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Regularity of the solutions to a nonlinear boundary problem with indefinite weight.

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ABSTRACT: In this paper we study the regularity of the solutions to the problem $\Delta_p u = |u|^{p-2}u$ in the bounded smooth domain $\Omega \subset \mathbb{R}^N$, with $|\nabla u|^{p-2}\frac{\partial u}{\partial \nu} = \lambda V(x)|u|^{p-2}u+h$ as a nonlinear boundary condition, where $\partial\Omega$ is $C^{2,\beta}$ with $\beta \in]0,1[$, and V is a weight in $L^s(\partial\Omega)$ and $h \in L^s(\partial\Omega)$ for some $s \geq 1$. We prove that all solutions are in $L^\infty(\partial\Omega)\bigcap L^\infty(\Omega)$, and using the D.Debenedetto's theorem of regularity in [1] we conclude that those solutions are in $C^{1,\alpha}\left(\overline{\Omega}\right)$ for some $\alpha \in]0,1[$.

Key Words: Regularity, p-Laplacian, nonlinear boundary conditions, weight.

Contents

1. Introduction

The following problem

$$\begin{array}{rclcl} \Delta_p u & = & |u|^{p-2}\, u & \text{on} & \Omega \\ |\nabla u|^{p-2}\dfrac{\partial u}{\partial \nu} & = & \lambda\, |u|^{p-2}\, uV(x) + h & \text{in} & \partial\Omega \end{array} \tag{1.1}$$

where Ω is a bounded domain in $\mathbb{R}^N$, with a $C^{2,\beta}$ boundary for some $\beta \in]0,1[$, $p > 1$, $\Delta_p u = div(|\nabla u|^{p-2}\nabla u)$ is the p-Laplacian, $\dfrac{\partial}{\partial \nu}$ is the outer normal derivative, and V is the weight such that $V^+ \neq 0$ on $\partial\Omega$ and $V \in L^s(\partial\Omega)$, and $h \in L^s(\partial\Omega)$, where $s > \frac{N-1}{p-1}$ if $1 < p \leq N$ and $s \geq 1$ if $N < p$, has been treated by Julián Fernández BONDER and Julio D.ROSSI in [2], they have proved that there exists a sequence of variational eigenvalues $\lambda_k \to +\infty$, and that the first eigenvalue is isolated, simple and monotone with respect to the weight.

In this work we will show the two theorems

Theorem 1.1 *If u is a solution of the problem (1.1) then $u \in L^\infty(\Omega)$ and $u/\partial\Omega \in L^\infty(\partial\Omega)$.*

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Theorem 1.2 *Any solution of (1.1) is in* $C^{1,\alpha}\left(\overline{\Omega}\right)$ *for some* $\alpha \in \left]0,1\right[$ *and* $\|u\|_{C^{1,\alpha}\left(\overline{\Omega}\right)} \leq K$ *where*

$$\begin{cases} K &= K\left(N,p,\|u/\partial\Omega\|_{L^{s'q_0}(\partial\Omega)},\|h\|_{L^{s}(\partial\Omega)},\|V\|_{L^{s}(\partial\Omega)}\right) \quad \text{for} \quad 1<p\leq N \\ K &= K\left(N,p,\|u\|_{L^{\infty}(\Omega)}\right) \quad \text{for} \quad N<p \end{cases}$$

and

$$\begin{cases} s'q_0 \in [s'p,p_0] & \text{if} \quad 1<p<N \\ q_0 = p+1 & \text{if} \quad p=N \end{cases}$$

$p_0 = \frac{p(N-1)}{N-p}$ *and* s' *is the conjugate of* s.

2. Proofs of theorems

2.1. Proof of theorem 1.1. One recall that u is a solution of the problem (1.1) if and only if for all v in $W^{1,p}(\Omega)$ one has

$$\int_{\Omega} |\nabla u|^{p-2} \nabla u \nabla v + \int_{\Omega} |u|^{p-2} uv = \lambda \int_{\partial\Omega} |u|^{p-2} uvV(x)\partial\sigma + \int_{\partial\Omega} hv\partial\sigma. \quad (2.1)$$

For $p>N$ we have $W^{1,p}(\Omega) \underset{cpct}{\hookrightarrow} C\left(\overline{\Omega}\right)$, then any solution of (1.1) is in $L^{\infty}(\Omega)$ and $u/\partial\Omega \in L^{\infty}(\partial\Omega)$.

For $1<p\leq N$ we show the following lemma

Lemma 2.1 *If* $u \in W^{1,p}(\Omega)$ *is a solution of the problem* (1.1) *then there exists a constant* $C>0$ *such that*

$$\left(\|u\|^{q_n}_{L^{q_n}(\Omega)} + \|u/\partial\Omega\|^{q_n s'}_{L^{q_n s'}(\partial\Omega)}\right)^{\frac{1}{q_n}} \leq C \quad \text{for all } n \geq n_0.$$

where the sequence $(q_n)_n$ *is defined as follows*

$$\begin{cases} s'q_0 \in [s'p,p_0] & \text{if} \quad 1<p<N, \\ q_0 = p+1 & \text{if} \quad p=N, \end{cases}, \quad p_0 = \frac{p(N-1)}{N-p} \quad \text{and} \quad q_{n+1} = \frac{q_0}{p} q_n.$$

In particular, $u \in L^{q_n}(\Omega)$ *and* $u/\partial\Omega \in L^{s'q_n}(\partial\Omega)$ *, for all* $n \geq 0$, *where* s' *is the conjugate of* s.

Proof: Let u be a solution of the problem (1.1) so $u \in W^{1,p}(\Omega)$.

One has $s > \frac{N-1}{p-1}$ so $1 < s' = \frac{s}{s-1} < \frac{N-1}{N-p}$, and $[p,p_0]\cap[s'p,s'p_0] = [s'p,p_0] \neq \emptyset$. Let q_0 be in $\left[p,\frac{p_0}{s'}\right]$, for this choice one has

$$W^{1,p}(\Omega) \underset{cont}{\hookrightarrow} L^{q_0}(\Omega) \quad \text{and} \quad W^{1,p}(\Omega) \underset{cont}{\hookrightarrow} L^{q_0 s'}(\partial\Omega)$$

then $u \in L^{q_0}(\Omega)$ and $u/\partial\Omega \in L^{q_0 s'}(\partial\Omega)$, so $u/\partial\Omega \in L^{q_0}(\partial\Omega)$ also.

We can suppose that $\|u/\partial\Omega\|_{L^{s'q_0}(\partial\Omega)} \geq 1$ if not one considers $u_0 = \frac{u}{\|u/\partial\Omega\|_{L^{s'q_0}(\partial\Omega)}}$ which is a solution of

$$\begin{array}{rcll} \Delta_p u & = & |u|^{p-2}\, u & \text{on} \quad \Omega \\ |\nabla u|^{p-2} \dfrac{\partial u}{\partial \nu} & = & \lambda\, |u|^{p-2}\, uV(x) + h' & \text{in} \quad \partial\Omega \end{array} \tag{2.2}$$

with $h' = \left(\|u/\partial\Omega\|_{L^{s'q_0}(\partial\Omega)}\right)^{p-1} h \in L^s(\partial\Omega)$.

By recurrence on n we suppose that $u \in L^{q_n}(\Omega)$, $u/\partial\Omega \in L^{s'q_n}(\partial\Omega)$ and $\|u/\partial\Omega\|_{L^{s'q_n}(\partial\Omega)} \geq 1$ and we show that $u \in L^{q_{n+1}}(\Omega)$, $u/\partial\Omega \in L^{q_{n+1}}(\partial\Omega)$, $u/\partial\Omega \in L^{q_{n+1}s'}(\partial\Omega)$ and $\|u/\partial\Omega\|_{L^{s'q_{n+1}}(\partial\Omega)} \geq 1$.

In what follows, one will indicate the norms $\|u\|_{L^p(\Omega)}$ and $\|u/\partial\Omega\|_{L^p(\partial\Omega)}$ by $\|u\|_{p(\Omega)}$ and $\|u\|_{p(\partial\Omega)}$.

Let us define the sequence $(v_k)_{k\geq 0}$ in $W^{1,p}(\Omega)$ by

$$v_k(x) = \begin{cases} k & \text{if} \quad u(x) \geq k \\ u(x) & \text{if} \quad -k \leq u(x) \leq k \\ -k & \text{if} \quad u(x) \leq -k \end{cases} \qquad \forall x \in \overline{\Omega}$$

and one poses $\delta = q_n - p > 0$. The following function test is taken $|v_k|^{\delta} v_k$ in the formula (2.1), we find on the one hand

$$\langle \Delta_p u, |v_k|^{\delta} v_k \rangle = \int_\Omega |u|^{p-2}\, u\, |v_k|^{\delta}\, v_k dx \geq \int_\Omega |v_k|^{\delta+p} = \int_\Omega |v_k|^{q_n}$$

and on the other hand

$$\begin{array}{rcl} \langle \Delta_p u, |v_k|^{\delta} v_k \rangle & = & -\displaystyle\int_\Omega |\nabla u|^{p-2}\, \nabla u \nabla(|v_k|^{\delta}\, v_k) + \int_{\partial\Omega} |\nabla u|^{p-2} \frac{\partial u}{\partial \nu}\, |v_k|^{\delta}\, v_k \partial\sigma \\ & = & -A_n \displaystyle\int_\Omega \left|\nabla(|v_k|^{\frac{\delta}{p}}\, v_k)\right|^p + \int_{\partial\Omega} \left(\lambda V(x)\, |u|^{p-2}\, u + h\right) |v_k|^{\delta}\, v_k \partial\sigma \\ & \leq & \lambda \displaystyle\int_{\partial\Omega} |u|^{q_n}\, |V(x)|\, \partial\sigma + H \left\|v_k^{\delta+1}\right\|_{s'(\partial\Omega)} - A_n \left\|\nabla(|v_k|^{\frac{\delta}{p}}\, v_k)\right\|_{p(\Omega)}^p \\ & \leq & \lambda \|u\|_{s'q_n(\partial\Omega)}^{q_n} \|V\| + H \|v_k\|_{(\delta+1)s'(\partial\Omega)}^{\delta+1} - A_n \left\|\nabla(|v_k|^{\frac{\delta}{p}}\, v_k)\right\|_{p(\Omega)}^p \end{array}$$

where $A_n = (\delta+1)\left(\frac{p}{q_n}\right)^p$, $\|V\| = \|V\|_{s(\partial\Omega)}$ and $H = \|h\|_{s(\partial\Omega)}$.

Then

$$\int_\Omega |v_k|^{q_n} \leq \lambda \|u\|_{s'q_n(\partial\Omega)}^{q_n} \|V\| + H \|v_k\|_{(\delta+1)s'(\partial\Omega)}^{\delta+1} - A_n \left\|\nabla\left(|v_k|^{\frac{\delta}{p}}\, v_k\right)\right\|_{p(\Omega)}^p \tag{2.3}$$

And since $W^{1,p}(\Omega) \underset{cont}{\hookrightarrow} L^{q_0}(\Omega)$ one has

$$\begin{array}{rcl} \left\|\nabla\left(|v_k|^{\frac{\delta}{p}}\, v_k\right)\right\|_{p(\Omega)}^p & \geq & C_1 \left\||v_k|^{\frac{\delta+p}{p}}\right\|_{q_0(\Omega)}^p - \left\||v_k|^{\frac{\delta+p}{p}}\right\|_{p(\Omega)}^p \\ & \geq & C_1 \|v_k\|_{q_{n+1}(\Omega)}^{q_n} - \|v_k\|_{\delta+p(\Omega)}^{\delta+p} \end{array}$$

Thus it is found that

$$\|v_k\|_{L^{q_{n+1}}(\Omega)}^{q_n} \leq R_n \left(\lambda \|u\|_{q_n s'(\partial\Omega)}^{q_n} \|V\| + H \|v_k\|_{(\delta+1)s'(\partial\Omega)}^{\delta+1} + S_n \|v_k\|_{q_n(\Omega)}^{q_n}\right)$$

where $R_n = \dfrac{1}{C_1 A_n}$ and $S_n = (A_n - 1)$. Then, since $\delta + 1 < q_n$, one has

$$\|v_k\|_{(\delta+1)s'(\partial\Omega)}^{\delta+1} \leq \|u\|_{(\delta+1)s'(\partial\Omega)}^{\delta+1} \leq \|u\|_{q_n s'(\partial\Omega)}^{\delta+1} \left(meas_\sigma(\partial\Omega)^{\frac{p-1}{s' q_n}}\right)$$

Supposing that $meas_\sigma(\partial\Omega) \leq 1$, and with the assumption $\|u\|_{L^{s' q_n}(\partial\Omega)} \geq 1$ we obtain

$$\|v_k\|_{(\delta+1)s'(\partial\Omega)}^{\delta+1} \leq \|u\|_{q_n s'(\partial\Omega)}^{\delta+1} \leq \|u\|_{q_n s'(\partial\Omega)}^{q_n}$$

And

$$\begin{aligned} \|v_k\|_{q_{n+1}(\Omega)}^{q_n} &\leq R_n \left((\lambda \|V\| + H) \|u\|_{q_n s'(\partial\Omega)}^{q_n} + |S_n| \|u\|_{q_n(\Omega)}^{q_n}\right) \\ &\leq R_n \max(B, |S_n|) \left(\|u\|_{q_n s'(\partial\Omega)}^{q_n} + \|u\|_{q_n(\Omega)}^{q_n}\right) \end{aligned}$$

where $B = \lambda \|V\| + H$

Then

$$\|u\|_{q_{n+1}(\Omega)}^{q_n} \leq \liminf_{|k|\to+\infty} \left(\|v_k\|_{q_{n+1}(\Omega)}^{q_n}\right) \tag{2.4}$$

$$\leq R_n \max(B, |S_n|) \left(\|u\|_{q_n s'(\partial\Omega)}^{q_n} + \|u\|_{q_n(\Omega)}^{q_n}\right) \tag{2.5}$$

Therefore $u \in L^{q_{n+1}}(\Omega)$.

We must now proof that $u/\partial\Omega \in L^{s' q_{n+1}}(\partial\Omega)$,(so $u/\partial\Omega \in L^{q_{n+1}}(\partial\Omega)$), and $\|u\|_{L^{s' q_{n+1}}(\partial\Omega)} \geq 1$.

Using (2.3) it is found that

$$\int_\Omega |v_k|^{q_n} + A_n \left\|\nabla(|v_k|^{\frac{\delta}{p}} v_k)\right\|_{p(\Omega)}^p \leq B \|u\|_{q_n s'(\partial\Omega)}^{q_n}$$

With the result $W^{1,p}(\Omega) \underset{cont}{\hookrightarrow} L^{s' q_0}(\partial\Omega)$ one has

$$\begin{aligned} \left\|\nabla(|v_k|^{\frac{\delta}{p}} v_k)\right\|_{p(\Omega)}^p &\geq C_2 \left\||v_k|^{\frac{\delta+p}{p}}\right\|_{s' q_0(\partial\Omega)}^p - \left\||v_k|^{\frac{\delta+p}{p}}\right\|_{p(\Omega)}^p \\ &\geq C_2 \|v_k\|_{s' q_{n+1}(\partial\Omega)}^{q_n} - \|v_k\|_{\delta+p(\Omega)}^{\delta+p} \end{aligned}$$

So

$$A_n \left(C_2 \|v_k\|_{s' q_{n+1}(\partial\Omega)}^{q_n} - \|v_k\|_{\delta+p(\Omega)}^{\delta+p}\right) \leq B \|u\|_{q_n s'(\partial\Omega)}^{q_n} - \int_\Omega |v_k|^{q_n}$$

What means that

$$\|v_k\|_{s' q_{n+1}(\partial\Omega)}^{q_n} \leq R'_n \left(B \|u\|_{q_n s'(\partial\Omega)}^{q_n} + |S_n| \|v_k\|_{q_n(\Omega)}^{q_n}\right)$$

where $R'_n = \dfrac{1}{C_2 A_n}$. Then we obtain

$$\|v_k\|^{q_n}_{s'q_{n+1}(\partial\Omega)} \le R'_n \max\left(B, |S_n|\right) \left(\|u\|^{q_n}_{q_n s'(\partial\Omega)} + \|u\|^{q_n}_{q_n(\Omega)}\right)$$

And

$$\begin{aligned} \|u\|^{q_n}_{s'q_{n+1}(\partial\Omega)} &\le \liminf_{|k|\to+\infty} \left(\|v_k\|^{q_n}_{s'q_{n+1}(\partial\Omega)}\right) &(2.6)\\ &\le R'_n \max\left(B, |S_n|\right) \left(\|u\|^{q_n}_{q_n s'(\partial\Omega)} + \|u\|^{q_n}_{q_n(\Omega)}\right) &(2.7)\end{aligned}$$

Consequently $u/\partial\Omega \in L^{s'q_{n+1}}(\partial\Omega)$ and $\|u\|_{L^{s'q_{n+1}}(\partial\Omega)} \ge \|u\|_{L^{s'q_n}(\partial\Omega)} \ge 1$.

Hence $u \in L^{q_n}(\Omega)$, $u/\partial\Omega \in L^{s'q_n}(\partial\Omega)$ and $\|u\|_{L^{s'q_n}(\partial\Omega)} \ge 1 \quad \forall n \ge 0$

It remains to be shown that there exists $C > 0$ such that

$$\left(\|u\|^{q_n}_{L^{q_n s'}(\partial\Omega)} + \|u\|^{q_n}_{L^{q_n}(\Omega)}\right)^{\frac{1}{q_n}} \le C \quad \text{for all } n \ge n_0$$

From the two preceding inequalities (2.5) and (2.7), one concludes that

$$\|u\|^{q_{n+1}}_{L^{s'q_{n+1}}(\partial\Omega)} + \|u\|^{q_{n+1}}_{L^{q_{n+1}}(\Omega)} \le D_n \left(\max\left(B, |S_n|\right)\left(\|u\|^{q_n}_{L^{q_n s'}(\partial\Omega)} + \|u\|^{q_n}_{L^{q_n}(\Omega)}\right)\right)^{\frac{q_0}{p}}$$

where $D_n = \left(\left(\dfrac{1}{C_1} + \dfrac{1}{C_2}\right)\dfrac{1}{A_n}\right)^{\frac{q_0}{p}}$. And since $A_n \underset{n\to+\infty}{\to} 0$ one has $|S_n| \underset{n\to+\infty}{\to} 1$, thus starting from a certain row n_0 of n one has $|S_n| \le 2$, consequently

$$\|u\|^{q_{n+1}}_{L^{s'q_{n+1}}(\partial\Omega)} + \|u\|^{q_{n+1}}_{L^{q_{n+1}}(\Omega)} \le C(q_n)^{q_0}\left(\|u\|^{q_n}_{L^{q_n s'}(\partial\Omega)} + \|u\|^{q_n}_{L^{q_n}(\Omega)}\right)^{\frac{q_0}{p}}$$

where $C = \dfrac{1}{p^{q_0}}\left(\left(\dfrac{1}{C_1} + \dfrac{1}{C_2}\right)\max\left(B, 2\right)\right)^{\frac{q_0}{p}}$.

Posing $v_n = \left(\|u\|^{q_n}_{L^{q_n s'}(\partial\Omega)} + \|u\|^{q_n}_{L^{q_n}(\Omega)}\right)^{\frac{1}{q_n}}$, one will have $v_{n+1}^{q_{n+1}} \le C(q_n)^{q_0}(v_n^{q_n})^{\frac{q_0}{p}}$ for all $n \ge n_0$, and

$$\ln(v_{n+1}) \le \frac{A}{q_{n+1}} + p\frac{\ln(q_n)}{q_n} + \ln(v_n) \quad \text{for all } n \ge n_0 \quad \text{where} \quad A = \ln(C)$$

Thus $\ln(v_{n+1}) \le A \sum\limits_{n_0+1\le k\le n+1}\left(\dfrac{1}{q_k}\right) + p\sum\limits_{n_0\le k\le n}\left(\dfrac{\ln(q_k)}{q_k}\right) + \ln(v_{n_0}) \quad \forall n \ge n_0$.

The sequence $\left(\dfrac{1}{q_k}\right)_{k\ge 0}$ is geometric of reason $0 < \dfrac{p}{q_0} < 1$, so

$$\sum_{n_0+1\le k\le n+1}\left(\frac{1}{q_k}\right) \le \frac{q_0}{q_0 - p}$$

It is checked easily that

$$\frac{\ln(q_k)}{q_k} = (\eta + \gamma k)\left(\frac{p}{q_0}\right)^k$$

and

$$\sum_{n_0 \leq k \leq n} \left(\frac{\ln(q_k)}{q_k}\right) \leq \sum_{0 \leq k} (\eta + \gamma k)\left(\frac{p}{q_0}\right)^k = \frac{\eta q_0}{q_0 - p} + \gamma \frac{p q_0}{(q_0 - p)^2}$$

where $\eta = \dfrac{\ln(q_0)}{q_0}$ and $\gamma = \dfrac{\ln(q_0) - \ln(p)}{q_0}$.

From where $\ln(v_n) \leq \dfrac{q_0}{q_0 - p}(A + p\eta) + \gamma \dfrac{p^2 q_0}{(q_0 - p)^2} + \ln(v_{n_0}) = D \quad \forall n \geq n_0$.

Consequently $v_n = \left(\|u\|^{q_n}_{L^{q_n s'}(\partial\Omega)} + \|u\|^{q_n}_{L^{q_n}(\Omega)}\right)^{\frac{1}{q_n}} \leq \exp(D) \;\; \forall n \geq n_0$. □

Let us return to the demonstration of the theorem 1.1. By the lemma 2.1 we conclude that $\|u\|_{L^{q_n}(\Omega)} \leq \exp(D)$ and $\|u/\partial\Omega\|_{L^{q_n s'}(\partial\Omega)} \leq \exp(D) \quad \forall n \geq n_0$. Then

$$\|u\|_{L^\infty(\Omega)} \leq \limsup_{n \to +\infty} \|u\|_{L^{q_n}(\Omega)} \leq \exp(D)$$

and

$$\|u/\partial\Omega\|_{L^\infty(\partial\Omega)} \leq \limsup_{n \to +\infty} \|u/\partial\Omega\|_{L^{q_n s'}(\partial\Omega)} \leq \exp(D)$$

Thus we proved that $u \in L^\infty(\Omega)$ and $u/\partial\Omega \in L^\infty(\partial\Omega)$.

2.2. PROOF OF THEOREM 1.2. The proof uses the following result of D.Debenedetto (see [1]).

Lemma 2.2 *Let u be in $W^{1,p}(\Omega) \cap L^\infty(\Omega)$ such that $\Delta_p u \in L^\infty(\Omega)$ and $\partial\Omega$ is $C^{2,\beta}$ with $\beta \in]0,1[$, thus*

$$u \in C^{1,\alpha}\left(\overline{\Omega}\right) \text{ for some } \alpha \in]0,1[$$

and

$$\|u\|_{C^{1,\alpha}(\overline{\Omega})} \leq K\left(N, p, \|u\|_{L^\infty(\Omega)}, \|\Delta_p u\|_{L^\infty(\Omega)}\right)$$

By the last theorem $u \in L^\infty(\Omega)$, $\Delta_p u = |u|^{p-2}u \in L^\infty(\Omega)$ and $\|\Delta_p u\|_{L^\infty(\Omega)} = \|u\|^{p-1}_{L^\infty(\Omega)}$, so u is in $C^{1,\alpha}\left(\overline{\Omega}\right)$ for some $\alpha \in]0,1[$ and $\|u\|_{C^{1,\alpha}(\overline{\Omega})} \leq K\left(N, p, \|u\|_{L^\infty(\Omega)}\right)$. Moreover for $1 < p \leq N$ one has $\|u\|_{L^\infty(\Omega)} \leq C$ where C depends on $\|u/\partial\Omega\|_{L^{s' q_0}(\partial\Omega)}$, $\|V\|_{L^s(\partial\Omega)}$ and $\|h\|_{L^s(\partial\Omega)}$, then $K = K\left(N, p, \|u/\partial\Omega\|_{L^{s' q_0}(\partial\Omega)}, \|h\|_{L^s(\partial\Omega)}, \|V\|_{L^s(\partial\Omega)}\right)$.

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