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Preliminary analysis in the study of conics: the case of the parabola in the context of mathematics undergraduate courses in Ceará

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ABSTRACT. This work aims to analyze different documents that provide information about the curriculum of the discipline of Analytical Geometry in Mathematics Licentiate courses, offered by public Higher Education Institutions in the state of Ceara, observing whether the notion of parabola is contemplated in the study of this discipline and how it is approached with undergraduates. To this end, the methodology adopted when structuring this work consists of the first phase of Didactic Engineering, which is the preliminary analysis. This first analysis was documentary, carried out from the syllabus of the discipline and curricular programs of the degree courses in Mathematics. The documents analyzed were made available by seven public institutions, six of which are Universities and one Federal Institute. In this study, aspects such as objectives of the discipline, form of evaluation, teaching procedures, bibliography used, among other information considered relevant, when available, were observed aiming to understand the gaps during the initial training of mathematics teachers in that subject. The results point to the demand for support materials aimed at an approach to the parabola that links its application both in the context of Analytical Geometry and the functions, as well as for the need to maintain the connection between these topics of study in the path between basic education and higher education.

Keywords: parabolas; analytical geometry; initial teacher training; mathematics teaching.

Análise preliminar no estudo das cônicas: o caso da parábola no contexto dos cursos de licenciatura em matemática no Ceará

RESUMO. Este trabalho tem como objetivo analisar diferentes documentos que forneçam informações sobre o currículo da disciplina de Geometria Analítica em cursos de licenciatura em Matemática, ofertados por Instituições de Ensino Superior públicas no estado do Ceará, observando se a noção de parábola é contemplada no estudo desta disciplina e de que forma ocorre sua abordagem com os graduandos. Para tal, a metodologia adotada ao estruturar este trabalho consiste na primeira fase da Engenharia Didática, que é a análise preliminar. Esta primeira análise foi documental, realizada a partir de ementários da disciplina e de programas curriculares dos cursos de licenciatura em Matemática. Os documentos analisados foram disponibilizados por sete instituições públicas, sendo seis Universidades e um Instituto Federal. Neste estudo foram observados aspectos como objetivos da disciplina, forma de avaliação, procedimentos de ensino, bibliografia utilizada, entre outras informações consideradas relevantes, quando disponíveis, almejando compreender as lacunas durante a formação inicial dos professores de matemática no referido tema. Os resultados apontam para a demanda de materiais de apoio direcionados a uma abordagem da parábola que vincule sua aplicação tanto no contexto da Geometria Analítica quanto das funções, bem como para a necessidade de manter a conexão destes tópicos de estudo no percurso entre a educação básica e o ensino superior.

Palavras-chave: parábolas; geometria analítica; formação inicial de professores; ensino de matemática.

Análisis preliminar en el estudio de las cónicas: el caso de la parábola en el contexto de los cursos de pregrado en matemáticas en Ceará

RESUMEN. Este trabajo tiene como objetivo analizar diferentes documentos que aportan información sobre el currículo de la disciplina Geometría Analítica en los cursos de licenciatura en Matemáticas ofrecidos por las Instituciones de Enseñanza Superior públicas en el estado de Ceará, observando si la noción de la

parábola está contemplado en el estudio de esta disciplina y cómo se produce su acercamiento a los estudiantes universitarios. Para ello, la metodología adoptada al estructurar este trabajo consta de la primera fase de Ingeniería Didáctica, que es el análisis preliminar. Este primer análisis fue documental, realizada a partir de los planes de estudios de la disciplina y programas curriculares de las carreras de grado en Matemáticas. Los documentos analizados fueron puestos a disposición por siete instituciones públicas, seis de las cuales son Universidades y un Instituto Federal. En este estudio, se observaron aspectos como los objetivos de la asignatura, el método de evaluación, los procedimientos de enseñanza, la bibliografía utilizada, entre otras informaciones consideradas relevantes, cuando esté disponible, con el objetivo de comprender los vacíos durante la formación inicial de los profesores de matemáticas en esa asignatura. Los resultados apuntan a la demanda de materiales de apoyo dirigidos a un acercamiento a la parábola que vincule su aplicación tanto en el contexto de la Geometría Analítica y funciones, así como la necesidad de mantener la conexión de estos temas de estudio en el trayecto entre la educación básica y educación superior.

Palabras-clave: parábolas; geometría analítica; formación inicial del profesorado; enseñanza de las matemáticas.

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Introduction

The parabola curve has its origins in the studies of Apollonius of Perga, in his treatise on conics, where its structural and conceptual aspects are examined. However, the research of Descartes and Fermat in the 16th century brought a connection between the parabola and the use of algebraic equations, widely utilized to this day (Bermúdez & Mesa, 2018).

The study of the parabola is pertinent to various fields of knowledge such as Physics, Engineering, and Architecture, with several important applications for society, including parabolic antennas and radars, vehicle headlight structures, suspension bridge construction, solar ovens, projectile launching, among others. In this sense, we understand that its mathematical understanding should be regarded as relevant for many technological advancements.

However, there are difficulties that permeate the teaching of the parabola, such as the little (or no) association between its geometric and algebraic views throughout Basic Education (Machado, 2007; Louzada, 2013; Siqueira, 2016). "In the case of the parabola, if the teacher does not take due care, the student may have the cyclical conception that this curve in the plane is the graph of the quadratic function and that the graph of the quadratic function is a parabola" (Louzada, 2013, p. 10), thus disregarding other forms of visualization and interpretation of this curve.

The first contact between the student and the parabola usually occurs at the end of Elementary School and the beginning of High School, with the study of quadratic functions, culminating at the end of this stage of education with notions of conics in Geometry, when such study occurs (Pereira, 2013). However, there is often a lack of approach by the teacher that establishes the relationship between these different ways of understanding the parabola (Siqueira, 2016; Siqueira & Silva, 2019). In general, a fragmented study is observed, which creates a gap in the understanding of this geometric entity in the trajectory between Basic Education and Higher Education, leading to difficulties in its comprehension and applications in disciplines such as Analytical Geometry, Differential and Integral Calculus, Linear Algebra, among other areas of knowledge.

Furthermore, another point to be observed is the little articulation between the notion of the parabola in the field of Analytical Geometry and teaching methodologies that advocate the use of technological resources (Richit, 2005; Lucena & Gitirana, 2016; Bermúdez & Mesa, 2018; Paiva & Alves, 2019; Sousa, Azevedo, & Alves, 2021), either due to a lack of material resources in schools, such as equipped computer laboratories, or even due to difficulties of teachers in developing lessons outside of traditional molds. As Lucena and Gitirana (2016, p. 25) affirm, the study of the parabola occurs through "[...] teaching that prioritizes algebraic treatment and leaves much to be desired in the geometric interpretation of the mathematical object represented, which hinders a deeper understanding of the concepts in focus".

To understand the difficulties of mathematics teachers in working with this theme, specifically in the field of Analytical Geometry, the objective of this work is to analyze different documents that provide information about the curriculum of the Analytical Geometry discipline in mathematics teaching degree programs offered by public Higher Education Institutions in the state of Ceará, observing whether the notion of the parabola is included in the study of this discipline and how its approach with undergraduates occurs.

The methodology adopted to structure this work was Didactic Engineering, in its first phase, which involves preliminary analyses (Artigue, 2020). In this stage, a documentary study was conducted based on an analysis of the syllabi of the Analytical Geometry course in seven public Higher Education Institutions (HEIs), including six (6) Universities and one (1) Federal Institute in the state of Ceará, offering the Mathematics Teaching degree. The documentary analysis aimed to observe characteristics, when available, such as the objectives of the course, assessment methods, teaching procedures, bibliography used, among other aspects considered relevant for understanding the approach to the parabola at this level of education.

Based on the above, in the following sections, we discuss the study of the parabola in higher education, the methodology of the work with the conducted survey, the results and discussions on the findings of the research, as well as the main considerations of the authors.

Theoretical framework

In order to understand the difficulties of teachers in teaching the notion of the parabola in Analytical Geometry, we sought references that present the format of studying this subject at the higher level, directed towards Mathematics Teaching degrees, as a way to identify didactic obstacles faced by teachers during their initial training journey.

The study of the parabola in higher education

Several studies indicate that, in general, the study of conics at the higher level prioritizes the use of equations and algebraic treatment, to the detriment of its geometric understanding (Oliveira, 2011; Pereira, 2013; Lima, 2015; Macedo, 2015; Siqueira & Silva, 2019). Pereira (2013, p. 12), regarding this study, asserts that:

> In basic education, conics typically only appear in the third year of high school, often being addressed solely with the center at the origin, thus overlooking conics with centers at other points and the possibility of rotated conics. Ellipses and hyperbolas are approached through parameters such as a, b, and c, while parabolas are characterized by the parameter p. In higher education, they are revisited in calculus for constructing surfaces in space, in analytical geometry with a focus on analytic equations, and in linear algebra, where they are linked to vectors and matrices.

In this regard, there is a concern about how the parabola as a conic section is grasped by undergraduate students throughout their initial education, as well as how they deal with this theme in their professional field. There exists a gap in the analytical study of the parabola between the trajectory that begins in Basic Education and continues to the higher education level (Macedo, 2015; Siqueira, 2016; Siqueira & Silva, 2019), which may result from various factors. These factors include the possibility that the teacher has not studied this topic within the field of Analytical Geometry at any educational level, epistemic difficulties stemming from a superficial study of the topic, or even didactic challenges in working with this theme in the classroom.

Lima (2015, p. 193) points out that in Mathematics textbooks, conic sections are addressed solely from the perspective of algebraic equations, while their historical approach, geometric properties, relationships between curves, and their applications are often overlooked or briefly mentioned in supplementary notes with abbreviated reading suggestions. According to this author:

> It is highly likely that this limited and superficial approach to conic sections in textbooks is one of the main reasons for students' lack of appreciation for the topic. This sentiment is shared by the majority of teachers themselves, who often rely solely on the textbook as their source of reference, thus failing to recognize the beauty, history, importance, and utility of the subject.

The gaps present in the basic education of undergraduate students accumulate up to the higher education level, resulting in difficulties in learning this topic during their initial training, if such study occurs at all (Siqueira, 2016). It is common for the parabola to be seen in some Analytical Geometry course syllabi as the last topic to be addressed, and in some cases, its study is not included at all, as pointed out by Siqueira (2016). These gaps and difficulties can have repercussions on their professional performance.

Bermúdez and Mesa (2018) explain that conic sections are usually presented in higher education textbooks as a didactic unit, and their demonstrations assume that undergraduate students have a previous foundation on their mathematical concept. This can be observed in the way the content is introduced in the classroom and in how its definitions are reached, through the generalization of concepts, without conducting a prior investigation that would allow for some judgments or basic geometric elements in the construction and derivation of the equation representing the curve of a conic section.

According to Sousa *et al*. (2021, cited in Bermúdez & Mesa, 2018), on many occasions, the study of the parabola in Analytical Geometry begins in the classroom with the use of algorithms and techniques that advocate for the identification and presentation of concepts through formal definitions, formula demonstrations, and gradual completion of exercises that do not initially provide stimulus for the student's geometric reasoning. Furthermore, according to Sousa *et al*. (2021, cited in Bermúdez & Mesa, 2018, p. 128), "[...] the recurrent use of analogies and inadequate exploration of intuition in knowledge construction, accompanied by various examples and counterexamples until a formal and generalized demonstration is obtained, constitutes the teaching format of the fundamental concepts of this subject/discipline".

Based on this overview, research over the years such as Santos (2004), Louzada (2013), Cerqueira (2015), Bermúdez and Mesa (2018), Vargas and Leivas (2019), Sousa, Alves, Aires, and Catarino (2023), Sousa, Alves, and Aires (2023), and Sousa, Alves, and Campos (2024) bring common aspects regarding the difficulties in teaching the parabola and understanding the relationship between conics and functions in a general panorama. These studies point out the shortcomings in the trajectory of the initial education of mathematics teachers, linked to the lack of appropriate materials on the subject for their work in Basic Education, recognizing the need for study and mathematical deepening of this theme by the teacher. Such didactic and epistemic obstacles directly reflect on the performance of students, and these difficulties have been gradually reported.

With that said, we can understand that many teachers, who possibly received their basic education in these patterns, bring with them didactic obstacles when working with this theme in the classroom, generating a cycle that has persisted and caused limitations in the evolution of teaching this subject. However, there are ways to overcome or at least mitigate such difficulties, such as the integration of technologies in working with this topic.

Methodology

The methodology adopted in this study was Didactic Engineering in its initial phase - preliminary analysis (Artigue, 2020). Regarding the preliminary analysis of Didactic Engineering, Artigue (2020, p. 33) explains that it "[...] generally includes three dimensions: epistemological analysis of the mathematical content at stake, an analysis of institutional conditions and constraints, an analysis of what didactic research offers to support the design".

In the case of this study, we sought to construct an epistemological analysis of the topic, as well as an examination of some conditions and/or restrictions imposed on the teaching of the topic within the Mathematics Teaching degree program, aiming to understand how teacher training has been occurring regarding the addressed theme.

As a way to construct this analysis, we conducted a documentary study. This methodological approach, according to Lima Junior, Oliveira, Santos, and Schnekenberg (2021, p. 37), can be adopted "[...] both as a qualitative and quantitative method, and is concerned with seeking concrete information in the various documents selected as the research corpus". Furthermore, according to Gil (2002, p. 46), documentary research has advantages, among them being the fact that "[...] firstly, it should be considered that documents constitute a rich and stable source of data. As documents persist over time, they become the most important source of data in any historical research".

In the meantime, we sought reliable information that would allow us to achieve the objective of this study, through an analytical treatment given to original documents recognized as public domain, while respecting the assumptions of Didactic Engineering and its structure.

For the purposes of this research, we delimited the number of institutions offering Mathematics Teaching degree programs in the state of Ceará to seven, comprising six Universities and one Federal Institute, namely:

- State University of Ceará (UECE)
- Vale do Acaraú State University (UVA)
- Federal University of Ceará (UFC)
- University of International Integration of Afro-Brazilian Lusophony (Unilab)
- Regional University of Cariri (URCA)
- Federal University of Cariri (UFCA)
- Federal Institute of Education, Science and Technology of Ceará (IFCE), Fortaleza *campus*.

In the specific case of this work, the documents analyzed pertain to syllabi, curriculum matrices, and course outlines of the Analytical Geometry (or Vectorial Analytical Geometry) discipline, as well as, in some cases, the course syllabus and the Pedagogical Political Project of the course itself, when made available by the institution. Based on the files and documents related to these institutions, we seek to identify which of them offer the Analytical Geometry (Vectorial) discipline and if the syllabus/course outline of the discipline includes the study of the parabola.

The documents analyzed were obtained through websites or direct contact with the coordination of such institutions, which are listed in Table 1:

Table 1. Institutions, analyzed documents, and electronic addresses (when available).

Institution	Analyzed document	Obtained from:
State University of Ceará (UECE)	Program of the Vectorial Analytical Geometry discipline	Obtained through direct contact with the course coordinator
Vale do Acaraú State University (UVA)	Curriculum 2018	http://www.matematicauva.org/matriz-curricular/
Federal University of Ceará (UFC)	Course plan of the discipline	Obtained through direct contact with the course coordinator
University of International Integration of Afro-Brazilian Lusophony (Unilab)	Pedagogical Political Project of the course	https://unilab.edu.br/matematica-licenciatura/
Regional University of Cariri (URCA)	Curriculum Matrix for the Degree in Mathematics	http://www.urca.br/portal2/matematica/
Federal University of Cariri (UFCA)	Pedagogical Course Project – Degree in Mathematics	Obtained through direct contact with the course coordinator
Federal Institute of Education, Science and Technology of Ceará (IFCE), Fortaleza *campus*	Curricular matrix – degree in Mathematics	https://ifce.edu.br/fortaleza/cursos/superiores/licenciatura/matematica/pdf/matriz-curricular-matematica.pdf/view

Source: Elaborated by the authors (2022).

The documents presented in Table 1 were analyzed through subjective reading and comprehension, aiming to understand how the notion of the parabola is treated in Mathematics Teaching degree programs of the mentioned institutions. As Gil (2002, p. 47) points out, it is worth noting that "[...] some research based on documents is important not because they definitively answer a problem, but because they provide a better insight into that problem or hypotheses that lead to its verification by other means".

Building upon this, we analyzed information about the discipline (when offered), such as its workload, objectives, assessment methods, teaching procedures, bibliography used, among other relevant aspects present in the analyzed documents. We highlighted our main observations about the topic of parabolas and their study. In the next section, we present some remarks about the findings of this research.

Analysis and Discussion of the Results

In this section, we present the main aspects and an analysis of the collected documents, providing a characterization of the Analytical Geometry course syllabi in the previously mentioned institutions, with a focus on the parabola theme, which is the core of this work.

State University of Ceará (UECE)

Regarding parabolas, they are studied in the Mathematics Teaching degree program at the State University of Ceará (UECE) in the Vectorial Analytical Geometry course, which is taken in the third semester, with a total workload of 68 class hours. The analyzed document was the syllabus of the course, obtained through direct contact with the course coordinator, as it is not a publicly available document on the institution's website. However, we found Opinion No. 0024/2009 (Ceará, 2009), which deals with the regulation of the syllabus of the Mathematics Teaching degree program at the institution, and we used it in conjunction with the document provided by the course coordinator.

According to the syllabus, the Mathematics Teaching degree program at UECE covers the following main topics in the Vectorial Analytical Geometry course: Operations with vectors in $\mathbb{R}^2$ and $\mathbb{R}^3$, Lines and planes, Distances, Conics and Quadrics, General equation of the second degree with two and three variables, and

Classification of conics and quadrics (Ceará, 2019). In this context, students study the notion of the parabola within the theme of conics.

One of the objectives indicated in the course syllabus is to understand the canonical equations of conics and quadrics. In Figure 1, we present the topic of conics and its respective division:

Cônicas:
3.1. Definições como lugar geométrico;
3.2. Mudança de coordenadas em R2: troca de eixos, translação e rotação.
3.3. Equação geral de segundo grau a duas variáveis.
3.4. Classificação via discriminante.

Figure 1. Parabolas in the Mathematics Teaching degree program / UECE. Source: Excerpt from the syllabus of the discipline (2012).

From the information presented in Figure 1, we can see that the parabola, developed within the content of conics, has an analytical focus and development based on algebraic procedures. This is reinforced by the bibliography used for the development of the discipline, as additionally indicated in Figure 2:

Livro-texto:
1. BOULOS, Paulo; CAMARGO, Ivan de. **Geometria Analítica** – Um Tratamento Vetorial. 3.ed. São Paulo: Pearson Education do Brasil Ltda., 2005.

Básica:
1. AZEVEDO FILHO, Manoel Ferreira de. **Geometria Analítica e Álgebra Linear**. Fortaleza: Edições Livro Técnico, 2003.
2. STEINBRUCH, Alfredo; WINTERLE, Paulo. **Geometria Analítica**. São Paulo: Makron Books do Brasil Editora LTDA., 1987.

Complementar:
1. EFIMOV, N. **Elementos de Geometria Analítica.** Belo Horizonte: Livraria Cultura Brasileira Editora Ltda., 1972.
2. KLÉTÉNIC. **Problemas de Geometria Analítica**. Belo Horizonte: Livraria Cultura Brasileira Editora Ltda., 1977.
3. LEHMANN, Charles H. **Geometria Analítica.** Porto Alegre: Editora Globo S.A., 1982.
4. LIMA, Elon Lages. **Coordenadas no Plano.** Rio de Janeiro: SBM, 1992.
5. LIMA, Elon Lages. **Coordenadas no Espaço.** Rio de Janeiro: SBM, 1992.

Figure 2. Bibliography used in the Vectorial Analytical Geometry discipline (UECE). Source: Excerpt from the syllabus of the discipline (2021).

Figure 2 shows us a list of references that approach conics from an analytical perspective, generally starting from their formal demonstrations. Depending on the teacher's methodology, there may be a focus on the geometric analysis of the parabola, exploring its characteristics and construction possibilities, or not.

Vale do Acaraú State University (UVA)

The Mathematics Teaching degree program at Vale do Acaraú State University (UVA) explicitly covers the content of parabolas within the Vectorial Analytical Geometry course, to be taken in the third period, with a total workload of 80 class hours. The syllabus of the course, built from Resolution No. 22/2017 - CEPE (Ceará, 2017), was obtained from the website www.matematicauva.org, under the Curriculum Matrix section, being a publicly accessible document.

According to its syllabus, the course aims to address some topics from Basic Education in order to prepare students for teaching in this educational modality and for subsequent disciplines in the course. Its main themes include the study of vectors, plane and line equations, angles and distances, coordinate changes, and conics (Mathematics UVA, 2018). In Figure 3, we have an excerpt from the syllabus of the discipline indicating the study of the parabola:

In Figure 3, when observing the syllabus of the Vectorial Analytical Geometry discipline, we see that the parabola appears as the last topic to be studied in this course. Furthermore, according to the document, teaching procedures consist of lectures, exercise resolution, and periodic written assessments (Mathematics UVA, 2018), emphasizing that there are no practical activities for this discipline, nor does it mention the use

of technology for pedagogical and didactic purposes, even if employed by the teacher. The bibliography used in the discipline is presented in Figure 4:

5. Cônicas
- 5.1. Cone Circular reto;
- 5.2. Circunferência;
- 5.3. Elipse;
- 5.4. Hipérbole
- 5.5. Parábola

Figure 3. Parabolas in the Mathematics Teaching degree program/UVA. Source: Syllabus of the discipline (Mathematics UVA, 2018).

Básica
[1] BOULOS, P.; CAMARGO, I. Geometria Analítica: Um tratamento vetorial. 3ª ed Prentice Hall. São Paulo, 2005.
[2] LIMA, E. L.; CARVALHO, P.C.P.; WAGNER, E.; MORGADO, A. C. A matemática no Ensino Médio, vol. 3, 6ª ed. SBM. Rio de Janeiro, 2001.
[3] FILHO, M. F. A. Geometria Analítica e Álgebra Linear. 2ª ed Edições Livro Técnico. Fortaleza, 2003.

Complementar
[4] LIMA, E. L. Coordenadas no espaço. 4ª ed SBM. Rio de Janeiro, 1998.
[5] LIMA, E. L. Geometria Analítica e Álgebra. IMPA. Rio de Janeiro, 2008.
[6] WINTERLE, P. Vetores e Geometria Analítica. 2ª. Edição. Makron Books, São Paulo, 2000.

Figure 4. Suggested bibliography for the course/UVA. Source: Excerpt from the syllabus of the discipline (Mathematics UVA, 2018).

The works mentioned in Figure 4 indicate a formal approach to the subject, with algebraic aspects of Analytical Geometry, but they can be worked on using technology, depending on the teacher's planning. An important observation concerns the basic work number 2, by Lima, Carvalho, Wagner, and Morgado (2001), 'Mathematics in high school', which, if approached to relate Analytical Geometry seen in Basic Education with what is studied in Higher Education, covering the relationship between the parabola, its function, equation, and types of graphs, can provide students with different knowledge and possibilities when working on this theme in their professional practice.

Federal University of Ceará (UFC)

The Mathematics Teaching degree program at the Federal University of Ceará (UFC) clearly includes the content of parabolas in its syllabus, within the Analytical Geometry course, to be taken in the third period, with a workload of 96 class hours. The analyzed document was the course syllabus obtained through direct contact with the course coordinator, as it is not publicly available.

According to the course syllabus of Analytical Geometry, its objectives are to develop intuitive notions of spaces and geometric shapes, understand the concept of vectors, study geometry using the notion of vectors, and solve problems involving equations and their respective geometric places (Brazil, 2014). In Unit IV of the document, there is a section called "Content Description" (Brazil, 2014, p. 1), which is directed towards the study of conics: circle, parabola, ellipse, and hyperbola. There is no detailed breakdown showing which specific subtopics are actually developed in the study of the parabola.

Another relevant information is found in Unit V of the document, related to the teaching methodology and evaluation, which states that the procedures for the course are "Lectures. Comprehension exercises. Classroom exams" (Brazil, 2014, p. 1), which constitutes a traditional teaching model.

In Figure 5, we have the recommended textbooks for the study of this discipline:

[1] AZEVEDO FILHO, M.F. de. Geometria analítica e álgebra linear. Ed. Livro Técnico.
[2] STEINBRUCH, A. e WINTERLE, P. Álgebra linear. Markon Books.
[3] BOULOS, P. e CAMARGO, I. Geometria analítica. McGraw-Hill

Figure 5. Suggested bibliography for the course/UFC. Source: Excerpt from the course syllabus (Brazil, 2014).

Some works mentioned in Figure 5 are common to the previously mentioned institutions and direct the study of Analytical Geometry towards a formal approach, as indicated by the teaching and evaluation methodology of lectures, comprehension exercises, and written assessments in the classroom. However, it is reiterated that this subject can be explored using technology, among other teaching methodologies, depending on the planning of the teacher.

University of International Integration of Afro-Brazilian Lusophony (Unilab)

The bachelor's degree in mathematics at the University of International Integration of Afro-Brazilian Lusophony (Unilab) includes Analytical Geometry in its curriculum in the second semester, with a total workload of 90 class hours, divided into 75 theoretical hours and 15 practical hours, with no prerequisite courses required. The analyzed document was the Pedagogical Political Project (Ceará, 2020) of the course, obtained from the institution's website, being publicly accessible.

So far, this was the only institution that included a practical component in the workload of the course. However, the document examined does not mention the format of these classes, whether they take place in a laboratory or in a traditional classroom, or even if they involve the use of technology, among other possibilities.

In the section regarding mandatory courses, there is a list of topics covered in Analytical Geometry (Ceará, 2020, p. 84):

(i) Cartesian plane: distance between two points, midpoint of a segment, equation of a line, distance between a point and a line, relative positions between lines, and equation of the circle.

(ii) Conics: parabola, ellipse, and hyperbola. Coordinate system in space: distance between points, midpoint, alignment condition of three points.

(iii) Vector study: vector addition, scalar multiplication, dot product, cross product, mixed product, orthogonal projection, and angle between vectors.

(iv) Line and plane in space: relative positions of lines and planes, angles, and distances.

(v) Quadric surfaces: ellipsoid, hyperboloids of one and two sheets, elliptic and hyperbolic paraboloids, cone, and right cylinder.

In this list of topics to be studied, we can observe the study of the notion of a parabola among the first subjects, which is a characteristic that can differentiate the approach to the theme in this course compared to other syllabi and curricular matrices. Moreover, it is worth noting that this list of study topics is broader than those presented previously. In Figure 6, we have the bibliography used/recommended for the study of the discipline:

Bibliografia Básica:

BOULOS, P.; CAMARGO, I. Geometria Analítica: Um Tratamento Vetorial, 3ª Edição. São Paulo: McGraw-Hill Ltda, 2005.
IEZZI, G. Fundamentos de Matemática Elementar – Volume 7: Geometria Analítica, 6ª Edição. São Paulo: Atual, 2013.
WINTERLE, P. Vetores e Geometria analítica. São Paulo: Makron Books, 2006.

Bibliografia Complementar:
CARVALHO, P. C. et al. A Matemática do Ensino Médio – Volume 3, 6ª Edição. Rio de Janeiro: SBM, 2001.
LIMA, E. L. Coordenadas no Plano. Rio de Janeiro: SBM, 1996.
LIMA, E. L. Coordenadas no Espaço. Rio de Janeiro: SBM, 1998.
MUNIZ NETO, A. C. Tópicos de Matemática Elementar – Volume 1: Geometria Plana, 2ª Edição. Rio de Janeiro: SBM, 2012.
STEINBRUCH, A. Geometria analítica, 2ª Edição. São Paulo: McGraw-Hill, 1987.

Figure 6. Recommended bibliography for the discipline/Unilab. Source: Excerpt from the Pedagogical Political Project of the course (Ceará, 2020).

The recommended books for the discipline are similar to those presented earlier, also approaching Analytical Geometry from a formal perspective, which emphasizes algebraic treatment. However, in this same list of works, there are books such as 'A Matemática no Ensino Médio' by Lima *et al.* (2001) and 'Fundamentos da Matemática Elementar' by Iezzi (2013) that can bridge the gap between the study of the parabola in high school and higher education, providing undergraduate students with different perspectives and a broader understanding of the topic.

These works and study topics are part of the seventh revision of the Curricular Program of the Course, which took place in June 2019. According to the document surveyed, the assessment method for each discipline is at the discretion of the teacher, who must present the criteria for assessing learning to the students in advance.

The mathematics degree program at Unilab has a structure that includes computer labs accessible to students for class activities and/or research, digital whiteboards, and projectors, which can facilitate the teaching of the subject with methods different from traditional teaching.

Regional University of Cariri (URCA)

The curriculum of the mathematics teaching degree program at the Regional University of Cariri (Urca) includes the Analytical Geometry course in the second semester, with a workload of 90 class hours and without any prerequisite courses required. The document analyzed was the course curriculum, obtained from the institution's website, which is publicly accessible.

However, we were unable to access the syllabus or any document containing information about the teaching plan, objectives, and teaching format of the instructor, only the Curriculum Framework (Ceará, 2004). We contacted the course coordination via email, but the document provided was the same as already obtained from the website, with insufficient information about the discipline, which made it impossible to analyze the teaching of the topic of parabolas within the said discipline.

Federal University of Cariri (UFCA)

The undergraduate Mathematics degree at the Federal University of Cariri (UFCA) offers the Vector Analytical Geometry course to be taken in the fourth semester, with a workload of 64 class hours. The analyzed document was the Course Pedagogical Project (Brazil, 2019), obtained through direct contact with the course coordination, as it is not freely accessible to the public. The analyzed document does not mention the evaluative methods of the course and/or information about the format of the classes.

According to the document provided by the course coordination, the objectives of the course are to present the concept of coordinates in space, discuss vectors in the plane and in space and their applications, define equations of lines and curves in $\mathbb{R}^2$ and apply techniques of coordinate changes (Brazil, 2019). For its study, the planned program content is:

(i) Coordinates in space.
(ii) Vectors in the plane and in space and applications.
(iii) Equations of lines and planes in space.
(iv) Relative positions of lines and planes.
(v) Curves in $\mathbb{R}^2$ and $\mathbb{R}^3$;
(vi) Coordinate changes;
(vii) Conics;
(viii) Quadrics.

In this case, we can observe that the study of conics occurs towards the end of the course, but there is no specificity about the subtopics within the conics theme. Therefore, we presume that the parabola may be studied, but we do not have enough information to describe how its study takes place.

The suggested bibliography for the study of the discipline is listed in Figure 7:

Bibliografia Básica
BOULOS, P.; CAMARGO, I. Geometria Analítica: um tratamento vetorial. 3. ed. São Paulo: Pearson, 2004.
MACHADO, A. dos S. Álgebra linear e Geometria analítica. 2ª edição. São Paulo. Atual, 1982. WINTERLE, P. Vetores e Geometria Analítica. São Paulo: Makron Books, 2000.

Bibliografia Complementar
CORRÊA, P. S. Q. Álgebra Linear e Geometria Analítica. Rio de Janeiro: Interciência, 2006.
LIMA, E. L. Geometria Analítica e Álgebra Linear. 2. ed. Rio de Janeiro: IMPA, 2015.
SANTOS, N. T. Vetores e Matrizes: uma introdução a álgebra linear. 4. ed. São Paulo: Cengage, 2007.
STEINBRUCH, A.; WINTERLE, P. Geometria Analítica. São Paulo: Pearson, 1995.
VALLADARES, R. J. C. Geometria analítica do Plano e do Espaço. 1. ed. Rio de Janeiro: LTC, 1990.

Figure 7. Recommended bibliography for the discipline/UFCA. Source: Excerpt from the Pedagogical Project of the Course (Brazil, 2019).

Among the works mentioned in Figure 7, the basic bibliography also coincides with other course syllabi analyzed in this study, with a similar approach.

Federal Institute of Education, Science and Technology of Ceará (IFCE)

In the case of the Mathematics Teaching degree program at the Federal Institute of Education, Science, and Technology of Ceará (IFCE) - Fortaleza campus, there isn't a specific course dedicated to the study of Analytical Geometry. We analyzed the Curriculum Framework of IFCE, and its last update occurred in 2012. The course that comes closest to the study of Analytical Geometry is the Linear Algebra course, taken in the third semester with a workload of 80 class hours (Brazil, 2012). However, there is no mention of the study of conics in this course, meaning the curriculum does not include the study of parabola.

Discussion

Given the above, we can observe that the format of classes and evaluation methods - when identified and mentioned in the documents -, the relationship between workload and the number of topics to be studied in the discipline, the recommended literature for learning, and the consequent study of conics, possibly present the parabola as quadratic equations in two variables, from an algebraic perspective. This has created a gap in education for a long time, as the mere algebraic treatment of the subject fails to remain vivid in students' memories.

Among the information obtained about the discipline in the seven institutions, we observe that the State University of Ceará (UECE) provides more detailed information about working with the topic of parabolas. However, the discipline has the shortest workload among all the observed institutions, which may make the study of this topic concise.

Regarding the adopted/suggested textbooks for their study, there is compatibility among the institutions, with many using corresponding works such as Boulos and Camargo (2005), Steinbruch and Winterle (1987; 1995), and Winterle (2000; 2006), with minor variations such as the edition of the book, either as basic or complementary bibliography of the discipline, with very similar lines of thought.

However, the list of works from the University of International Integration of Afro-Brazilian Lusophony (Unilab) includes suggestions that caught our attention, such as the works ‘Fundamentos de Matemática Elementar, volume 7’ (Iezzi, 2013) and ‘A Matemática do Ensino Médio, volume 3’ (Lima *et al*., 2001), with the latter also listed as bibliography for study by the State University of Vale do Acaraú (UVA). These works have great potential to establish a connection between Analytical Geometry in Basic Education and Higher Education, serving as support for the undergraduate student and future teacher in teaching parabolas, bringing a new perspective to the work with the subject, if this connection is reinforced during the work of the teacher responsible for the discipline during initial training.

Absolutely, it's essential to consider conducting further research involving participants from various Mathematics Education programs so they can share their experiences and expand this study. This would allow us to better understand their main difficulties in comprehending the topic.

Final Considerations

The conic parabola is present in students' daily lives and is part of other areas of knowledge beyond Mathematics; however, its study still presents weaknesses. Based on the collected data, we can infer that the structural templates of the curriculum matrices of the aforementioned teaching degree programs present teaching models focused on formal approaches, prioritizing Algebra over Geometry—despite the name of the discipline carrying the word “Geometry”, its focus is algebraic.

Based on the study conducted, we understand that the format of classes and the adopted textbooks for the study of Analytical Geometry (or Vectorial Analytical Geometry) in the mentioned institutions approach the subject from an algebraic perspective, with the necessary mathematical rigor. However, the study also shows us that there is no clear association of the notion of a parabola with the topics previously covered in Basic Education, such as quadratic functions (or quadratics) and conics, particularly the conic parabola—which, when addressed in Basic Education, occurs in an abbreviated manner. This could create a gap in the initial training of Mathematics teachers when bringing the discussion of the topic to their teaching practice in Basic Education, or it could even be a factor influencing the dynamics and teaching work.

In a perspective of deepening this work, we expanded this research into the other phases of Didactic Engineering, where we conducted a study with Mathematics undergraduates, aiming to collect their impressions, difficulties, and experiences with the topic of parabola in the scope of Analytical Geometry, both in their trajectory during Basic Education and in undergraduate studies. This study was validated and can be verified through the references Sousa *et al.* (2023), Sousa, Alves, and Aires (2023), and Sousa, Alves, and Campos (2024), proposing didactic situations, alternative approaches, and materials that can support Mathematics teachers in working with this theme.

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