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MATHEMATICS

Difference Gai Sequences of Interval numbers with respect to Orlicz Function

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ABSTRACT. In this article we have introduced some classes generalized difference (Δ_m) Gai sequences of interval numbers with Orlicz functions *M*. We have studied some algebraic and topological properties of the classes of sequences like, linearity, completeness, solid, symmetric and convergence free.

Keywords: Solidness; monotone; symmetric; convergence free.

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Introduction

Most of the mathematical structures are constructed with real or complex numbers. Many important mathematical evolutions were took place under those structures. After the introduction of fuzzy number there was a sudden revolution and thereafter interval numbers. The idea of interval arithmetic was first suggested by Dwyer in1951. Thereafter Moore in 1959 and Moore and Yang (1962, 1958), further developed it and used as a computational device. (Chiao, 2002) introduced the sequence of interval numbers and defined usual convergence of sequences with interval numbers. (Şengönül & Eryilmaz, 2010) introduced and studied the concept of bounded and convergent sequence spaces of interval numbers and showed that these spaces are complete metric space. Furthermore, the concept has been applied by some researcher namely (Dutta & Tripathy, 2016; Devnath, Dutta, & Saha, 2014; Esi, 2014a; Esi, 2011; Esi, 2014b; Esi, & Braha, 2013; Esi & Esi, 2013; Kizmaz, 1981; Mikail, Lee, & Tripathy, 2006; Tripathy & Mahanta, 2007; Tripathy & Borgogain, 2011; Tripathy & Dutta, 2012; Tripathy & Dutta, 2014; Tripathy, Braha, & Dutta, 2014), to study different properties of class of sequences. Recently (Baruah & Dutta, 2020) studied on Quasi-Cauchy sequence spaces of interval numbers and established some important results. In recent time interval algorithms are used to solve different problems in numerical analysis, global optimization, and several engineering field.

Material and Methods

An interval number is defined by a closed interval of real numbers x such that $a \le x \le b$.Thus an interval number $\bar{x}$ is a closed subset of real numbers and therefore a real interval can also be considered as a set. We denote the set of all real valued closed intervals by $R(I)$. Any elements of $R(I)$ is called closed interval and denoted by $\bar{x}$, we write it as $\bar{x} = \{x \in R: a \le x \le b\}$. By x_l and x_r we denote first and last points of interval number$\bar{x}$. For $x_1, x_2 \in R(I)$, we define the arithmetic operations on $R(I)$ a follows:

$\bar{x}_1 = \bar{x}_2 \Leftrightarrow x_{1_l} = x_{2_l}$ and $x_{1_r} = x_{2_r}$

$\bar{x}_1 + \bar{x}_2 = \{x \in R : x_{1_l} + x_{2_l} \le x \le x_{1_r} + x_{2_r}\}$

$\alpha\bar{x} = \{x \in R: \alpha x_{1_l} \le x \le \alpha x_{1_r}\}$, if $\alpha \ge 0$;

$\alpha\bar{x} = \{x \in R: \alpha x_{1_r} \le x \le \alpha x_{1_l}\}$ and $\alpha < 0$

$$\bar{x}_1\bar{x}_2 = \left\{ \begin{array}{l} x \in R: min\{x_{1_l}x_{2_l},\ x_{1_l}x_{2_r},\ x_{1_r}x_{2_l},\ x_{1_r}x_{2_r}\} \le x \\ \le max\{x_{1_l}x_{2_l},\ x_{1_l}x_{2_r},\ x_{1_r}x_{2_l},\ x_{1_r}x_{2_r}\} \end{array} \right\}$$

The set of all interval numbers $R(I)$ is a complete metric space defined by

$d(\bar{x}_1, \bar{x}_2) = \max \{|x_{1_l} - x_{2_l}|, |x_{1_r} - x_{2_r}|\}$

In the special case, $\bar{x}_1 = [a, a]$ and $\bar{x}_2 = [b, b]$, we obtain usual metric of R.

Let us consider the transformation $f : N \to R(I)$ by $k \to f(k) = \bar{x}$, where $x = (x_k)$. Then $\bar{x} = (\bar{x}_k)$ is called sequence of interval numbers. The $\bar{x}_k$ is the k^{th} term of sequence $\bar{x} = (\bar{x}_k)$. By w^i, we denotes the set of all interval numbers with real terms.

Definition1. An interval valued sequence space $\bar{E}$ is said to be solid if $\bar{y} = (\bar{y}_k) \in \bar{E}$ whenever $|\bar{y}_k| \leq |\bar{x}_k|$,or all $k \in N$ and $\bar{x} = (\bar{x}_k) \in \bar{E}$.

Definition2. An interval valued sequence space $\bar{E}$ is said to be monotone if $\bar{E}$contains the canonical pre-image of all its step spaces.

Definition3. An interval valued sequence space $\bar{E}$ is said to be convergence free if $\bar{y} = (\bar{y}_k) \in \bar{E}$ whenever $\bar{x} = (\bar{x}_k) \in \bar{E}$ and $\bar{x}_k = 0$ implies $\bar{y}_k = 0$.

Definition4. A sequence $\bar{x} = (\bar{x}_k)$ of interval numbers is said to be convergent to the interval number $\bar{x}_0$ if for each $\varepsilon > 0$ there exists a positive integer k_o such that $d(\bar{x}_k, \bar{x}_o) < \varepsilon$ for all $k \geq k_0$, we write it as $\lim_k \bar{x}_k = \bar{x}_o$. Thus $\lim_k \bar{x}_k = \bar{x}_0 \Leftrightarrow \lim_k x_{k_l} = x_{0_l}$ and $\lim_k x_{k_r} = x_{0_r}$.

Esi (2010) define the following interval valued sequence space:

$$\ell_\infty(p) = \left\{ \bar{x} = (\bar{x}_k) : \sum_{k=1}^{\infty} [d(\bar{x}_k, \bar{0})]^{p_k} < \infty \right\}$$

and if $p_k = 1$ for all $k \in \mathbb{N}$, then we have

$$\ell_\infty = \left\{ \bar{x} = (\bar{x}_k) : \sum_{k=1}^{\infty} [d(\bar{x}_k, \bar{0})] < \infty \right\}$$

An Orlicz function M is continuous, convex, non-decreasing function define for $x > 0$ such that $M(0) = 0$ and $M(x) > 0$ for $x > 0$ and $M(x) \to 0$ as $x \to \infty$.

If convexity of Orlicz function is replaced by $M(x + y) \leq M(x) + M(y)$, then the function is called the modulus function.

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An Orlicz function M is said to satisfy Δ_2 condition for all values u, if there exits $K>0$, such that $M(2u) \leq KM(u), u \geq 0$.

Lindenstrauss and Tzafriri (1971) used the idea of Orlicz function to construct the sequence space

$$\ell_M = \left\{ x_k \in w : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho}\right) < \infty : \text{ for some } \rho > 0 \right\}.$$

The space ℓ_M becomes a Banach space, with the norm

$$\|x\| = \left\{ \inf \rho > 0 : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho}\right) \leq 1 \right\}$$

which is called an Orlicz sequence space.

Let ℓ_∞, c and c_0 be the sequence space of bounded, convergent and null sequences respectively. In respect of ℓ_∞, c and c_0 the norm is given by $\|x\| = \sup_k |x_k|$, where $x = (x_k) \in \ell_\infty \subset c \subset c_0$.

A sequence $x = (x_k)$ is called analytic sequence if $\sup_k |x_k|^{\frac{1}{k}} < \infty$. The vector space of all analytic sequences will be denoted by $Z2$.

A sequence $x = (x_k)$ is called gai sequence if $(k!|x_k|)^{\frac{1}{k}} \to 0$, as $k \to \infty$.The vector space of all gai sequences will be denoted by Z_3.

Subramanian and Mishra (2010) introduced the concept of generalised double difference of Gai Sequence.

Now we introduce the following sequence of interval numbers.

A sequence of interval number $\bar{x} = (\bar{x}_k)$ is called gai sequence if. $\left(k!|\bar{x}_k|\right)^{1/k} \to 0$ as $k \to \infty$.

A sequence $\bar{x} = (\bar{x}_k)$ is said to be an analytic sequence of interval number if $\left(\sup_k |\bar{x}_k|\right)^{1/k} < \infty$.

Kizmaz defined the difference sequence space for crisp set. This concept further generalized by Tripathy and Esi (2006). Baruah and Dutta (2018) introduce the concept in terms of interval number as follows:

Let $m \geq 0$ be an integer then $Z(\Delta_m) = \{(\bar{x}_k) \in w^i : (\Delta_m \bar{x}_k) \in Z\}$, for $Z = \bar{l}_p(\Delta_m),\ \bar{c}(p)(\Delta_m)\ and\ \bar{c_0}(p)(\Delta_m)$. Where $\Delta_m \bar{x}_k = \bar{x}_k - \bar{x}_{k+m}$, for all $k \in \mathbb{N}$.

In this paper we introduce the following class of gai sequence of interval number with generalized difference operator Δ_m in terms of Orlicz functions M.

$$\bar{\chi}(\Delta_m) = \left\{(\bar{x}_k) \in W : M\left(d\left(\left(k!\left|\Delta_m \bar{x}_k\right|\right)^{1/k}, 0\right)\right) \to 0, \text{as } k \to \infty\right\}.$$

Results and Discussion

Theorem 1: The spaces $\bar{\chi}(\Delta_m)$ is complete metric space with the following metric

$$\rho(\bar{x}, \bar{y}) = d(\bar{x}_1, \bar{y}_1) + \sup_k \left[M\left(d\left(\left(\underline{|k}\left(\left|\Delta_m \bar{x}_k\right|, \left|\Delta_m \bar{y}_k\right|\right)\right)^{\frac{1}{k}}\right)\right)\right]$$

Proof : Let $(\bar{x}^i)$ be a Cauchy sequence in $\bar{\chi}(\Delta_m)$ such that

$$(\bar{x}^i) = (\bar{x}^i_k) = (\bar{x}^i_1, \bar{x}^i_2, \bar{x}^i_3, \ldots\ldots) \in \bar{\chi}(\Delta_m), \text{ for each } i \in N.$$

Then for a given ε > 0, there exists $n_0 \in N$.

$$\rho(\bar{x}^i, \bar{x}^j) = d(\bar{x}^i_1, \bar{x}^j_1) + \sup_k \left[M\left(d\left(\left(\underline{|k}\left(\left|\Delta_m \bar{x}^i_k\right|, \left|\Delta_m \bar{x}^j_k\right|\right)\right)^{\frac{1}{k}}\right)\right)\right] < \varepsilon \text{ for all } i, j \geq n_0.$$

Then

$$d(\bar{x}^i_1, \bar{x}^j_1) < \varepsilon, \text{ for all } i, j \geq n_0 \quad \ldots. (1.1)$$

$$M\left[d\left(\left(\underline{|k}\left(\left|\Delta_m \bar{x}^i_k\right|, \left|\Delta_m \bar{x}^j_k\right|\right)\right)^{\frac{1}{k}}\right)\right] < \varepsilon, \text{ for all } i, j \geq n_0 \quad \ldots. (1.2)$$

$$\Rightarrow d\left(\left(\underline{|k}\left(\left|\Delta_m \bar{x}^i_k\right|, \left|\Delta_m \bar{x}^j_k\right|\right)\right)^{\frac{1}{k}}\right) < \varepsilon, \text{ for all } i, j \geq n_0$$

$$\Rightarrow \underline{\lfloor k}\ \ d\left(\left(\left|\Delta_m \bar{x}_k^i\right|,\left|\Delta_m \bar{x}_k^j\right|\right)^{\frac{1}{k}}\right) < \varepsilon, \text{ for all} i, j \geq n_0$$

$$\Rightarrow\ d\left(\left(\left|\Delta_m \bar{x}_k^i\right|,\left|\Delta_m \bar{x}_k^j\right|\right)^{\frac{1}{k}}\right) < \varepsilon, \text{ for all} i, j \geq n_0$$

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$$\Rightarrow\ \left(d\left(\left|\Delta_m \bar{x}_k^i\right|,\left|\Delta_m \bar{x}_k^j\right|\right)\right)^{\frac{1}{k}} < \varepsilon, \text{ for all} i, j \geq n_0$$

$$\Rightarrow\ d\left(\left|\Delta_m \bar{x}_k^i\right|,\left|\Delta_m \bar{x}_k^j\right|\right) < \varepsilon, \text{ for all} i, j \geq n_0$$

$$\Rightarrow\ d\left(\Delta_m \bar{x}_k^i, \Delta_m \bar{x}_k^j\right) < \varepsilon, \text{ for all} i, j \geq n_0$$

Now $\left(\bar{x}_1^i\right)$ and $\left(\Delta_m \bar{x}_k^i\right)$ for all $k \in N$ are Cauchy sequence in ℝ. Since ℝ is complete therefore $\left(\bar{x}_1^i\right)$ and $\left(\Delta_m \bar{x}_k^i\right)$ are convergent in ℝ for all $k \in N$.

Let $\lim_{i\to\infty} \bar{x}_1^i = \bar{x}_1$ . . . . (1.3)

and

$\lim_{i\to\infty} \Delta \bar{x}_1^j = \bar{x}_1$ for all $k \in N$ . . . . (1.4)

From (1.3) and (1.4) we have

$\lim_{i\to\infty} \bar{x}_k^i = \bar{x}_k$ for all $k \in N$.

Now we fix $i \geq n_0$ and taking $j \to \infty$ in (1.1) and (1.2)

We have

$$d\left(\bar{x}_1^i,\ \bar{x}_1\right) < \varepsilon$$

and

$$d\left(\left(\underline{\lfloor k}\left(\left|\Delta_m \bar{x}_k^i\right|,\left|\Delta_m \bar{x}_k\right|\right)\right)^{\frac{1}{k}}\right) < \varepsilon,$$

for all $i \geq n_0$ . . . (1.5)

this imply

$$\rho\left(\bar{x}^i,\ \bar{x}\right) < \varepsilon \text{ for all} i \geq n_0$$

i.e. $\bar{x}^i \to \bar{x}, \quad \text{as } i \to \infty$

Now, we shall show that, $\bar{x} \in \bar{\chi}(\Delta_m)$.

From (1.5) for all $i \geq n_0$

$$d\left(\left(\underline{|k}\left(\left|\Delta_m \bar{x}_k^i\right|, \left|\Delta_m \bar{x}_k\right|\right)\right)^{\frac{1}{k}}\right) < \varepsilon,$$

However, for all $i \in N$,

$$\bar{x}^i = \left(\bar{x}_k^i\right) \in \bar{\chi}(\Delta_m)$$

$$\Rightarrow d\left(\left(\left|\Delta_m \bar{x}_k^i\right|, \bar{0}\right)^{\frac{1}{k}}\right) < \infty,$$

Now for all $i \geq n_0$ we have

$$\Rightarrow d\left(\left(\left|\Delta_m \bar{x}_k\right|, \bar{0}\right)^{\frac{1}{k}}\right) < d\left(\left(\left|\Delta_m \bar{x}_k\right|, \left|\Delta_m \bar{x}_k^i\right|\right)^{\frac{1}{k}}\right) + \left(\left(\left|\Delta_m \bar{x}_k^i\right|, \bar{0}\right)^{\frac{1}{k}}\right) < \infty$$

.

Thus $\bar{x} \in \bar{\chi}(\Delta_m)$.

This the proves of completeness of $\bar{\chi}(\Delta_m)$.

Theorem 2: The sequence space $\bar{\chi}(\Delta_m)$ is closed under the operations, addition and scalar multiplication.

Proof: Let $\bar{x} = (\bar{x}_k) \in \bar{\chi}(\Delta_m)$ and $c \in R$, Then we have

$$M\left[d\left(\left(\underline{|k}\, c\left|\Delta_m \bar{x}_k\right|\right)^{\frac{1}{k}}, \bar{0}\right)\right] \leq \max(1, c^{\frac{1}{k}}) M\left[d\left(\left(\underline{|k}\, \left|\Delta_m \bar{x}_k\right|\right)^{\frac{1}{k}}, \bar{0}\right)\right]$$

$$= \max(1, c^{\frac{1}{k}}).\bar{0}$$
$$= \bar{0}$$

Thus $(c\bar{x}_k) \in \bar{\chi}(\Delta_m)$.

Again we suppose

$\bar{x}_k, \ \bar{y}_k \in \bar{\chi}(\Delta_m)$

Then we have

$$M\left[d\left(\left(\underline{|k}\, \left(\left|\Delta_m \bar{x}_k\right| \oplus \left|\Delta_m \bar{y}_k\right|\right)\right)^{\frac{1}{k}}, \bar{0}\right)\right]$$

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$$\leq K\left[M\left[d\left(\left(\underline{\lfloor k}\left|\Delta_m \bar{x}_k\right|\right)^{\frac{1}{k}},\bar{0}\right)\right]+M\left[d\left(\left(\underline{\lfloor k}\left|\Delta_m \bar{y}_k\right|\right)^{\frac{1}{k}},\bar{0}\right)\right]\right]$$

$$=\bar{0}$$

Thus $\bar{x_k} \oplus \bar{y_k} \in \bar{\chi}(\Delta_m)$

Hence the proof.

Theorem 3: The sequence space $\bar{\chi}(\Delta_m)$ is solid and hence monotone.

Proof: We consider the sequence $\bar{x} = (\bar{x_k}) \in \bar{\chi}(\Delta_m)$ and let $\bar{y} = (\bar{y_k})$ be any sequence such that $|\bar{y_k}| \leq |\bar{x_k}|$ for all $k \in N$. Then by the property of the space

$$M\left[d\left(\left(\underline{\lfloor k}\left|\Delta_m \bar{x}_k\right|\right)^{\frac{1}{k}},\bar{0}\right)\right]\to 0,\ \text{as } k\to\infty$$

Since $|\bar{y_k}| \leq |\bar{x_k}|$, therefore we have

$$M\left[d\left(\left(\underline{\lfloor k}\left|\Delta_m \bar{y}_k\right|\right)^{\frac{1}{k}},\bar{0}\right)\right]\leq M\left[d\left(\left(\underline{\lfloor k}\left|\Delta_m \bar{x}_k\right|\right)^{\frac{1}{k}},\bar{0}\right)\right]\to 0,\ \text{as } k\to\infty$$

Therefore $\bar{y} = (\bar{y_k}) \in \bar{\chi}(\Delta_m)$

This proves that the class of sequence is a solid space and hence monotone.

Theorem 4: The sequence space $\bar{\chi}(\Delta_m)$ is not symmetric.

Proof: The proof follows from the following example.

Example 1 Let $m=2$ and consider the sequence defined by

$$\bar{x}_k=(\bar{1},\ \bar{2},\ \bar{3},\ \bar{4},\ \bar{5},\ \bar{6},\ .\ .\ .\ .\ .)$$

Clearly, $\bar{x_k} \in \bar{\chi}(\Delta_m)$.

Now consider the rearrangement $\bar{y}_k$ of $\bar{x}_k$ defined by

$$\bar{y}_k=(\bar{1},\ \bar{7},\ \bar{3},\ \bar{2},\ \bar{5},\ \bar{4},\ \bar{6},\ .\ .\ .\ .)$$

Then it is clear that $\bar{y_k} \notin \bar{\chi}(\Delta_m)$.

Hence we can conclude that the sequence spaces defined, is not symmetric.

This completes the proof.

Theorem 5: The sequence space $\bar{\chi}(\Delta_m)$ is not convergence free.

Proof: The proof follows from the following example.

Example 2 For each $k \in N$ and$m$=1 let us consider the sequence defined by

$$\bar{x}_k=\left[-\frac{\bar{1}}{k},\frac{\bar{1}}{k}\right]$$

Therefore,

$$\bar{x}_{k+1}=\left[-\frac{\bar{1}}{k+1},\frac{\bar{1}}{k+1}\right]$$

and

$$\Delta \bar{x}_k = \left[-\overline{\frac{1}{k}} - \overline{\frac{1}{k+1}}, \overline{\frac{1}{k}} + \overline{\frac{1}{k+1}} \right]$$

Then it is clear that $\lim_{k\to\infty} \Delta \bar{x}_k = 0$.

Thus $\bar{x_k} \in \bar{\chi}(\Delta_m)$.

Now we consider

$$\bar{y}_k = \left[-\bar{k}, \bar{k} \right] \text{ and } \bar{y}_{k+1} = \left[-(\overline{k+1}), (\overline{k+1}) \right]$$

and therefore we have

$$\Delta \bar{y}_k = \left[-(\overline{2k+1}), (\overline{2k+1}) \right]$$

Thus $\bar{y_k} \notin \bar{\chi}(\Delta_m)$.

Hence the sequence space $\bar{\chi}(\Delta_m)$ is not convergence frees.

This completes the proof.

Conclusion

In this article we have introduced Gai sequence of interval number using the difference operator and studied the class of sequence over certain domain. The properties discussed in this context, shows different algebraic and topological behaviour of the class of sequences which will be helpful to extend the results into double sequence and also in terms of fuzzy real numbers. Besides, by replacing Orlicz function with some other functions, one may obtain some interesting results. Thus there is an ample scope of further study in different directions using the results derived in this article.

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