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Some generalizations in certain classes of rings with involution

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ABSTRACT: Let R be a 2-torsion free σ-prime ring with an involution σ, I a nonzero σ-ideal of R. In this paper we explore the commutativity of R satisfying any one of the properties: *(i)* $d(x) \circ F(y) = 0$ for all $x, y \in I$. *(ii)* $[d(x), F(y)] = 0$ for all $x, y \in I$. *(iii)* $d(x) \circ F(y) = x \circ y$ for all $x, y \in I$. *(iv)* $d(x)F(y) - xy \in Z(R)$ for all $x, y \in I$. We also discuss (α, β)-derivations of σ-prime rings and prove that if G is an (α, β)-derivation which acts as a homomorphism or as an anti-homomorphism on I, then $G = 0$ or $G = \beta$ on I.

Key Words: σ-prime ring; derivation; generalized derivation; (α, β)-derivation; commutativity.

Contents

1 Introduction **9**

2 Some preliminaries **10**

3 Generalized derivations of σ-prime rings **11**

4 (α, β)-derivations of σ-prime rings **14**

1. Introduction

Throughout the present paper R will denote an associative ring with center $Z(R)$. For any $x, y \in R$, the symbol $[x, y]$ stands for the commutator $xy - yx$ and the symbol $x \circ y$ denotes the anti-commutator $xy + yx$. In all that follows the symbol $Sa_\sigma(R)$, first introduced by Oukhtite, will denote the set of symmetric and skew symmetric elements of R, i.e. $Sa_\sigma(R) = \{x \in R \mid \sigma(x) = \pm x\}$. An involution σ of a ring R is an anti-automorphism of order 2 (i.e. an additive mapping satisfying $\sigma(xy) = \sigma(y)\sigma(x)$ and $\sigma^2(x) = x$ for all $x, y \in R$.) An ideal I of R is said to be a σ-ideal if $\sigma(I) = I$. An example, due to Rehman: Let Z be the ring of integers. Set $R = \left\{ \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} \mid a, b, c \in Z \right\}$. We define a map $\sigma : R \to R$ as follows: $\sigma \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} = \begin{pmatrix} c & -b \\ 0 & a \end{pmatrix}$. It is easy to check that $I = \left\{ \begin{pmatrix} 0 & b \\ 0 & 0 \end{pmatrix} \mid b \in Z \right\}$ is a σ-ideal of R. Note that an ideal I of a ring R may be not a σ-ideal: Let Z be the ring of integers and let $R = Z \times Z$. Consider a map $\sigma : R \to R$ defined by $\sigma((a, b)) = (b, a)$ for all $(a, b) \in R$. For an ideal $I = Z \times \{0\}$ of R, I is not a σ-ideal

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of R since $\sigma(I) = \{0\} \times Z \neq I$. A ring R is called 2-torsion free, if whenever $2x = 0$, with $x \in R$, then $x = 0$. Recall that a ring R is prime if for any $a, b \in R$, $aRb = 0$ implies $a = 0$ or $b = 0$. A ring R equipped with an involution σ is said to be a σ-prime ring if for any $a, b \in R$, $aRb = aR\sigma(b) = 0$ implies $a = 0$ or $b = 0$. It is worthwhile to note that every prime ring having an involution σ is σ-prime but the converse is in general not true. Such an example due to Oukhtite is as following: Let R be a prime ring, $S = R \times R^\circ$ where R° is the opposite ring of R, define $\sigma(x, y) = (y, x)$. From $(0, x)S(x, 0) = 0$, it follows that S is not prime. For the σ-primeness of S, we suppose that $(a, b)S(x, y) = 0$ and $(a, b)S\sigma((x, y)) = 0$, then we get $aRx \times yRb = 0$ and $aRy \times xRb = 0$, and hence $aRx = yRb = aRy = xRb = 0$, or equivalently $(a, b) = 0$ or $(x, y) = 0$. This example shows that every prime ring can be injected in a σ-prime ring and from this point of view σ-prime rings constitute a more general class of prime rings. An additive mapping $d : R \longrightarrow R$ is called a derivation if $d(xy) = d(x)y + xd(y)$ holds for all $x, y \in R$. An additive mapping $F : R \longrightarrow R$ is called a generalized derivation associated with d if there exists a derivation $d : R \longrightarrow R$ such that $F(xy) = F(x)y + xd(y)$ holds for all $x, y \in R$. Let α and β be homomorphisms of R, an additive mapping $G : R \longrightarrow R$ is called an (α, β)-derivation if $G(xy) = G(x)\alpha(y) + \beta(x)G(y)$ holds for all $x, y \in R$. Obviously, every $(1, 1)$-derivation on R is just a derivation on R, where 1 is the identity mapping. Let S be a nonempty subset of R and G an (α, β)-derivation of R. If $G(xy) = G(x)G(y)$ or $G(xy) = G(y)G(x)$ for all $x, y \in S$, then G is called an (α, β)-derivation which acts as a homomorphism or anti-homomorphism on S.

Recently, some well-known results concerning prime rings have been proved for σ-prime rings by Oukhtite et al. (see[1-9], where further references can be found). Over the past thirty years, there has been an ongoing interest concerning the relationship between the commutativity of a prime ring R and the behavior of a special mapping on that ring ([13], where further references can be found). In the year 2005, Ashraf et al. [10] proved some commutativity theorems for prime rings. In Section 3, we will generalize these results to generalized derivations on rings with involution.
On the other hand, Bell and Kappe [11] proved that if d is a derivation of a prime ring R which acts as a homomorphism or an anti-homomorphism on a nonzero ideal I of R, then $d = 0$ on R. In [12], Albas and Argac extended this result to generalized derivations. Further, Oukhtite [8] proved the above result is also true for σ-prime rings. In Section 4, we extend the mentioned result in the setting of (α, β)-derivations of σ-prime rings.

2. Some preliminaries

In all that follows, we assume that R is a 2-torsion free σ-prime ring, where σ is an involution of R. We begin with the following results which will be used to prove our theorems.

Lemma 2.1 (1, Lemma 3.1) . *Let R be a 2-torsion free σ-prime ring and I a nonzero σ-ideal of R. If $a, b \in R$ such that $aIb = aI\sigma(b) = 0$, then $a = 0$ or $b = 0$.*

Lemma 2.2 (2, Lemma 2.3) . *Let R be a 2-torsion free σ-prime ring, I a nonzero σ-ideal and d a derivation on R commuting with σ. If $d^2(I) = 0$, then $d = 0$.*

Lemma 2.3 (1,Theorem 3.2) . *Let R be a 2-torsion free σ-prime ring, d a nonzero derivation and I a nonzero σ-ideal of R. If $d(I) \subseteq Z(R)$, then R is commutative.*

Lemma 2.4 (2,Theorem 1.2)). *Let R be a 2-torsion free σ-prime ring, I a nonzero σ-ideal and d a nonzero derivation on R commuting with σ. If $[d(x), d(y)] = 0$ for all $x, y \in I$, then R is commutative.*

3. Generalized derivations of σ-prime rings

Theorem 3.1 *Let R be a 2-torsion free σ-prime ring with an involution σ, I a nonzero σ-ideal. If R admits a nonzero generalized derivation F associated a nonzero derivation d commuting with σ such that $d(x) \circ F(y) = 0$ for all $x, y \in I$, then R is commutative.*

Proof: By hypothesis, we have $d(x) \circ F(y) = 0$ for all $x, y \in I$. Replacing y by yr to get $d(x) \circ F(yr) = 0$, which implies that

$$(d(x) \circ y)d(r) - y[d(x), d(r)] + (d(x) \circ F(y))r - F(y)[d(x), r] = 0 \tag{1}$$

for all $x, y \in I$ and $r \in R$. Now using that $d(x) \circ F(y) = 0$, the relation (1) yields that $(d(x) \circ y)d(r) - y[d(x), d(r)] - F(y)[d(x), r] = 0$, which can reduce to

$$(d(x) \circ y)d^2(x) - y[d(x), d^2(x)] = 0 \tag{2}$$

if we replace r by $d(x)$, for all $x, y \in I$ and $r \in R$. Replacing y by zy in (2) to get $(d(x) \circ zy)d^2(x) - zy[d(x), d^2(x)] = 0$, which implies that

$$z(d(x) \circ y)d^2(x) + [d(x), z]yd^2(x) - zy[d(x), d^2(x)] = 0$$

for all $x, y, z \in I$. In view of (2), the above relation leads to the following

$$[d(x), y]zd^2(x) = 0 \tag{3}$$

for all $x, y, z \in I$.

Since I is a σ-ideal and $d\sigma = \sigma d$, for all $x \in I \bigcap Sa_\sigma(R)$, we have either $[d(x), y] = 0$ or $d^2(x) = 0$ by Lemma 2.1. Using the fact that $x - \sigma(x) \in I \bigcap Sa_\sigma(R)$ for all $x \in I$, then $[d(x - \sigma(x)), y] = 0$ or $d^2(x - \sigma(x)) = 0$ for all $y \in I$.

If $[d(x - \sigma(x)), y] = 0$, then $[d(x), y] = [\sigma(d(x)), y]$, for all $y \in I$. As I is a σ-ideal, it follows from (3) that $[d(x), y]zd^2(x) = 0 = \sigma([d(x), y])zd^2(x)$, and hence Lemma 2.1 yields that $[d(x), y] = 0$ or $d^2(x) = 0$.

If $d^2(x - \sigma(x)) = 0$, then $d^2(x) = \sigma(d^2(x))$ and (6) gives $[d(x), y] = 0$ or $d^2(x) = 0$. Consequently, for all $x \in I$, either $[d(x), I] = 0$ or $d^2(x) = 0$.

Now let $I_1 = \{x \in I \mid [d(x), I] = 0\}$ and $I_2 = \{x \in I \mid d^2(x) = 0\}$. Then I_1, I_2 are both additive subgroups of I and $I_1 \bigcup I_2 = I$. But a group can't be a union of its two proper subgroups, and hence $I_1 = I$ or $I_2 = I$. On the one hand, if $I_1 = I$, then

$$[d(x), y] = 0, \tag{4}$$

for all $x, y \in I$. Replacing y by ry in (4) to get $[d(x), r]y = 0$ for all $x, y \in I$ and $r \in R$. As d commutes with σ, the fact that I is a σ-ideal gives us $[d(x), r] = 0$ i.e. $d(I) \subseteq Z(R)$, and hence R is commutative by Lemma 2.3. Of course, we can also replace y by $yd(z)$ in (4) and use (4) to get $y[d(x), d(z)] = 0$ for all $x, y, z \in I$. As d commutes with σ, the fact that I is a σ-ideal shows that $[d(x), d(z)] = 0$ for all $x, z \in I$, and hence R is commutative by Lemma 2.4. On the other hand, if $I_2 = I$, then $d^2(x) = 0$ for all $x \in I$. In other words, $d^2(I) = 0$ and hence $d = 0$ by Lemma 2.2, a contradiction. □

Theorem 3.2 *Let R be a 2-torsion free σ-prime ring with an involution σ, I a nonzero σ-ideal. If R admits a nonzero generalized derivation F associated a nonzero derivation d commuting with σ such that $[d(x), F(y)] = 0$ for all $x, y \in I$, then R is commutative.*

Proof: We are given that

$$[d(x), F(y)] = 0 \tag{5}$$

for all $x, y \in I$. Replacing y by yz in (5) and using (5) to get

$$F(y)[d(x), z] + y[d(x), d(z)] + [d(x), y]d(z) = 0 \tag{6}$$

for all $x, y, z \in I$. Replacing z by $zd(x)$ in (6) and using (6) to get

$$yz[d(x), d^2(x)] + y[d(x), z]d^2(x) + [d(x), y]zd^2(x) \tag{7}$$

for all $x, y, z \in I$. Replacing y by wy in (7) and using (7) to get

$$[d(x), w]yzd^2(x) = 0 \tag{8}$$

for all $x, y, z, w \in I$.

For all $x \in I \bigcap Sa_\sigma(R)$, (8) yields that $[d(x), w]yId^2(x) = 0 = [d(x), w]yI\sigma(d^2(x))$ for all $x, y, w \in I$. Thus, we have either $[d(x), w]y = 0$ or $d^2(x) = 0$ by Lemma 2.1. Suppose that $[d(x), w]y = 0$ i.e. $[d(x), w]I = 0$, then it is easy to see $[d(x), w] = 0$. Consequently, for all $x \in I$, either $[d(x), I] = 0$ or $d^2(x) = 0$. Note that the arguments used in the proof of Theorem 3.1 are still valid in the present situation, as required. □

Theorem 3.3 *Let R be a 2-torsion free σ-prime ring with an involution σ, I a nonzero σ-ideal. If R admits a generalized derivation F associated a nonzero derivation d commuting with σ such that $d(x) \circ F(y) = x \circ y$ for all $x, y \in I$, then R is commutative.*

Proof: If $F = 0$, then $x \circ y = 0$ for all $x, y \in I$. Replacing y by yz and using that $x \circ y = 0$ to get $y[x, z] = 0$ for all $x, y, z \in I$. In particular, $[x, z]I[x, z] = 0 = [x, z]I\sigma([x, z])$, then $[x, z] = 0$ in view of Lemma 2.1. From ([8], proof of Theorem 1.1) this yields that R is commutative.

If $F \neq 0$, then $d(x) \circ F(y) = x \circ y$ for all $x, y \in I$. Replacing y by yr to get

$$(d(x) \circ y)d(r) - y[d(x), d(r)] + (d(x) \circ F(y))r - F(y)[d(x), r] = (x \circ y)r - y[x, r]$$

which reduces to

$$(d(x) \circ y)d(r) - y[d(x), d(r)] - F(y)[d(x), r] + y[x, r] = 0 \quad (9)$$

for all $x, y \in I$ and $r \in R$. In (9), replacing r by $d(x)$ to get

$$(d(x) \circ y)d^2(x) - y[d(x), d^2(x)] + y[x, d(x)] = 0 \quad (13)$$

for all $x, y \in I$. Replacing y by zy in (10) and using (10) to get

$$[d(x), z]yd^2(x) = 0 \quad (11)$$

for all $x, y, z \in I$. Now again use the arguments used in the proof of Theorem 3.1, we get the required result. □

Theorem 3.4 *Let R be a 2-torsion free σ-prime ring with an involution σ, I a nonzero σ-ideal. If R admits a generalized derivation F associated a nonzero derivation d commuting with σ such that $d(x)F(y) - xy \in Z(R)$ for all $x, y \in I$, then R is commutative.*

Proof: If $F = 0$, then $xy \in Z(R)$ for all $x, y \in I$. In particular, $[xy, z] = 0$ and hence $x[y, z] + [x, z]y = 0$ for all $x, y, z \in I$. Replacing x by wx to get $[w, z]xy = 0$ for all $w, x, y, z \in I$ and therefore $[w, z]Iy = 0 = [w, z]I\sigma(y)$. Applying Lemma 2.1, we get $[w, z] = 0$ for all $w, z \in I$ and from ([8], proof of Theorem 1.1) we get the required result. □

If $F \neq 0$, then $d(x)F(y) - xy \in Z(R)$ for all $x, y \in I$. Replacing y by yz to get $(d(x)F(y) - xy)z + d(x)yd(z) \in Z(R)$, which implies $[d(x)yd(z), z] = 0$ for all $x, y, z \in I$. Hence it follows that $d(x)[yd(z), z] + [d(x), z]yd(z) = 0$ for all $x, y, z \in I$. Replacing y by $d(x)y$ in the above to get

$$[d(x), z]d(x)yd(z) = 0 \quad (12)$$

for all $x, y, z \in I$. For all $z \in I \bigcap Sa_\sigma(R)$, (12) yields that $[d(x), z]d(x) = 0$ or $d(z) = 0$ by Lemma 2.1. For any $z \in I$, the fact $z - \sigma(z) \in I \bigcap Sa_\sigma(R)$ yields that either $d(z - \sigma(z)) = 0$ or $[d(x), z - \sigma(z)]d(x) = 0$. If $d(z - \sigma(z)) = 0$, then $d(z) = \sigma(d(z))$ and hence (12) yields that $[d(x), z]d(x) = 0$ or $d(z) = 0$. If $[d(x), z - \sigma(z)]d(x) = 0$, using that $z + \sigma(z) \in I \bigcap Sa_\sigma(R)$ then $[d(x), z + \sigma(z)]d(x) = 0$ or $d(z + \sigma(z)) = 0$. Assume that $[d(x), z + \sigma(z)]d(x) = 0$, then $2[d(x), z]d(x) = 0$

and hence $[d(x), z]d(x) = 0$. Assume that $d(z + \sigma(z)) = 0$, then $d(z) = -\sigma(d(z))$ and hence (12) yields that $[d(x), z]d(x) = 0$ or $d(z) = 0$. Consequently, for all $z \in I$, either $[d(x), z]d(x) = 0$ or $d(z) = 0$.

Now let $I_1 = \{z \in I \mid [d(x), z]d(x) = 0\}$ and $I_2 = \{z \in I \mid d(z) = 0\}$. Then I_1, I_2 are both additive subgroups of I and $I_1 \bigcup I_2 = I$. By Brauer's trick, either $I_1 = I$ or $I_2 = I$.

On the one hand, if $I_1 = I$ then $[d(x), z]d(x) = 0$, and hence $[d(x), yz]d(x) = 0$, from ([5], proof of Theorem 2.1) R is commutative.

On the other hand, if $I_2 = I$ then $d(I) = 0$ and R is commutative by Lemma 2.3.

The following example demonstrates that the above results are not true in the case of arbitrary rings.

Example 3.1. Let Z be the ring of integers. Set $R = \left\{ \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} \mid a, b, c \in Z \right\}$ and $I = \left\{ \begin{pmatrix} 0 & b \\ 0 & 0 \end{pmatrix} \mid b \in Z \right\}$. We define the following maps: $\sigma \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} = \begin{pmatrix} c & -b \\ 0 & a \end{pmatrix}$. $F \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} = \begin{pmatrix} a & 2b \\ 0 & 0 \end{pmatrix}$. $d \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} = \begin{pmatrix} 0 & b \\ 0 & 0 \end{pmatrix}$. Then it is easy to see that I is a σ-ideal of R with an involution σ and F is a generalized derivation associated with a nonzero derivation d commuting with σ. Moreover, it is straightforward to check that F satisfies the properties: (i) $d(x) \circ F(y) = 0$ (ii) $[d(x), F(y)] = 0$ (iii) $d(x) \circ F(y) = x \circ y$ (iv) $d(x)F(y) - xy \in Z(R)$ for all $x, y \in I$. However, R is not commutative.

Remark 3.1. Some more concrete examples showing the hypothesis of σ-primeness is necessary for R in literature appear in the works of Oukhtite [14], [15] and [16].

4. (α, β)-derivations of σ-prime rings

Theorem 4.1 *Let R be a 2-torsion free σ-prime ring with an involution σ, I a nonzero σ-ideal and G an (α, β)-derivation commuting with σ, where β is a automomorphism of R such that $\sigma\beta = \beta\sigma$. If G acts as an homomorphism or as an anti-homomorphism on I, then $G = 0$ or $G = \beta$ on I.*

Proof: Assume that G acts as a homomorphism on I. By our hypothesis, we have $G(xy) = G(x)G(y)$, which can be rewritten as

$$G(x)G(y) = G(x)\alpha(y) + \beta(x)G(y) \tag{13}$$

for all $x, y \in I$.
Replacing x by xz in (13), to get

$$G(xz)G(y) = G(xz)\alpha(y) + \beta(xz)G(y) = G(x)G(z)\alpha(y) + \beta(xz)G(y)$$

for all $x, y, z \in I$.

And hence

$$G(xz)G(y) = G(x)G(z)\alpha(y) + \beta(xz)G(y) \tag{14}$$

for all $x, y, z \in I$. Note that G is a homomorphism on I, we have also

$$G(xz)G(y) = G(x)G(z)G(y) = G(x)G(zy) = G(x)G(z)\alpha(y) + G(x)\beta(z)G(y)$$

for all $x, y, z \in I$. An hence

$$G(xz)G(y) = G(x)G(z)\alpha(y) + G(x)\beta(z)G(y) \tag{15}$$

for all $x, y, z \in I$. Combing (14) with (15), we have $(G(x) - \beta(x))\beta(z)G(y) = 0$ for all $x, y, z \in I$, and hence $(G(x) - \beta(x))\beta(I)G(y) = 0$. Set $J = \beta(I)$, it is easy to see that J is a nonzero σ-ideal. In other words, we have

$$((G(x) - \beta(x))JG(y) = 0 \tag{16}$$

Now (16) yields $((G(x) - \beta(x))JG(y) = 0 = ((G(x) - \beta(x))J\sigma(G(y))$ since both G commutes with σ, and hence by Lemma 2.1 either $G(x) - \beta(x) = 0$ or $G(y) = 0$ for all $x, y \in I$, namely, $G = \beta$ or $G = 0$ on I.

Now assume that G acts as an anti-homomorphism on I, then $G(xy) = G(y)G(x)$, which can be rewritten as

$$G(y)G(x) = G(x)\alpha(y) + \beta(x)G(y) \tag{17}$$

for all $x, y \in I$. Replacing x by xy in (17) to get $G(y)G(xy) = G(xy)\alpha(y) + \beta(xy)G(y)$, which implies that $G(y)G(x)\alpha(y) + G(y)\beta(x)G(y) = G(y)G(x)\alpha(y) + \beta(xy)G(y)$, hence we have

$$G(y)\beta(x)G(y) = \beta(xy)G(y) \tag{18}$$

for all $x, y \in I$. Replacing x by rx in (18) and using (18) to get

$$[G(y), \beta(r)]\beta(x)G(y) = 0 \tag{19}$$

or equivalently, if we set $J = \beta(I)$ then we have

$$[G(y), \beta(r)]JG(y) = 0 \tag{20}$$

for all $y \in I$ and $r \in R$. For all $y \in I \bigcap Sa_\sigma(R)$, we have $[G(y), \beta(r)]JG(y) = 0 = [G(y), \beta(r)]J\sigma(G(y))$ from (20), and hence $[G(y), \beta(r)] = 0$ or $G(y) = 0$ by Lemma 2.1. But $G(y) = 0$ also implies that $[G(y), \beta(r)] = 0$. Accordingly, for all $y \in I \bigcap Sa_\sigma(R)$ we have $[G(y), \beta(r)] = 0$ for all $r \in R$. For all $y \in I$, as $y - \sigma(y) \in I \bigcap Sa_\sigma(R)$ yields that $[G(y - \sigma(y)), \beta(r)] = 0$. Therefore $[G(y), \beta(r)] = [G(\sigma(y)), \beta(r)]$ and the relation (24) gives us $[G(y), \beta(r)]JG(y) = 0 = [G(\sigma(y)), \beta(r)]JG(y) = \sigma([G(\sigma(y)), \beta(r)])JG(y)$. Using Lemma 2.1, we get $[G(y), \beta(r)] = 0$ or $G(y) = 0$, in which case $[G(y), \beta(r)] = 0$. Consequently, for all $y \in I$ we have $[G(y), \beta(r)] = 0$ i.e., $[G(y), R] = 0$ and then $G(I) \subseteq Z(R)$. Hence G acts as a homomorphism on I so that $G = 0$ or $G = \beta$ on I. □

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