

# Metamodeling on uncertainty quantification in the behavior of the tire/road interaction of vehicles

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**ABSTRACT.** In recent years, the automotive industry has been developing applied research to meet customer's needs; considering safety, vehicle comfort and energetic efficiency. In particular, automotive tires have a prominent position in this research area, ensuring good vehicle handling, comfort and safety. In the vehicle dynamics performance, excellent gripping and reduced rolling resistance in the tires are crucial to maximize the energetic efficiency in a reliable way. However, to ensure such a level of reliability in vehicle operation, the inherent uncertainties of tires, as well as other factors subject to variability must be taken into account in the vehicle design. In this way, the present paper analyzes in detail the effect of variations in some parameters such as ambient temperature, ground conditions, vertical load, speed and tire inflation on the analysis of vehicle dynamics. A Metamodeling approach associated with the Monte Carlo Simulation was employed to develop the mathematical models to analyze the effect of uncertain parameters on the tire rolling resistance; traction, centripetal and lateral forces, using experimental data from the literature, in the longitudinal and lateral vehicle dynamics. Therefore, the present research brings as an innovation an integrated approach to the input parameters of the system with the rolling resistance through the developed metamodels. There was a substantial variability of up to 15% both up and down in the Maximum Traction Force of a vehicle in response to variations in the vehicle's weight and the coefficient of tire rolling resistance. In contrast, the Lateral Force exhibited a greater variability, with a 25 downward and 10% upward variation associated with the weight and friction coefficient variability of the vehicle. Further investigations into the sensitivity analysis highlight the significant influence of the friction coefficient and temperature on the Traction Forces of the vehicle.

**Keywords:** rolling resistance; uncertainty quantification; metamodels; automotive tires; energy efficiency; Monte Carlo method.

Received on September 26, 2022.

Accepted on May 29, 2023.

## Introduction

In the last decades, the automotive industry has been highly concerned with energy efficiency, comfort and vehicle safety. Particularly, the automotive tires due to the increase in vehicle power and speed bring up a prominent role in new research and developments, since they intensively participate in vehicle performance. The tires are crucial elements for the transmission of longitudinal, lateral and vertical forces between the vehicle and the road. Their statical and dynamical properties must be well known to provide the vehicle with good drivability associated with energy efficiency (Gillespie, 1992; Reimpell, Stoll, & Betzler, 2001; Kabe, Rachi, Takahashi, & Kaga, 2005; Savitski, Hoepping, Ivanov, & Augsburg, 2015; Farroni, Sakhnevych, & Timpone, 2016; Farroni, Sakhnevych, & Timpone, 2018; d'Ambrosio & Vitolo, 2019).

The literature presents a correlation between the tire's properties and the vehicle dynamics, and their variability can deeply affect vehicle performance in the most diverse aspects such as rolling resistance coefficient (RRC) and, consequently, energy efficiency, drivability, safety, comfort, braking and tractive force. It is important to emphasize that the tires are also a vehicle safety component, with a great impact on the vehicle handling dynamics. Therefore, tire manufacturers seek to achieve a balance among energetic efficiency, gripping, safety and vehicle reliability during the tire project. Any change in its properties directly affects the vehicle dynamics performance. This can be verified, for example, in a scenario with a flat tire or low inflation pressure. Consequently, the increase in the rolling resistance negatively influences the energetic efficiency of the vehicle, not mentioning the comfort (Gillespie, 1992; Pacejka, 2012; Savitski et al., 2015; Farroni et al., 2016; 2018; d'Ambrosio & Vitolo, 2019; Strigel, Peckelsen, Unrau, & Gauterin, 2019).

Particularly, the rolling resistance is associated with energy dissipation capable to keep the tire in motion, due to the viscoelastic behavior of the elastomer. In the tire's deformation cycles, variations in the tire's inflation pressure, the vertical load applied, ambient temperature, vehicle speed, material properties and the type of road surface can also influence this energy consumption value (Gillespie, 1992; Wong, 2001; Rao, Kumar, & Bohara, 2006; Yokota, Higuchi, & Kitagawa, 2012; Cho, Lee, Jeong, Jeong, & Kim, 2013; Taghavifar & Mardani, 2013; Ejsmont, Taryma, Ronowski, & Swieczko-Zurek, 2016; 2018; d'Ambrosio & Vitolo, 2019; Ejsmont & Owczarzak, 2019; Sina, Yazdi, & Esfahanian, 2019).

Rolling resistance is a widely discussed and studied topic since it is closely linked to vehicle dynamics issues. In this way, several engineering methods are adopted to analyze the tire's rolling resistance, in order to guarantee an improvement in the efficiency of obtaining this parameter. Thus, tests related to several parameters, such as temperature, inflation pressure, vertical load, speed, and road surface roughness are performed to analyze and investigate the tire and vehicle behavior (Stutts & Soedel, 1992; Sandberg, Ramdén, & Gamberg, 2004; Cho et al., 2013; Taghavifar & Mardani, 2013; Hoever & Kropp, 2015; Behnke & Kaliske, 2015; Li et al., 2018; Ejsmont & Owczarzak, 2019; Zhai et al., 2021).

The dynamic performances of the vehicle movement are fully determined by the interaction forces between the tires and the road. In addition, accurate estimates of tire/road interface information play a key role in vehicle control (Qi, Taheri, Wang, & Yu, 2015). The tire/road friction force has an essential role in maintaining the stability and controllability of the vehicle's dynamic behavior. Thus, the estimation of the tire/road friction coefficient is indispensable for the dynamic behavior of the vehicle (Hong, Erdogan, Hedrick, & Borrelli, 2013). Therefore, if the value of the friction coefficient is low, the risk of jackknifing exists and involves the non-linear effects of the tire characteristic curve. On the other hand, if the friction coefficient is sufficiently high, the level of lateral acceleration is high and yaw instability may not occur, as rollover occurs first (Mendes, Fleury, Ackermann, Leonardi, & Bortolussi, 2019).

To study in detail this subject, a metamodeling approach can be used. Metamodels for longitudinal and lateral vehicle dynamics were constructed, considering the experimental data from the literature, taking into account the tire rolling resistance; traction, centripetal and lateral forces. Basically, metamodeling techniques were developed from different subjects such as statistics, mathematics, computer science, and various engineering knowledge areas. Metamodels are initially developed as replacement tools for the expensive simulation process, in order to improve overall computation efficiency, moreover, they are considered a valuable tool to support a wide scope of activities in modern engineering design, especially in the optimization of projects (Wang & Shan, 2007; Dey, Mukhopadhyay, & Adhikari, 2017).

Metamodeling is the construction of models that act as surrogates for complex problems. Therefore, currently, the state of the art creates metamodels based on adaptive and active learning. Adaptive metamodeling refers to methods that somehow use an improvement measure to increase the ability to replace a given function. Thus, adaptive metamodeling has gained significant importance in reliability analysis in recent years (Khodak, Balcan, & Talwalkar, 2019; Teixeira, Nogal, & O'Connor, 2021). Now considering mathematical models obtained from the metamodeling approach, the uncertainties could be quantified using the Monte Carlo Method in the input parameters subjected to variability. The Monte Carlo Method consists of a numerical method based on random sampling used in probabilistic simulations, which requires the generation of samples composed of random variables in stochastic fields, considering previous probability distributions fixed in the stochastic model. Monte Carlo approaches use random sampling as a tool to produce observations that can be used to perform spectral analysis. The Maximum Entropy technique could be used to obtain the probability density function, with the available information, in the Monte Carlo simulation (Orkoulas, 2009; Cursi & Sampaio, 2015).

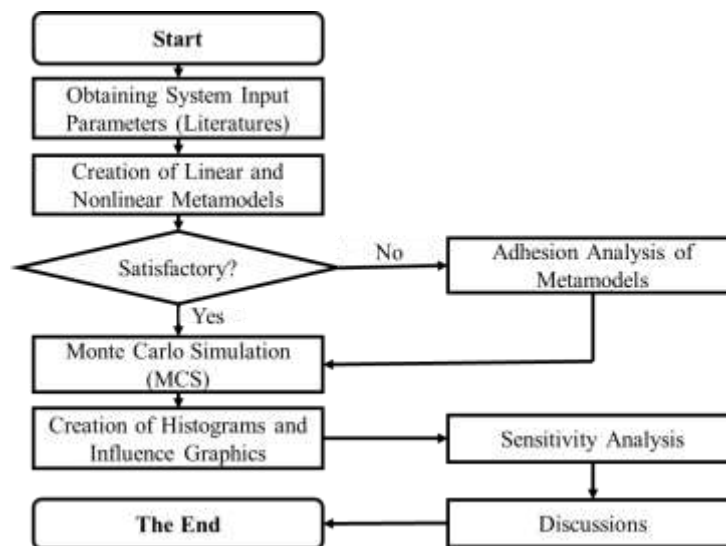
In this way, the present research aimed to investigate the effect of the inherent variability of input parameters, such as ambient temperature, inflation pressure, vertical load, vehicle speed, and road roughness on the performance of automotive tires. Therefore, the article promotes a qualitative analysis of the relationship between the influence and the relationship between the investigated parameters of the system. Additionally, the inherent variability of the tire manufacturing process and its use in the most diverse types of applications are taken into account here, from the combined approach between the Monte Carlo Method and Metamodeling techniques, using the data available in the literature to construct the mathematical models. In this way, the present study runs a unified approach to these parameters with vehicular performance, being an innovative contribution.

## Material and methods

### Applied methodology

The methodology proposed in the present study is summarized in Figure 1. For the construction of the tire/road interaction models, the uncertainty quantification considered the influence of several parameters with inherent variability that effectively influence the behavior of the longitudinal and lateral dynamics of a vehicle. Input parameters such as ambient temperature, inflation pressure, load, speed, and friction coefficient were used to develop the metamodels, using experimental data from the literature (Yokota et al., 2012; Cho et al., 2013; Ejsmont et al., 2016; Ejsmont & Owczarzak, 2019).

First, in the computer software MATLAB®, metamodels were developed using experimental data from the literature related to rolling resistance, to check the influence of the system input parameters on the rolling resistance of automotive tires. After, these metamodels were used in the Monte Carlo Simulation. The quantification of uncertainties associated with the variability aspects of each parameter was performed. Histograms and influence graphs associated with uncertain parameters were presented considering the models to study their influence on the vehicle dynamics performance. Finally, a sensitivity analysis was carried out on the vehicle dynamics equations to check the contribution of each uncertain input parameter.



**Figure 1.** Flowchart of the proposed methodology used in the metamodeling approach.

### Metamodel creation

Intensive computer-aided design problems are increasingly common in the automotive industry. The computational burden is usually caused by expensive analysis and simulation processes to achieve a comparable level of precision and reliable physical test data, with approximation or metamodeling techniques frequently used to address this challenge. Metamodeling techniques have been developed from various subjects such as statistics, mathematics, computer science, and various engineering areas. Metamodels are initially developed as replacement tools for expensive simulation processes to improve overall computational efficiency and are considered a valuable tool to support a wide range of activities in modern engineering design, especially in design optimization (Wang & Shan, 2007; Dey et al., 2017).

In the past two decades, approximation methods and approximation-based optimization have attracted much attention, with this approach approximating intensive computational functions with simple analytical models. The simple model is often called a metamodel, while the process of building a metamodel is called metamodeling. With a metamodel, optimization methods can be applied to search for the optimum, known as metamodel-based design optimization (MBDO). In addition, the benefits of MBDO are associated with various factors, such as the easier connection of proprietary and frequently expensive simulation codes, simple parallel computation involving running the same simulation at many points in the design, better filtering of numerical noise than gradient-based methods (Wang & Shan, 2007). In this way, the metamodeling technique has thus been employed in various engineering fields, such as in processes for bioethanol production (Freitas, Olivo, & Andrade, 2017) and sucrose crystallization models (Gonzales, Peloso Jr., Olivo, & Andrade, 2020).

Linear and nonlinear metamodels were developed to replace the lack of analytical models or high computational cost simulations of the tire/road interaction of the vehicular dynamics. Subsequently, metamodel domain analysis was performed. The metamodel was compared with experimental data to minimize errors.

An option to reduce the percentage error of metamodels is to refine the mathematical model in the interest area, thus promoting a more detailed analysis in the region of interest, or redefining the function in the new subdomain, which requires another function in the metamodel. Metamodels referring to the vehicle dynamics system input parameters were developed according to Equations 1, 2, 3, 4 and 5.

$$RRC_{TEMP} \cong F^{MM}(x) = c_0 + \sum_{i=1}^n c_i T_i + \sum_{j=1}^p \sum_{k=1}^q a_j T_j^k \quad (1)$$

$$RRC_{PRESSURE} \cong F^{MM}(x) = c_0 + \sum_{i=1}^m c_i P_i \quad (2)$$

$$RRC_{LOAD} \cong F^{MM}(x) = \sum_{i=1}^n \frac{c_0}{W_i} + \sum_{i=1}^m \frac{c_i C_i}{W_i} \quad (3)$$

$$RRC_{SPEED} \cong F^{MM}(x) = \sum_{i=1}^m \frac{c_0}{W_i} + \sum_{i=1}^n \frac{c_i V_i}{W_i} + \sum_{j=1}^p \sum_{k=1}^q \frac{a_j V_j^k}{W_j} \quad (4)$$

$$RRC_{TOTAL} = \sum \left[ RRC_{TEMP} + RRC_{PRESSURE} + \left[ \frac{1}{n} \sum_{i=1}^n RRC_{LOAD_i} \right] + RRC_{SPEED} \right] \quad (5)$$

The metamodels were developed by fitting experimental data available in the literature. A detailed inspection was carried out in their linearity and non-linearity, minimizing the percentage error between the developed metamodel and the experimental data. From a thorough analysis of the formulations related to the metamodels, the analysis of variability using the Monte Carlo Simulation Method becomes more feasible considering the uncertainties arising only from the input parameters.

### Metamodel adherence

In order to analyze the numerical errors arising from the suggested methodology, an analysis of the metamodel's adherence to experimental data was performed. Coefficients  $c_0$ ,  $c_i$ ,  $a_j$  and  $a_0$  in Equation 1 to 4 were determined by minimizing their quadratic error, according to Equation 6, from the metamodels  $x^{(K)}$ , where  $K = 1, 2, \dots, N^D$ , known and physically possible in the interaction between tire and road.

$$\min e = \sum_{t=1}^{N^D} (F^{MM}(x^K) - F(x^K))^2 \quad (6)$$

In Equation 6, the term  $e$  refers to the minimization of the error between the metamodel and the original function;  $F(x^K)$  is related to the function obtained from the literature, while  $F^{MM}(x^K)$  is related to the function related to the metamodel of each parameter.

Therefore, a detailed analysis of the adherence of the metamodels with experimental data was carried out, portraying that in the domain of the function, where the Monte Carlo Simulation will be coherently generated. Thus, it was possible to quantify the uncertainties using the appropriate metamodels since there was no change in the physical meaning of the analyzed phenomenon. In this way, the present research minimizes the errors between the experimental data and the values obtained by virtual simulation, without incurring non-parametric uncertainties.

Therefore, to develop the rolling resistance metamodel referring to the action of temperature, the study by Yokota et al. (2012) was used, and the variation between the data collected from the literature and the developed metamodel is less than 0.05%. On the other hand, for the case of the metamodel of rolling resistance due to vertical load, the study by Ejsmont et al. (2016) was used as a reference, and the error obtained between the metamodel obtained and the literature data is less than 0.90%.

In addition, for the development of the rolling resistance metamodel referring to the effect of speed, the study by Cho et al. (2013) was used as a database. For the effect of the vehicle speed variation on the rolling resistance, the percentage errors between the data collected from the literature and the developed metamodel

are minimal, less than 0.05%. Finally, to develop the rolling resistance metamodel due to the effect of inflation pressure, the study by Ejsmont and Owczarzak (2019) was used as a database. Thus, no error was found between the literature data and the proposed metamodel. Simulations about rolling resistance with the system input parameters were performed with tires of similar characteristics and the same application (Yokota et al., 2012; Cho et al., 2013; Ejsmont et al., 2016; Ejsmont & Owczarzak, 2019). In this way, the results, even from different studies, present compatibility with the obtained results.

### Monte Carlo simulation

The Monte Carlo method is a powerful numerical technique for probabilistic simulations that involve generating samples from random variables, vectors, or processes, based on probability distributions defined in a stochastic model. This approach uses random sampling as a tool to produce observations, which are subsequently used in statistical analyses and inference to extract information about quantities of interest. The deterministic and stochastic models of the system play crucial roles in obtaining accurate results in Monte Carlo simulations and directly affect the responses obtained (Cursi & Sampaio, 2015; Castelo & Ritto, 2016).

To generate studies of stochastic objects, which are subjected to certain probability densities, the Monte Carlo method uses random sampling as a tool to produce observations on which statistical inferences are made. The method has proven to be a valuable tool in obtaining numerical approximations for complex problems. The implementation of the Monte Carlo method is intrinsically linked to the problem to which it is applied. Therefore, the precision of the simulation results heavily depends on the accurate definition of the system and the inclusion of all critical parameters into the model, along with their statistical or probabilistic characteristics (Cursi & Sampaio, 2015; Castelo & Ritto, 2016).

With the developed metamodels, the Monte Carlo Simulation was implemented considering the following uncertain parameters (Figure 2): ambient temperature, inflation pressure, load, vehicle speed, and friction coefficient. To develop the probabilistic models in the uncertain input parameters, the Principle of Maximum Entropy was used with the available information (Kapur, 1989; Piovan & Sampaio, 2015; Cursi & Sampaio, 2015; Scinocca & Nabarrete, 2020). The principle consists of maximizing the system entropy, as defined by Shannon (1948), using the available information. Thus, the Probability Density Function (PDF) of the uncertain parameters is obtained consistently with the available information and the physics of the problem. With the obtained PDF, the analysis of the uncertainty propagation can be conducted. The principle is formulated as follows:

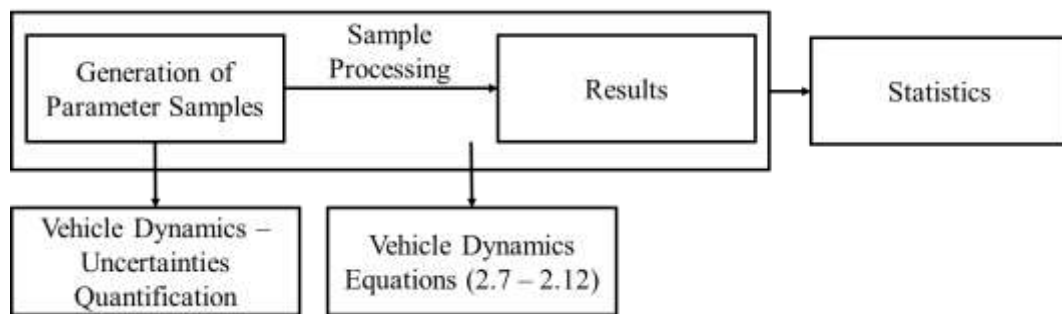


Figure 2. Monte Carlo simulation (MCS).

‘Among all probability distributions that satisfy the constraints given by the available information, select the one that maximizes the entropy’.

The probabilistic distributions that maximize the entropy of the system under analysis were selected, as listed in Table 1. Thus, incorporating the inherent variability of input parameters into the vehicle dynamics equations, represented according to Equations 7, 8, 9, 10 and 11, generates the related histograms and influence graphs.

$$F_{TRACTION_{Max}} = \frac{\mu_p W_t \left[ \frac{L_b + \left[ \frac{1}{4} \sum_{i=1}^4 RRC_{TOTAL} \right] h}{L} \right]}{1 + \frac{\mu_p h}{L}} \quad (7)$$



$$F_{TRACTION_{Front}} = \mu_p W_t \cos \theta \left[ \frac{1 - x + \frac{\left[ \frac{1}{2} \sum_{i=1}^2 RRC_{TOTAL} \right] h}{L}}{1 + \frac{\mu_p h}{L}} \right] \quad (8)$$

$$F_{CENT} = M \frac{V^2}{\rho} \quad (9)$$

$$F_Y = \mu_p W_t \left[ 1 - \frac{\mu_p W_t}{4 K_L \tan(\alpha)} \right] \quad (10)$$

$$K_L \cong F^{MM}(c) = a_0 + \sum_{j=1}^m a_j P_j \quad (11)$$

Also, a sensitivity analysis of the input parameters was performed. The largest contributors in the analysis of the vehicle's dynamic behavior could be identified.

**Table 1.** Probabilistic distribution of the system input parameters.

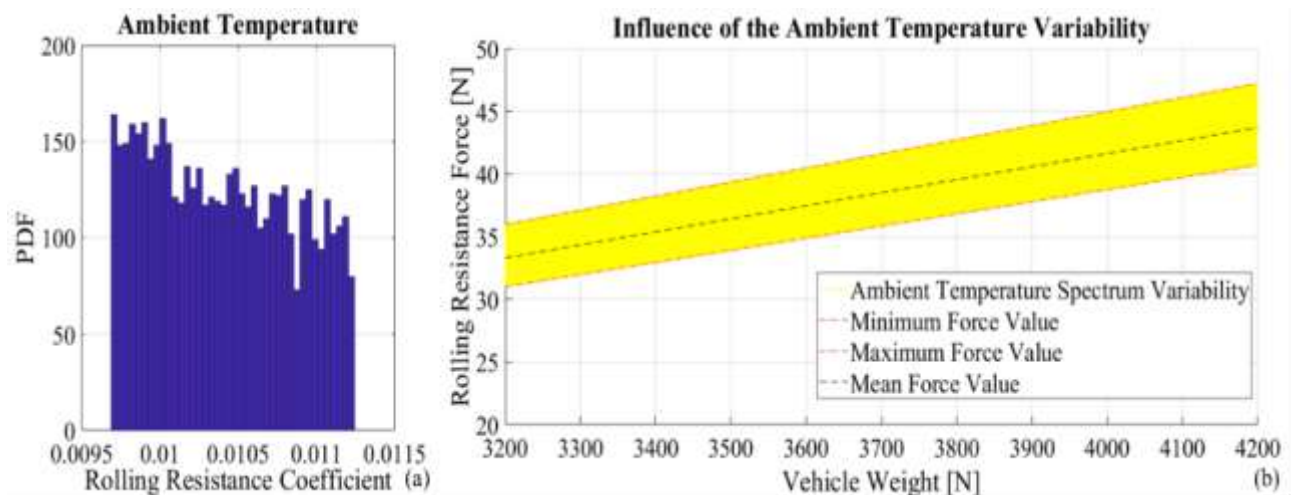
Parameters	Probability Distribution	Minimum Value	Maximum Value
Temperature	Uniform	12°C	28°C
Inflation Pressure	Logarithmic Decay	0.1 MPa	0.25 MPa
Vertical Load	Normal	3200 N	4200 N
Vehicle Speed	Normal	0	≈150 km hour
Friction Coefficient	Uniform	0.5	0.9

## Results and discussion

### Uncertainty quantification in the rolling resistance

The uncertainties that influence the system input parameters referring to rolling resistance were analyzed in terms of the developed metamodel equations, in particular, related to temperature, inflation pressure, load, speed, and final case. In addition, with a large number of parameters susceptible to the influence of uncertainties during tire manufacturing and the simulation, a thorough data analysis is required. Therefore, multiple studies are currently underway to quantify uncertainties in tires, to improve their production and performance (Fathollahi-Fard et al., 2021; Böttcher, Graf, & Kaliske, 2022; Liu, Wang, Cai, Wei, & Marburg, 2023).

The first analysis performed was related to the tire rolling resistance due to the influence of ambient temperature, according to Equation 1, with data obtained from the literature (Yokota et al., 2012). Figure 3 illustrates the behavior of rolling resistance influenced by temperature.



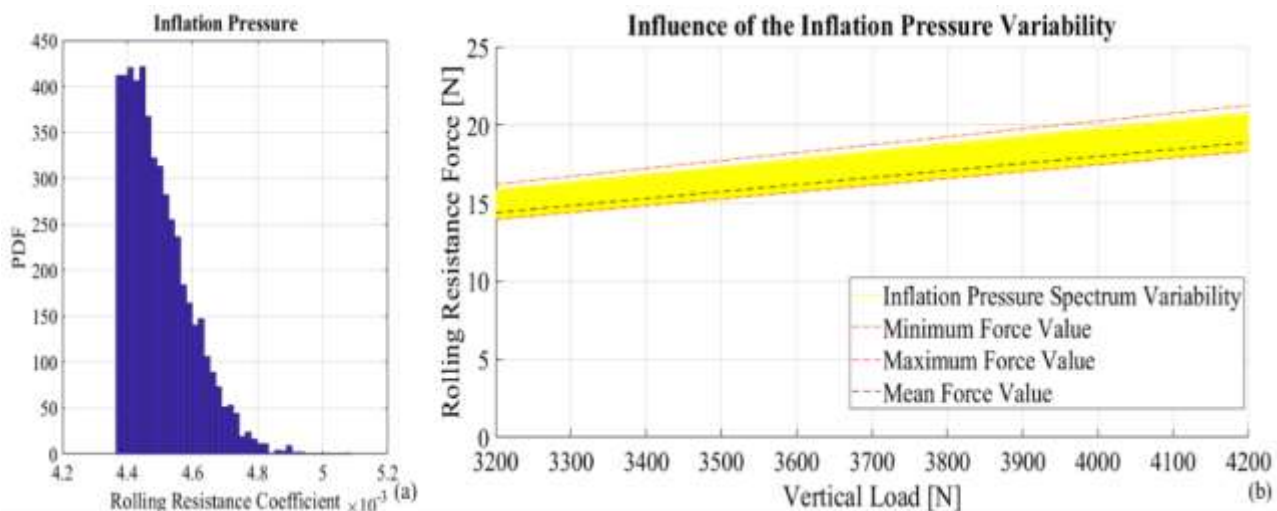
**Figure 3.** Temperature variability in the rolling resistance: (a) histogram; (b) influence graph.

The asymmetry of the histogram in Figure 3a is due to the input parameters becoming a distinct statistical distribution, once those random variables algebra results in a scatter plot with this asymmetric format

(Springer, 1979). Importantly, values referring to the rolling resistance coefficient vary between 0.0095 and 0.0115. According to Figure 3b, the variation in ambient temperature drastically affects the relationship between rolling resistance force per load on the tire. In addition, when the number of passengers in the vehicle increases, the temperature variability region proportionally grows, more strongly influencing the relationship in question. Therefore, for load values equal to 3,800 N, their respective rolling resistance force varies from 35 to 45 N.

For the tire inflation pressure, as presented in Equation 2, a similar analysis was made using data from the literature (Ejsmont & Owczarzak, 2019). The histogram and the influence graph were obtained, as shown in Figure 4.

The shape of the histogram in Figure 4a is directly related to its constituent parameters since hypotheses such as logarithmic decay are considered, showing the result that the rolling resistance coefficient varies between 0.0042 and 0.0052. In Figure 4b, it is possible to observe that the effect of variation in the inflation pressure of the vehicle tire can affect this relationship between force per load, in approximately 4% referring to the minimum and average values, and in about 10% comparing the maximum and average values, obtaining a maximum value greater than 20 N, when applying the maximum load to the tire.



**Figure 4.** Inflation pressure variability in the rolling resistance: (a) histogram; (b) influence graph.

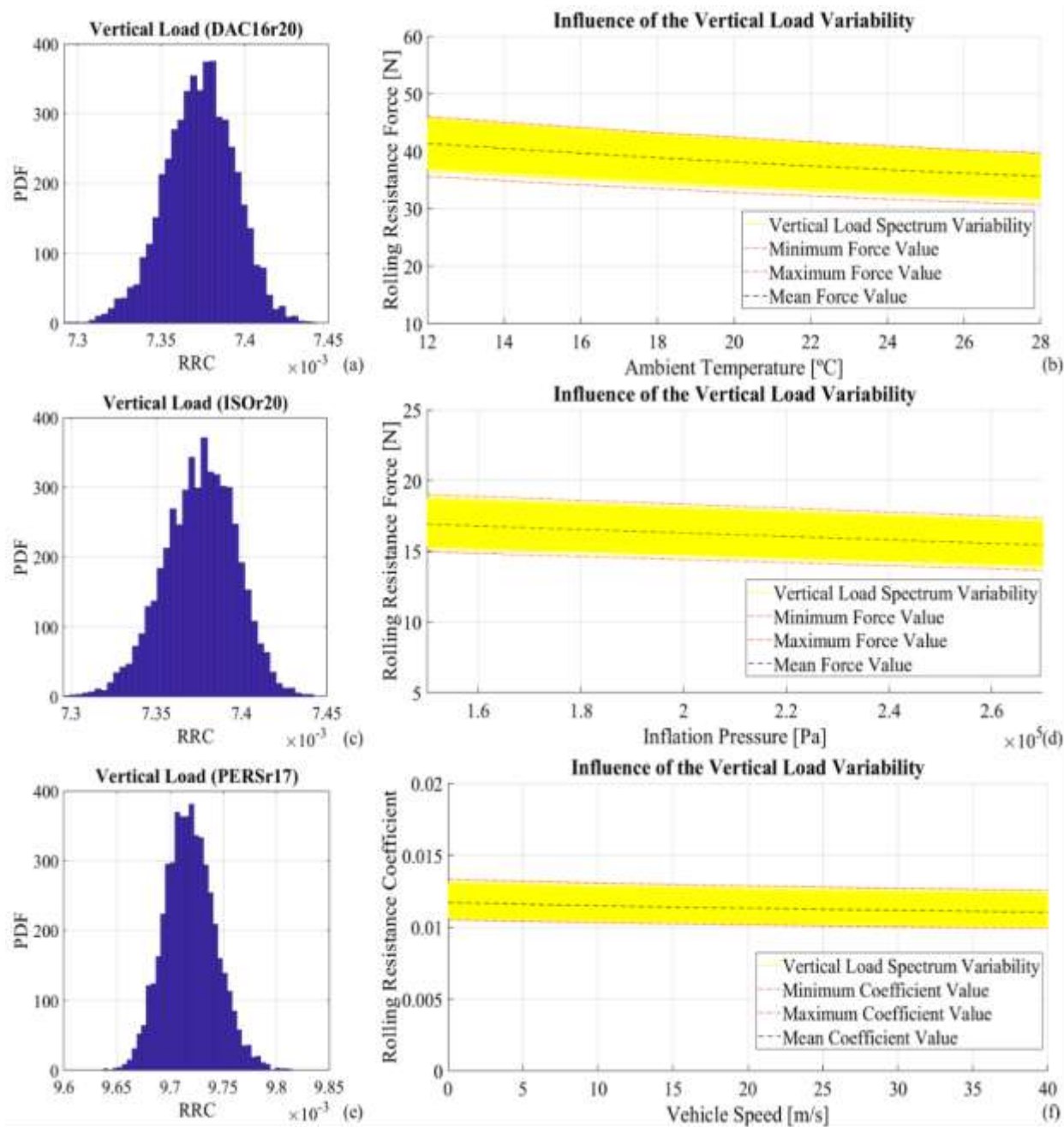
Figure 5 presents the histograms and graphs of the influence of the relationship between vertical load and rolling resistance. Subsequently, the influence of the vertical load on three different types of road surfaces was analyzed, according to Equation 3. The influence of the number of passengers and the different road conditions on the automotive tire rolling resistance, obtaining experimental data from the literature, was investigated by Ejsmont et al. (2016).

The characteristics of histograms in Figure 5a, c, and e are intrinsic to the number of occupants in the vehicle, with the hypothesis that the car varies in weight as the number of passengers increases or decreases. The histograms referring to the DAC16r20 and ISOr20 roads have similarities in the behavior regarding the values of the rolling resistance coefficient, with values between  $7.3 \times 10^{-3}$  and  $7.45 \times 10^{-3}$ . However, for the case of PERSr17 road, there was a large increase in the coefficient, ranging from  $9.6 \times 10^{-3}$  to  $9.85 \times 10^{-3}$ . Thus, this phenomenon may be related to the irregularities and textures of each type of road, represented by the friction coefficient parameter of the road (Ejsmont et al., 2016).

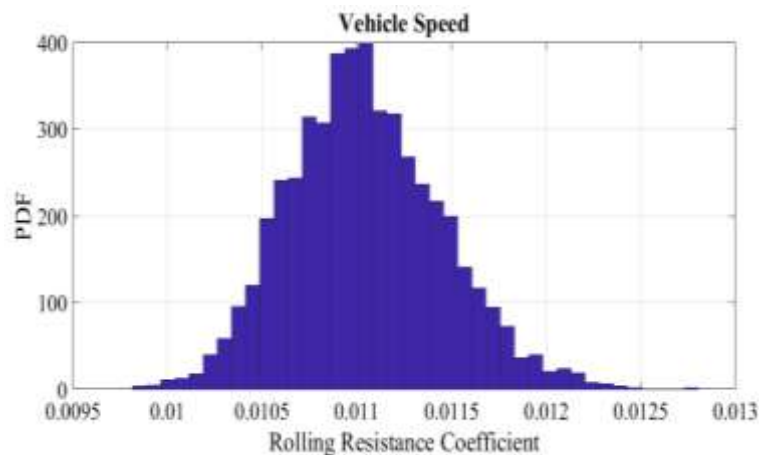
Analyzing Figure 5b and d, it is possible to observe that the effect of the number of vehicle occupants affected the relationship between the rolling resistance force by ambient temperature and inflation pressure, presenting a variation of approximately 10% more or less in their resistance force. In particular, in Figure 5b, it is possible to observe a decrease in the rolling resistance with increasing temperature.

Furthermore, in Figure 5f, the influence of the number of vehicle occupants on the tire's rolling resistance totaled a variation of approximately 10% of the maximum and minimum values compared to the average value of the coefficient. In this case, Figure 5f presents the increase in the vehicle speed, when the rolling resistance is reduced, confirming the physical phenomenon addressed by the metamodels.

Therefore, an approach was carried out regarding the influence of vehicle speed variability on tire rolling resistance, according to Equation 4, obtaining data from the literature (Cho et al., 2013). Therefore, Figure 6 shows a histogram of the relationship between speed and rolling resistance.



**Figure 5.** Vertical load variability in the rolling resistance: (a, c, e) histogram; (b, d, f) influence graph.



**Figure 6.** Histogram of speed variability in the rolling resistance.



Figure 6 shows the range between 0.0095 and 0.013, in which the coefficient of rolling resistance due to the influence of speed is included. The shape of the histogram obtained is directly related to the type of probability distribution referring to the input parameter of the system.

Finally, we analyzed the behavior of the rolling resistance through the superposition of the effects of all input parameters, according to Equation 5, and the total rolling resistance is presented in Figure 7. In addition, a hybrid factor was considered for the case of vertical load, since in this scenario there were three different types of roads.

In the histogram in Figure 7, the coefficient of rolling resistance assumed values between 0.0325 and 0.0365. This can be explained by the influence of overlapping parameters, making this analysis of great interest to understand how such factors are capable of interfering with the dynamic behavior of the vehicle.

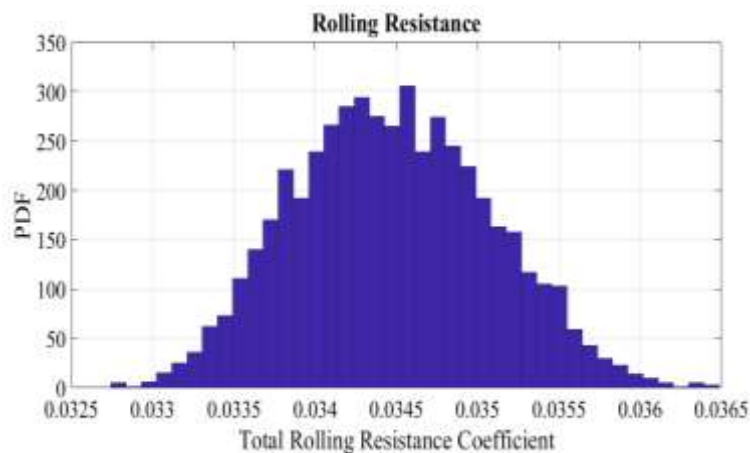


Figure 7. Histogram of the total rolling resistance variability.

### Uncertainty quantification in the vehicle dynamics

The uncertainties that influenced the rolling resistance are analyzed using equations that govern the vehicle dynamics, more specifically, the longitudinal and lateral dynamics.

The first analysis was based on the maximum traction force of the vehicle, through the variability factors of its parameters, as well as vehicle weight, friction coefficient, and the coefficient of total rolling resistance of the tire. Therefore, from Equation 7, we obtained the histogram and the influence graph associated with variation in the maximum traction force related to the intrinsic uncertainties of its constituent parameters, as shown in Figure 8.

As seen in the histogram in Figure 8a, this characteristic shape and its probability density function had an unconventional configuration. Such behavior occurs because the histogram result is intrinsically obtained by Monte Carlo modeling, in which the input parameters had different probability distributions and, when performing the algebra with random variables, resulted in a scatter plot with this shape (Shewhart & Wilks, 1979). In addition, it was possible to identify the variation of this force between approximately 2,500 and 5,500 N.

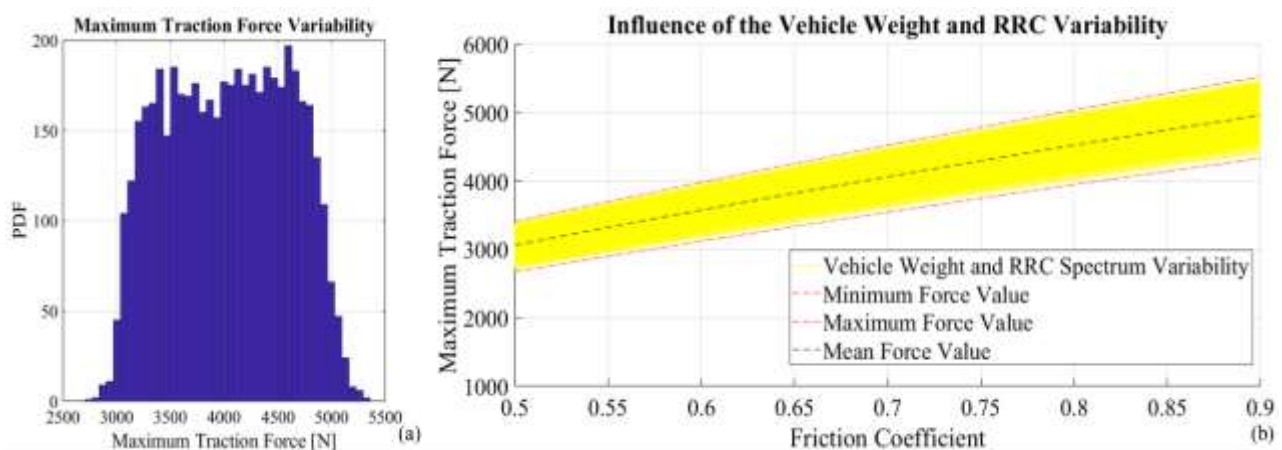


Figure 8. Maximum traction force: (a) histogram; (b) influence graph.

Figure 8b illustrates the effect of variations in vehicle weight on the total rolling resistance coefficient, factors with high influence on the maximum traction force ratio by friction coefficient, with a variation of approximately 15% up or down, drastically influencing this relationship. In addition, with the increasing friction coefficient of the track, combined with the texture and irregularities of the soil, there is a proportional increase in this variability. Therefore, the maximum traction force assumes a maximum value between 5,000 and 6,000 N, when the friction coefficient in the analyzed domain is maximum.

Subsequently, we sought to analyze the dynamic behavior of a vehicle with front-wheel drive in a situation of inclined road, considering some of the dimensional parameters of the vehicle, as represented by Equation 8. Therefore, Figure 9 shows the aspects of the variability of the constituent parameters of this case.

The shape of the histogram in Figure 9a is unconventional due to factors related to the intrinsic issues of the histogram obtained by the Monte Carlo simulation, resulting in a scatter plot with this shape (Shewhart & Wilks, 1979). In addition, values referring to the force varied between 2,500 and 5,500 N. Analyzing Figure 9b, the variation in vehicle weight and the total rolling resistance coefficient drastically affect this relationship between maximum traction and friction coefficient, with a variation between 10 and 15% referring to the average value of this force. In addition, the maximum value of this analyzed force can reach values between 5,000 and 6,000 N when the friction coefficient is at its maximum value in the studied domain.

The analysis of lateral dynamics was based on centripetal and lateral forces in order to verify how the variability of stochastic parameters influences the lateral performance of the vehicle. Thus, the first approach considered the centripetal force, using Equation 9, obtaining the scatter plot and influence graph, as shown in Figure 10.

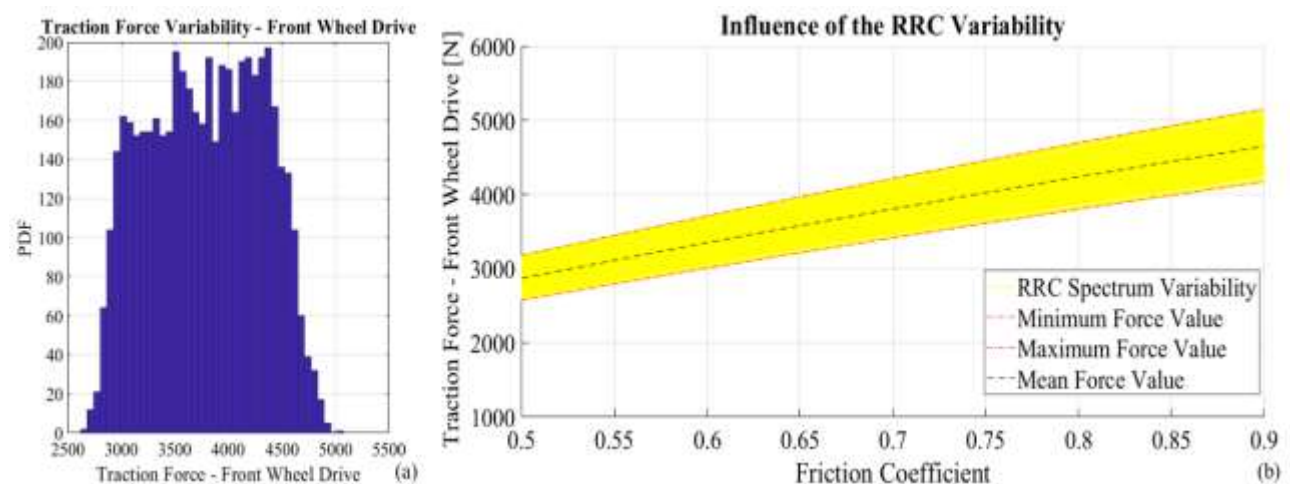


Figure 9. Traction force – front-wheel drive: (a) histogram; (b) influence graph.

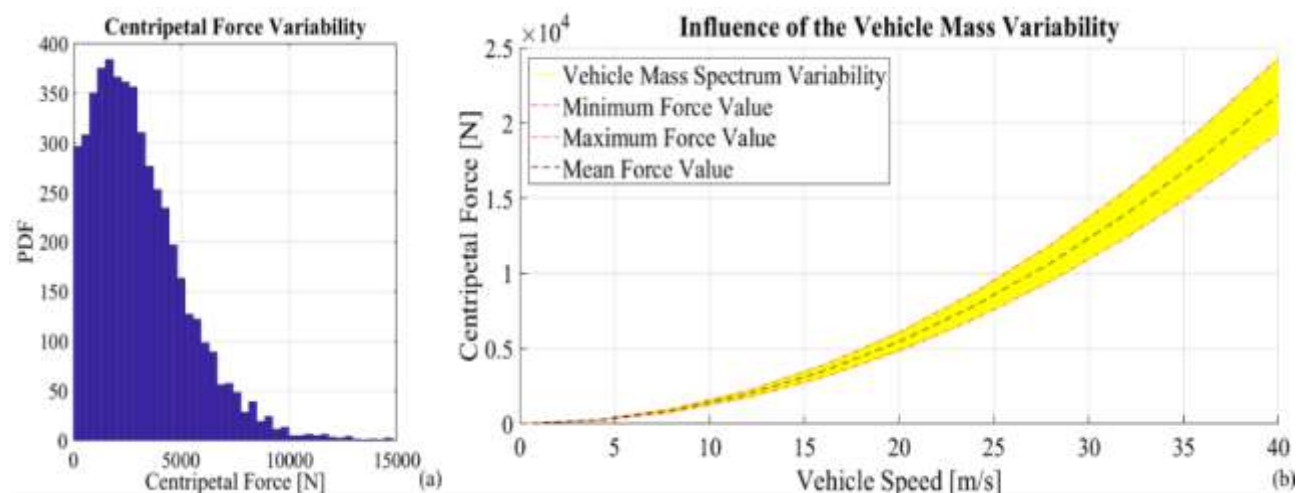


Figure 10. Centripetal force: (a) histogram; (b) influence graph.

In Figure 10a, the centripetal force is considered within a domain interval with a maximum value of 15 kN. In Figure 10b, it is possible to observe that the vehicle mass variability drastically affects the relationship between centripetal force per vehicle speed, with a variation of approximately 10 up and 15% down in its centripetal force at its highest velocity value in the studied domain. As a result, the centripetal force showed a small sensitivity to the vehicle mass at low speeds, however at higher speeds, there was a significant increase in the sensitivity to this force, as the thickness of the mass variability spectrum increased.

Therefore, an approach was developed for the lateral force of the vehicle, in order to approach the behavior of the vehicle in an analysis related to the vehicular lateral dynamics, represented by Equation 10 and 11, obtaining the scatterplot and influence graph represented in Figure 11.

With the histogram in Figure 11a, the value of the vehicle's lateral force is within the domain range from 5,000 to 8,000 N. In Figure 11b, a great influence can be observed for the variability of the friction coefficient and the weight of the vehicle exerted on the lateral force. Thus, there was a high variability of these parameters along the tire inflation pressure domain, in which the variation between the average and minimum values of lateral force is approximately 25%, while between the average and maximum values, it is around 10% when analyzed at the minimum value of the inflation pressure domain. Thus, Table 2 lists the main results referring to histograms obtained to enrich and provide a better understanding of the results presented.

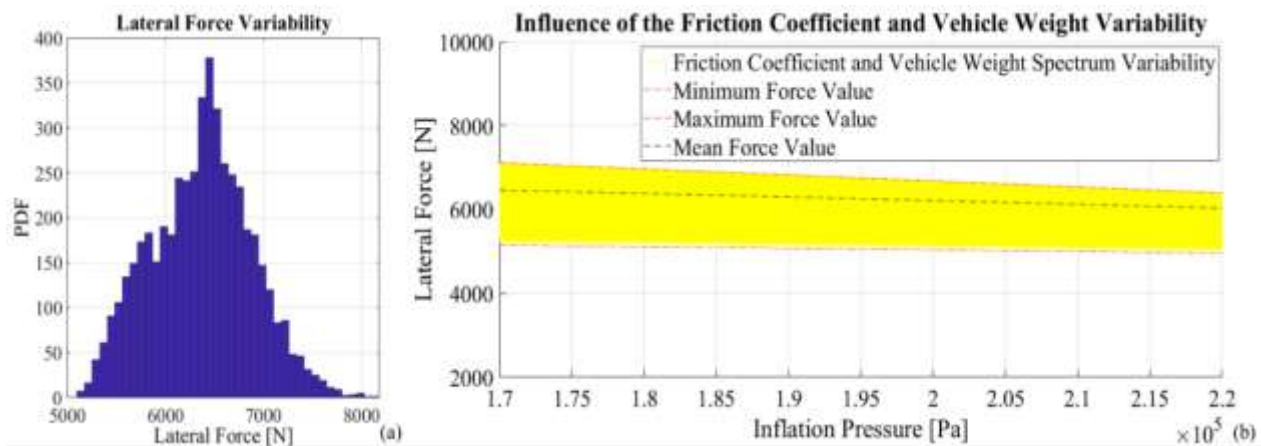


Figure 11. Lateral force: (a) histogram; (b) influence graph.

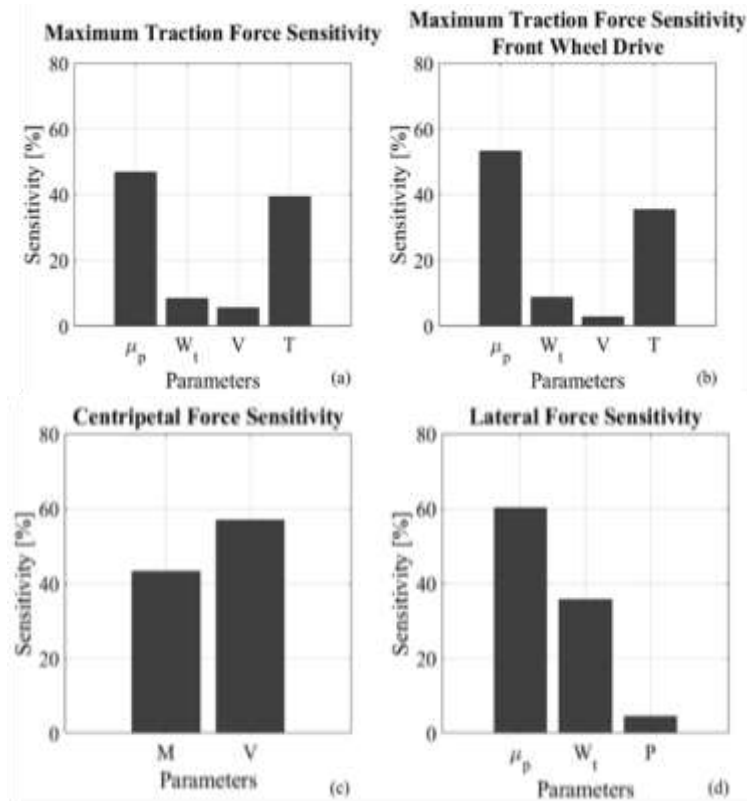
Table 2. Main results.

Parameters			Minimum	Maximum
RRC (Temperature)			0.00970	0.01125
RRC (Inflation Pressure)			0.00435	0.0049
RRC (Vertical Load)	DAC16r20		0.00732	0.00743
	ISOr20		0.00730	0.00743
	PERSr17		0.00964	0.00982
RRC (Speed)			0.00980	0.01275
RRC (Total)			0.03275	0.0365
Maximum Traction Force [N]			2,750	5,250
Traction Force – Front-Wheel Drive [N]			2,750	5,100
Centripetal Force [N]			0	15,000
Lateral Force [N]			5,100	8,000

### Sensitivity analysis

Sensitivity analyses were carried out using Equation 7 to 11 that govern the vehicle dynamics, especially in the scenarios of longitudinal and lateral dynamics, as shown in Figure 12.

In Figure 12a and b, it is possible to observe a high influence of temperature on the maximum traction forces. In vehicles with front-wheel drive, this parameter contributes approximately 40%. In addition, the friction coefficient had a strong influence in cases of traction and lateral forces. Its sensitivity is circa 40 to 60%, compared with the other parameters. The other parameters such as total vertical load and speed have a lower influence, 10%, according to Figure 12a, b, and d. Due to the very low influence of the inflation pressure, it is disregarded in Figure 12a and b. Finally, as to the centripetal force, the contribution relative to the sensitivity of the mass is approximately 40%, while the speed is responsible for circa 60%, as shown in Figure 12c.



**Figure 12.** Sensitivity analysis: (a) maximum traction force; (b) traction force in front-wheel drive; (c) centripetal force; (d) lateral force.

### List of nomenclature

$a_0, c_0, c_i$  and  $a_j$ : coefficients of the metamodels (obtained through experimental data collected from the literature);

$F_{TRACTION_{front}}$ : maximum traction force in front-wheel drive vehicles;

$F_{TRACTION_{Max}}$ : maximum traction force of the vehicle;

$F_{centr}$ : centripetal force;

$F^{MM}(x)$ : function relative to the metamodel of the analyzed parameter in question;

$F_y$ : lateral force;

$K_L$ : lateral tire stiffness;

$L_b$ : distance between the center of gravity and the vehicle's rear wheel;

$RRC_{LOAD}$ : rolling resistance coefficient related to the vertical load;

$RRC_{PRESSURE}$ : rolling resistance coefficient related to the inflation pressure;

$RRC_{SPEED}$ : rolling resistance coefficient related to the vehicle speed;

$RRC_{TEMP}$ : rolling resistance coefficient related to the temperature;

$RRC_{TOTAL}$ : total rolling resistance coefficient;

$W_t$ : vehicle weight;

$\mu_p$ : friction coefficient;

$C$ : vertical load;

$h$ : height of the vehicle's center of gravity;

$L$ : wheelbase;

$M$ : vehicle mass;

$P$ : tire inflation pressure;

$T$ : temperature;

$V$ : vehicle speed;

$W$ : applied tire load;

$x$ : portion of load on the rear axle;

$\alpha$ : tire slip angle;

$\theta$ : slope angle of the track;

$\rho$ : turn radius of the road.

## Conclusion

In conclusion, changes made in the properties and characteristics of tires can drastically influence the behavior of the vehicle in different situations, such as fuel consumption, safety, comfort, braking, and transmission of forces.

Concerning the analyses carried out on the uncertainty quantification and variability of the system's input parameters, we found satisfactory results since the developed metamodels effectively and reliably fit the experimental data collected from the analyzed literature.

There was a variability of 15% up and down in the maximum traction force associated with the vehicle weight variability and the tire rolling resistance coefficient. On the other hand, a 25 downward and 10% upward variability in lateral force is related to the variability of vehicle weight and friction coefficient.

The sensitivity analysis indicated the great interference of the friction coefficient and the temperature with the vehicle's traction forces, the great influence of the speed on the centripetal force, and the great interference of the friction coefficient and total vehicle weight with the lateral force.

Therefore, the type of road surface, combined with the friction coefficient, has a great influence on issues related to vehicle performance, confirming the importance of in-depth and complex studies on the surface pattern. These studies aim to guarantee the development of economical tires, from the point of view of fuel consumption, and adhering to the road, associated with vehicle safety.

## References

- Behnke, R., & Kaliske, M. (2015). Thermo-mechanically coupled investigation of steady state rolling tires by numerical simulation and experiment. *International Journal of Non-Linear Mechanics*, 68, 101-131. DOI: <https://doi.org/10.1016/j.ijnonlinmec.2014.06.014>
- Böttcher, M., Graf, W., & Kaliske, M. (2022). Quantification of data and production uncertainties for tire design parameters in the frame of robustness evaluation. *Probabilistic Engineering Mechanics*, 70, 103357. DOI: <https://doi.org/https://doi.org/10.1016/j.pro bengmech.2022.103357>
- Castelo, D. A., & Ritto, T. G. (2016). *Quantificação de incertezas e estimação de parâmetros em dinâmica estrutural: uma introdução a partir de exemplos computacionais* (Vol. 81). São Carlos, SP: SBMAC.
- Cho, J. R., Lee, H. W., Jeong, W. B., Jeong, K. M., & Kim, K. W. (2013). Numerical estimation of rolling resistance and temperature distribution of 3-D periodic patterned tire. *International Journal of Solids and Structures*, 50(1), 86-96. DOI: <https://doi.org/10.1016/j.ijsolstr.2012.09.004>
- Cursi, E. S., & Sampaio, R. (2015). *Uncertainty quantification and stochastic modeling with Matlab*. Great Britain and the United States by ISTE Press Ltd and Elsevier Ltd
- d'Ambrosio, S., & Vitolo, R. (2019). Potential impact of active tire pressure management on fuel consumption reduction in passenger vehicles. *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, 233(4), 961-975. DOI: <https://doi.org/10.1177/0954407018756776>
- Dey, S., Mukhopadhyay, T., & Adhikari, S. (2017). Metamodel based high-fidelity stochastic analysis of composite laminates: A concise review with critical comparative assessment. *Composite Structures*, 171, 227-250. DOI: <https://doi.org/10.1016/j.compstruct.2017.01.061>
- Ejsmont, J., & Owczarzak, W. (2019). Engineering method of tire rolling resistance evaluation. *Measurement*, 145, 144-149. DOI: <https://doi.org/10.1016/j.measurement.2019.05.071>
- Ejsmont, J., Taryma, S., Ronowski, G., & Swieczko-Zurek, B. (2016). Influence of load and inflation pressure on the tyre rolling resistance. *International Journal of Automotive Technology*, 17(2), 237-244. DOI: <https://doi.org/10.1007/s12239-016-0023-z>
- Ejsmont, J., Taryma, S., Ronowski, G., & Swieczko-Zurek, B. (2018). Influence of temperature on the tyre rolling resistance. *International Journal of Automotive Technology*, 19(1), 45-54. DOI: <https://doi.org/10.1007/s12239-018-0005-4>
- Farroni, F., Sakhnevych, A., & Timpone, F. (2016). Physical modelling of tire wear for the analysis of the influence of thermal and frictional effects on vehicle performance. *Proceedings of the Institution of Mechanical Engineers, Part L: Journal of Materials: Design and Applications*, 231(1-2), 151-161. DOI: <https://doi.org/10.1177/1464420716666107>



- Farroni, F., Sakhnevych, A., & Timpone, F. (2018). A three-dimensional multibody tire model for research comfort and handling analysis as a structural framework for a multi-physical integrated system. *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, 233(1), 136-146. DOI: <https://doi.org/10.1177/0954407018799006>
- Fathollahi-Fard, A. M., Dulebenets, M. A., Hajiaghahi-Keshteli, M., Tavakkoli-Moghaddam, R., Safaeian, M., & Mirzahosseini, H. (2021). Two hybrid meta-heuristic algorithms for a dual-channel closed-loop supply chain network design problem in the tire industry under uncertainty. *Advanced Engineering Informatics*, 50, 101418. DOI: <https://doi.org/https://doi.org/10.1016/j.aei.2021.101418>
- Freitas, H. F. S., Olivo, J. E., & Andrade, C. M. G. (2017). Optimization of bioethanol in silico production process in a fed-batch bioreactor using non-linear model predictive control and evolutionary computation techniques. *Energies*, 10(11), 1763. DOI: <https://doi.org/10.3390/en10111763>
- Gillespie, T. D. (1992). *Fundamentals of vehicle dynamics*. Warrendale, USA: SAE International.
- Gonzales, P. E. M., Peloso Jr., M. A. S., Olivo, J. E., & Andrade, C. M. G. (2020). Fed-batch sucrose crystallization model for the b massecuite vacuum pan, solution by deterministic and heuristic methods. *Processes*, 8(9), 1145. DOI: <https://doi.org/10.3390/pr8091145>
- Hoefer, C., & Kropp, W. (2015). A model for investigating the influence of road surface texture and tyre tread pattern on rolling resistance. *Journal of Sound and Vibration*, 351, 161-176. DOI: <https://doi.org/10.1016/j.jsv.2015.04.009>
- Hong, S., Erdogan, G., Hedrick, K., & Borrelli, F. (2013). Tyre-road friction coefficient estimation based on tyre sensors and lateral tyre deflection: modelling, simulations and experiments. *Vehicle System Dynamics*, 51(5), 627-647. DOI: <https://doi.org/10.1080/00423114.2012.758859>
- Kabe, K., Rachi, K., Takahashi, N., & Kaga, Y. (2005). Tire design methodology based on safety factor to satisfy tire life (simulation approach to truck and bus tire design). *Tire Science and Technology*, 33(4), 195-209. DOI: <https://doi.org/10.2346/1.2174343>
- Kapur, J. N. (1989). *Maximum entropy models in science and engineering*. New Delhi, IN: Wiley-Blackwell.
- Khodak, M., Balcan, M.-F., & Talwalkar, A. (2019). Adaptive gradient-based meta-learning methods. In *33rd International Conference on Neural Information Processing Systems* (p. 5917-5928). Vancouver, Canada.
- Li, F., Liu, F., Liu, J., Gao, Y., Lu, Y., Chen, J., ... Zhang, L. (2018). Thermo-mechanical coupling analysis of transient temperature and rolling resistance for solid rubber tire: Numerical simulation and experimental verification. *Composites Science and Technology*, 167, 404-410. DOI: <https://doi.org/10.1016/j.compscitech.2018.08.034>
- Liu, Z., Wang, F., Cai, Z., Wei, Y., & Marburg, S. (2023). Stochastic analysis for in-plane dynamic responses of low-speed uniformity of tires due to geometric defects. *Mechanical Systems and Signal Processing*, 197, 110377. DOI: <https://doi.org/https://doi.org/10.1016/j.ymssp.2023.110377>
- Mendes, A. S., Fleury, A. T., Ackermann, M., Leonardi, F., & Bortolussi, R. (2019). Assessing the influence of the road-tire friction coefficient on the yaw and roll stability of articulated vehicles. *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, 233(12), 2987-2999. DOI: <https://doi.org/10.1177/0954407018812957>
- Orkoulas, G. (2009). Computer simulations: an overview of monte carlo methods for fluid simulation. *Computing in Science & Engineering*, 11(5), 76-87. DOI: <https://doi.org/10.1109/mcse.2009.135>
- Pacejka, H. B. (2012). Tire characteristics and vehicle handling and stability. In H. B. Pacejka (Ed.), *Tire and vehicle dynamics* (3rd ed., p. 1-58). (DOI:10.1016/B978-0-08-097016-5.00001-2). Elsevier.
- Piovan, M. T., & Sampaio, R. (2015). Parametric and non-parametric probabilistic approaches in the mechanics of thin-walled composite curved beams. *Thin-Walled Structures*, 90, 95-106. DOI: <https://doi.org/10.1016/j.tws.2014.12.018>
- Qi, Z., Taheri, S., Wang, B., & Yu, H. (2015). Estimation of the tyre-road maximum friction coefficient and slip slope based on a novel tyre model. *Vehicle System Dynamics*, 53(4), 506-525. DOI: <https://doi.org/10.1080/00423114.2014.1002795>
- Rao, K. V. N., Kumar, R. K., & Bohara, P. C. (2006). A sensitivity analysis of design attributes and operating conditions on tyre operating temperatures and rolling resistance using finite element analysis. *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, 220(5), 501-517. DOI: <https://doi.org/10.1243/09544070JAUTO170>

- Reimpell, J., Stoll, H., & Betzler, J. W. (2001). *Automotive chassis: engineering principles* (2nd ed.). Butterworth-Heinemann
- Sandberg, T., Ramdén, C., & Gamberg, M. (2004). Tire temperature measurements for validation of a new rolling resistance model. *IFAC Advances in Automotive Control*, 37(22), 589-594. DOI: [https://doi.org/10.1016/s1474-6670\(17\)30407-x](https://doi.org/10.1016/s1474-6670(17)30407-x)
- Savitski, D., Hoeppeing, K., Ivanov, V., & Augsburg, K. (2015). Influence of the tire inflation pressure variation on braking efficiency and driving comfort of full electric vehicle with continuous anti-lock braking system. *SAE International Journal of Passenger Cars - Mechanical Systems*, 8(2), 460-467. DOI: <https://doi.org/10.4271/2015-01-0643>
- Scinocca, F., & Nabarrete, A. (2020). Parametric stochastic analysis of a piezoelectric vibration absorber applied to automotive body structure. *Journal of Vibration Engineering & Technologies*, 8, 199-213. DOI: <https://doi.org/10.1007/s42417-019-00099-z>
- Shannon, C. E. (1948). A mathematical theory of communication. *The Bell System Technical Journal*, 27(3), 379-423. DOI: <https://doi.org/10.1002/j.1538-7305.1948.tb01338.x>
- Sina, N., Yazdi, M. R. H., & Esfahanian, V. (2019). A novel method to improve vehicle energy efficiency: Minimization of tire power loss. *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, 234(4), 1153-1166. DOI: <https://doi.org/10.1177/0954407019861241>
- Springer M. D. (1979). *The algebra of random variables: wiley series in probability and mathematical statistics.*, MI: John Wiley & Sons Inc.
- Strigel, A., Peckelsen, U., Unrau, H.-J., & Gauterin, F. (2019). Estimation of feasible ranges of functional tire characteristics based on tire dimension, inflation pressure, and wheel load. *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, 233(14), 3700-3706. DOI: <https://doi.org/10.1177/0954407019831575>
- Stutts, D. S., & Soedel, W. (1992). A simplified dynamic model of the effect of internal damping on the rolling resistance in pneumatic tires. *Journal of Sound and Vibration*, 155(1), 153-164. DOI: [https://doi.org/10.1016/0022-460x\(92\)90652-e](https://doi.org/10.1016/0022-460x(92)90652-e)
- Taghavifar, H., & Mardani, A. (2013). Investigating the effect of velocity, inflation pressure, and vertical load on rolling resistance of a radial ply tire. *Journal of Terramechanics*, 50(2), 99-106. DOI: <https://doi.org/10.1016/j.jterra.2013.01.005>
- Teixeira, R., Nogal, M., & O'Connor, A. (2021). Adaptive approaches in metamodel-based reliability analysis: a review. *Structural Safety*, 89, 102019. DOI: <https://doi.org/10.1016/j.strusafe.2020.102019>
- Wang, G. G., & Shan, S. (2007). Review of metamodeling techniques in support of engineering design optimization. *Journal of Mechanical Design*, 129(4), 370-380. DOI: <https://doi.org/10.1115/1.2429697>
- Wong, J. Y. (2001). *Theory of ground vehicles* (3rd ed.). Ottawa, CA: John Wiley & Sons Inc.
- Yokota, K., Higuchi, E., & Kitagawa, M. (2012). Estimation of tire temperature distribution and rolling resistance under running conditions including environmental factors. *SAE Technical Paper*. DOI: <https://doi.org/10.4271/2012-01-0796>
- Zhai, X., Chen, Y., Han, D., Zheng, J., Wu, X., Wang, Z., ... Zhang, L. (2021). New designed coupling agents for silica used in green tires with low VOCs and low rolling resistance. *Applied Surface Science*, 558, 149819. DOI: <https://doi.org/10.1016/j.apsusc.2021.149819>