formulations of the Gini coefficient also may lead to the perception of different intensities of inequality.

The ability of a given inequality measure to assign a value to a specific distribution, which allows the characterization of that distribution in order to provide direct and objective comparisons between different distributions, is closely related to the fulfillment of certain properties by that measure.

There are four properties that appear in the literature as the basic requirements for an inequality measure. This basic criteria for measures of inequality are symmetry, scale invariance, the population principle, and the principle of transfers (Jenkins & Van Kerm, 2009). The compliance of these four properties ensure that the inequality measures behave in a reasonable manner, however not all inequality measures meet all of these requirements (e.g. Allison, 1978; Costa & Pérez-Duarte, 2019).

The Gini coefficient has been the default choice in many works addressing inequality. The good properties of this coefficient may be the main motivation for this use, however, the similarity and relation between different formulations of the Gini coefficient can be misleading as to the assumption that the properties are common to the entire family of Gini coefficients.

In the literature, it is widely assumed that the 'Gini coefficient' meets the basic criteria for inequality measures, however, as we will see in this work, there are formulations of the Gini coefficient that do not fulfil all basic criteria for inequality measures.

A widely used formulation of the Gini coefficient for discrete distributions considers the inclusion of self-on-self differences in its calculation. Based on this detail, Bowles and Carlin (2020) reject this formulation of the Gini coefficient as a measure of inequality, despite being a Lorenz based measure, and present an alternative that excludes the self-on-self differences, which they call the True Gini coefficient.

Our interest lies in studying the True Gini coefficient compliance with the four basic criteria for measures of inequality.

The outline of the current paper is as follows. In next section we mention the mean difference between n quantities, as a basis for the introduction of the Gini coefficient, we address the Gini coefficient associated with the Lorenz curve and we emphasize the formulation of the Gini coefficient, called the 'True Gini coefficient'. Following we present the four basic criteria for measures of inequality. In the central section of this work, we assess fulfilment of the basic criteria for measures of inequality by the True Gini coefficient. The paper is closed with some concluding remarks.

The true Gini coefficient

Measures of statistical dispersion describe the inequality between the values of a frequency distribution, revealing how concentrated or diffuse the distribution is. The Gini coefficient, like other measures of dispersion, such as the variance, the standard deviation, or the coefficient of variation, summarizes the information on the entire distribution in a single figure. This is considered one of the most advantageous features of the Gini coefficient (Morton & Blair, 2015). When compared with those and other measures, such as the standard deviation of logarithms, the Gini coefficient has the advantages of not focusing on differences from the mean, and avoiding the arbitrary squaring procedure (Sen, 1973). However, the Gini coefficient is not ideal in all circumstances, for example, it is not suitable for heavy-tailed distributions (Sitthiyot & Holasut, 2020) and it has the great disadvantage of being able to have the same value in two very different distributions (Bendel, Higgins, Teberg, & Pyke, 1989; González Abril, Velasco Morente, Gavilán Ruiz, & Sánchez-Reyes Fernández, 2010).

Since its inception, the Gini coefficient has been refined, improved and frequently modified to suit the numerous applications for which it has been adopted, however the essence of this coefficient was, and continues to be, the fact that it provides a direct measurement of the differences between individuals, as is so well shown by the 'mean of differences between n quantities' that constituted the starting point for the introduction of the Gini coefficient.

Covering the evolution of the Gini coefficient or exploring its numerous variants is beyond the scope of this work, however, it seems important to begin by presenting the measure that triggered it, to better understand its essence and potential.

Let us consider n quantities, $x_1, x_2, \ldots, x_n$, assumed in ascending order such that $x_{i-1} < x_i$, $i = 1,2,\ldots,n$. The mean difference between the n quantities, is given by

$$\Delta = \frac{2}{n(n-1)} \sum_{i=1}^{\frac{n+1}{2}} (n+1-2i)\,(x_{n-i+1} - x_i) \quad \text{(Gini, 1912).} \qquad (1)$$

This formulation only considers the differences between each quantity and the others, however we can include the difference between each quantity and itself, since these self-on-self differences are null. So, the sum of the n^2 differences between all possible pairs of quantities will be equal to the sum of the $n(n-1)$ differences between each quantity and the other. Taking this into consideration, Gini presented the mean difference with repetition between n quantities,

$$\Delta_R = \frac{2}{n^2} \sum_{i=1}^{\frac{n+1}{2}} (n+1-2i)\,(x_{n-i+1} - x_i) \qquad (2)$$

As Gini (1912) pointed out, both measures are relevant, adapting to particular situations, and the relationship between the two measures is

$$\Delta = \frac{n}{n-1} \Delta_R. \qquad (3)$$

The measure we know today as Gini coefficient, introduced two years later, is an updated version of the mean difference between n quantities, obtained by dividing it by twice the mean value of the n quantities, $\bar{x}$, taking the form of a measure of relative dispersion, which in Ceriani and Verme (2015) appears as

$$G = \frac{1}{2n^2\bar{x}}\sum_{i=1}^{n}(n+1-2i)\,(x_{n-i+1}-x_i). \qquad (4)$$

Over time, many other ways of representing the Gini coefficient have emerged, both for discrete and continuous distributions. Some different ways to express the Gini coefficient can be found in Dorfman (1979), Berrebi and Silber (1987), Ceriani and Verme (2015), and Xu (2004).

Let us consider n individuals, $n \geq 2$, and $X = (x_1, x_2, \dots, x_n)$ the distribution of a variable (e.g. income) over these individuals, with x_i the value of the variable for the i-th individual. One of the most usual formulations to measure inequality is the Gini coefficient given by

$$G = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n}|x_i - x_j|}{2\bar{x}n^2}, \qquad (5)$$

where: n is the sample size, x_i and x_j are the values of the variable for the i-th and j-th individuals, respectively, and $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$ is the mean value of X, which can be found, for example, in Kendall (1948), Dagum (1997) and Mussard, Terraza, and Seyte (2003).

As argued by Bowles and Carlin (2020) the Gini coefficient in (Equation 5) may not be considered a true measure of inequality, since 'it includes the fictitious self-on-self zero differences'. To measure inequality between real pairs of individuals, a corrected version of this coefficient can be obtained using the unique configuration of non-identical pairing. This corrected Gini coefficient, which Bowles and Carlin (2020) call the True Gini coefficient, is express as follows:

$$G_T = \frac{\sum_{i<j}\sum_{j=1}^{n}|x_i - x_j|}{\bar{x}\; n(n-1)} \qquad (6)$$

where: n is the number of individuals, x_i and x_j are the values of the variable for the i-th and j-th individuals, respectively, and $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$ is the mean value of X.

The geometric interpretation of the Gini coefficient translates the 'distance' between the measured distribution and a perfectly equal distribution (Molander, 2022).

Deriving the Gini coefficient from the Lorenz Curve (Figure 1), we have the ratio

$$G_L = \frac{A_b}{A_u}, \qquad (7)$$

where: A_b is the shadowed area between the Lorenz curve and the absolute equality line (diagonal), andA_u is the total area under the perfect equality line, corresponding to the triangular region underneath the diagonal (Gini, 1914; Golden, 2008). Considering this association, Bowles and Carlin (2020) adopt the designation of Lorenz-based Gini coefficient to refer to a Gini coefficient that, like the one in (Equation 4), can be deduced from the Lorenz curve.

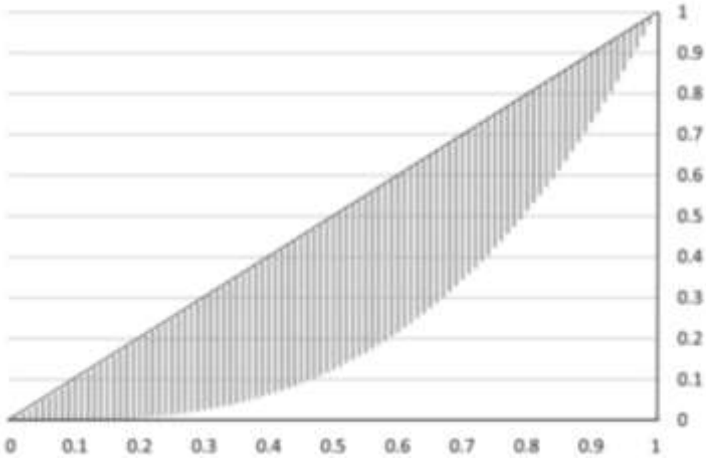

Figure 1. The Gini coefficient and the Lorenz curve (Molander, 2022).

As it follows from the derivation of the Gini coefficient from the Lorenz curve, the value of the Gini coefficient is lower the smaller the inequality, its maximum value is 1 and its minimum value is 0.

Since the Gini coefficient may misbehave for variables that assume negative values, falling outside the range [0, 1], we assume $x_i \geq 0$, $i = 1, \dots, n$.

Proposition 1: When all the individuals have the same income, $G_T = 0$.

Proof: When all the incomes are equal, $x_i = a,\ i = 1, \dots, n$, the differences $x_i - x_j,\ i = 1, \dots, n,\ j = 1, \dots, n$, are all null, so $G_T = 0$.

Proposition 2: When all the individuals have null income except one, $G_T = 1$.

Proof: When $x_k = b$, with $b \neq 0$ and $x_j = 0,\ j = 1, \dots, n,\ j \neq k$, the mean value is

$\bar{x} = \frac{\sum_{j=1}^{n} x_j}{n} = \frac{(n-1)\times 0+b}{n} = \frac{b}{n}$.

The differences $x_i - x_j$ are all null except those which involve x_k, *so*

$G_T = \frac{0+(n-1)b}{\frac{b}{n}n(n-1)} = 1$.

A null Gini coefficient represents perfect equality and a Gini coefficient of one represents maximal inequality.

Basic criteria for measures of inequality

Although several authors discuss the difficulty of conceptualizing inequality (e.g. Coulter, 1989), considering Sen (1973) proposal, where measures of inequality are classified as positive measures (statistical measures of dispersion) and normative measures (connected with a social welfare function), the use of positive measures allows to objectively quantify the extent of inequality, making it possible to establish a link between the two categories of measures since some positive measures are special cases of normative ones and it is not easy to establish a determined line between them. Thus, a starting point for defining criteria we want to be satisfied by measures of inequality has been to study the properties of different measures of dispersion.

Allison (1978) studied different candidates for inequality measures, establishing three criteria to determine their acceptability, the scale invariance, which states that when all incomes increase (decrease) in the same proportion the value of the inequality measure does not change, the ‘principal of transfers’, which translates the requirement established by Dalton (1920) that inequality measures must reflect, through an decrease in its value, the existence of income transfer from a higher-income individual to a lower-income individual, and decomposition, which makes sense in approaches that, instead of focusing on measuring inequality among all individuals, consider the population partitioned into relevant sub-groups.

Chakravarty (1999) states that a possible criterion for choosing a measure of inequality is to derive it from a social welfare function, considering a set of axioms that equality or an ordering of social welfare should comply with. Referring to the approach adopted in pioneering and relevant development works for inequality measures by Dalton (1920), Atkinson (1970), Sen (1973), and others, Chakravarty (1999, p. 1) states that these axioms involve “[…] the behavior of the welfare function in relation to changes in income levels, income distributions, and population size”.

According to Foster and Lustig (2019), an admissible approach to considering a measure of inequality as ‘good’ involves the four basic criteria for measures of inequality that gather broad consensus, although some authors add other advisable properties to inequality measures.

Symmetry (anonymity principle)

This principle states that does not matter who is earning the income, which means that the degree of inequality remains unchanged if there is a shift in the income values of pairs of individuals.

Pigou-Dalton transfer principle

This criterion establishes that a progressive [regressive] transfer of a positive amount of income, this is, a transfer from a richer [poorer] individual to a poorer [richer] individual without reversing the ranking between both, must lead to a decrease [increase] in inequality.

Scale invariance (relative income principle)

This criterion establishes that if everyone's income changes by the same proportion then there is no essential change in income distribution and therefore the value of the measure of inequality remains the same, so the measure cannot be affected by the absolute values of the income, only their relative values matter.

Principle of population

This criterion establishes that the inequality value remains the same if the population is replicated one or more times, this is, when a population with n individuals is combined with another similar population resulting in a population of Kn individuals and the same proportion of the population receiving any income.

The properties of Symmetry, Scale invariance, and the Pigou-Dalton transfer principle are concerned with the inequality in a population which has a fixed size, in contrast the Principle of Population deals with the inequality of different groups (Ebert, 1988).

In this set of desirable properties for inequality measures, the degree of importance attributed to properties varies at various levels, and failure to comply with one of these properties, by a given inequality measure, is not an impediment to the use of this measure (United Nations Development Programme [UNDP], 2019).

The diversity of measures of inequality, and the different degree of importance given to their properties, has provided attempts to collect some of these measures of inequality in classes considering some common characteristics. An important class is the class of relative inequality indexes (RII), which includes the measures that satisfy the principle of transfers, scale invariance and the principle of population at the same time (Calia, 2017).

In general, the option for a certain measure of inequality aims to obtain the maximum level of information on how to rank income distributions, therefore, choice decisions can be guided through other approaches besides the axiomatic one. Being outside the scope of our work, we refer those interested in knowing a general procedure to select inequality measures to the flow chart proposed by Bellú and Liberati (2006).

Checking the fulfilment of the basic criteria for measures of inequality by the True Gini Coefficient

The adequacy of inequality measures, and, in particular, the adequacy of the different formulations of the Gini coefficient, has been widely discussed, namely regarding their advantages, disadvantages and compliance with the set of desirable properties for inequality measures. Strangely, despite the vastness of literature that addresses the Gini coefficient, the approach to these properties is often incomplete or superficial or simply based on illustrative examples of these properties (e.g. Trannoy, 1986; Lemelin, 2005; Yitzhaki & Schechtman, 2013).

To ensure an objective analysis of the properties of the True Gini coefficient, in this work we present the proofs related to the verification of its fulfilment of the set of desirable properties for the inequality measures.

For the convenience of exposure, we will, from now on, restrict our mention to the Gini coefficient as a measure of income inequality.

Let us consider (a random sample of) n non-negative values (incomes) $x_1, x_2, \dots, x_n$, $(n \geq 2)$, x_i the income of the i-th person, $i = 1, \dots, n$, and $G_T(x_1, x_2, \dots, x_n) = G_T$ the Gini coefficient given in (Equation 7).

Proposition 3: G_T verifies the anonymity principle (Symmetry).

Proof: Rearranging the observations in the sample, for example, by placing them in ascending order $x_1' \leq x_2' \leq \cdots \leq x_n'$, or considering that two of the individuals, say A and B, who previously had incomes of x_{l_1} and x_{l_2}, respectively, now have incomes x_{l_2} and x_{l_1}, respectively, the Gini coefficient remains the same, due to the commutative property of addition, since all the absolute differences, $|x_i - x_j|$, $j = 1, \dots, n$, $i < j$, will occur (although in 'another order') when each value is replaced by the one that took its position, and also since the mean value, $\bar{x}$, is not influenced by order, $\sum_{j=1}^{n} x_j = \sum_{j=1}^{n} x_j'$, $j = 1, \dots, n$.

Proposition 4: G_T verifies the Pigou-Dalton transfer principle.

Proof: Let us consider $x_1 \leq \cdots \leq x_i \leq \cdots \leq x_j \leq \cdots \leq x_n$ and a transfer of an amount $k > 0$ from x_j to x_i, without changing the position of any of the elements of the sequence. Let us also consider $p < i$, $i < h < j$ and $m > j$.

After transferring the amount k from x_j to x_i, the sum in the numerator of (Equation 7) will have $i - 1$ addends equal to $|x_i - x_p + k|$, $i - 1$ addends equal to $|x_j - x_p - k|$, $j - i - 1$ addends equal to $|x_h - x_i - k|$, 1 addend equal to $|x_j - x_i - 2k|$, $j - i - 1$ addends equal to $|x_j - x_h - k|$, $j - n - j$ addends equal to $|x_m - x_i - k|$, $n - j$ addends equal to $|x_m - x_j + k|$, and $n - j$ addends equal to $|x_m - x_i - k|$, in addition to the addends corresponding to the differences between the observations not involved in the transfer. These addends corresponding to the differences between the observations not involved in the transfer remain unchanged after the transfer, so the difference in the value of the Gini coefficient before and after the transfer (G_{T_b} e G_{T_a}, respectively) is originated by changes in the parcels involving x_i and/or x_j. The difference between the numerator of (Equation 4) before and after the transfer is given by $-2(j-i-1)k-2k = -2(j-i)k$ and, therefore, $G_{T_b} - G_{T_a} = \frac{-2(j-i)k}{\bar{x}n(n-1)} < 0$.

As a result, the transfer caused a decrease in inequality between individuals.

Proposition 5: G_T is scale invariant.

Proof: Given two income distributions $x_1, x_2, \ldots, x_n$ and $y_1, y_2, \ldots, y_n$ corresponding to two random variables X and $Y = KX$, with k a positive constant. The Gini coefficient of the distribution Y is $G_{T_Y} = \frac{\sum_{i<j}\sum_{j=1}^{n}|y_i - y_j|}{\bar{y}n(n-1)} = \frac{\sum_{i<j}\sum_{j=1}^{n}|kx_i - kx_j|}{k\bar{x}n(n-1)} = \frac{k\sum_{i<j}\sum_{j=1}^{n}|x_i - x_j|}{k\bar{x}n(n-1)} = \frac{\sum_{i<j}\sum_{j=1}^{n}|x_i - x_j|}{\bar{x}n(n-1)} = G_{T_X}$

so, when all data points increased (decreased) by the same proportion their relative differences remain the same, so the value of the Gini coefficient remains the same too.

Proposition 6: G_T does not verify the principle of population.

Proof: Let us consider replication of an income distribution $x_1, x_2, \ldots, x_n$ by order k, for $k > 2$, $x^{[k]} = x_1, \ldots, x_1, x_2, \ldots, x_2, \ldots, x_n, \ldots, x_n$ where each $x_i, i = 1, \ldots, n$, is repeated k times.

The Gini coefficient of the distribution $x^{[k]}$ is $G_{T_{x^{[k]}}} = \frac{\sum_{i<j}\sum_{j=1}^{n} k^2|x_i - x_j|}{\bar{x}kn(kn-1)}$.

So $G_{T_{x^{[k]}}} = \frac{k(n-1)}{kn-1} G_{T_x}$, where G_{T_x} is the Gini coefficient of the 'before replication' income distribution, $x_1, x_2, \ldots, x_n$.

Since $k(n-1) < kn - 1$, we have $G_{T_x} > G_{T_{x^{[k]}}}$, concluding that replication reduces difference-based inequality.

The replication invariance property, reflected in the Principle of Population, ensures that inequality measures are independent of the size of the population, thus, allowing direct comparison of inequality in different sizes populations, regardless of their sizes. It is easy to see that the Gini coefficient in (Equation 5) is invariant by replication. As we have shown, the True Gini coefficient, expressed by (Equation 6), is not. Thus, it does not belong to the class of relative inequality indexes.

Conclusion

There are several ways to measure inequality, however there are rules that must be respected. Different legitimate choices of inequality measures can lead to different evaluations, so choosing the appropriate measure for a given object of study is crucial.

Choice decisions guided through an axiomatic approach, which involves Allison's criteria for inequality measures, is a relevant alternative.

The Gini coefficient, or rather the family of Gini coefficients that aggregate the variety of Gini coefficient formulations, has been the default choice in many works dealing with inequality. References to the main motivation for opting for this coefficient have pointed to its good properties, however, it often goes without saying that the similarity and relationship between different formulations of the Gini coefficient, in terms of its properties as a measure of inequality, can be misleading.

In this work we evaluated the fulfillment of the four basic criteria for inequality measures by the True Gini coefficient, having concluded that this formulation of the Gini coefficient does not comply with the principle of population and that, therefore, it cannot be integrated into the class of relative inequality indexes.

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