Bol. Soc. Paran. Mat. (3s.) v. 29 1 (2011): 55–66.
©SPM –ISSN-2175-1188 on line ISSN-00378712 in press
SPM: www.spm.uem.br/spm doi:10.5269/bspm.v29i1.11471

Remarks on $\widetilde{g}_\alpha$-irresolute maps

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abstract: Only a few of the class of generalized closed sets form a topology. The class of $\widetilde{g}_\alpha$-closed sets is one among them. The aim of this paper is to introduce the different notions of irresolute function using $\widetilde{g}_\alpha$-closed sets and study some of their basic properties.We also study the relation between strongly $\widetilde{g}_\alpha$- continuous and perfectly $\widetilde{g}_\alpha$-continuous functions.We also introduce $\widetilde{g}_\alpha$-compact and $\widetilde{g}_\alpha$-connected spaces and study their properties using $\widetilde{g}_\alpha$-continuous and $\widetilde{g}_\alpha$-irresolute functions.

Key Words: $^{\#}gs$-closed sets,$^{\#}gs$-open sets , $\widetilde{g}_\alpha$-closed and $\widetilde{g}_\alpha$-open sets.

Contents

1. Introduction

Levine[5] offered a new and useful notion in General Topology, that is the notion of a generalized closed set. This notion has been studied extensively in recent years by many topologists. The investigation of generalized closed sets has led to several new and intersting concepts, e.g. new covering properties and new separation axioms weaker than T_1. Some of these separation axioms have been found to be useful in computer science and digital topology. After the introduction of generalized closed sets there are many research papers which deal with different types of generalized closed sets. Devi et al.[2] and Maki et al.[1] introduced semi-generalised closed sets (briefly sg-closed), generalised semi-closed sets (briefly gs-closed), generalised α-closed (briefly gα-closed) sets and α-generalised closed (briefly αg-closed) sets respectively. Jafari et al.[3] have introduced $\widetilde{g}_\alpha$-closed set and studied their properties using $^{\#}$gs-open[10]. In this paper we have introduced $\widetilde{g}_\alpha$-irresolute functions .Using these new types of functions ,several chracterizations and its properties have been obtained. Also the relationship between strongly $\widetilde{g}_\alpha$-continuous function and perfectly $\widetilde{g}_\alpha$-continuous function have been established.We have also introduced $\widetilde{g}_\alpha$-compact and $\widetilde{g}_\alpha$-connected spaces.

2000 *Mathematics Subject Classification:* 54C08,54D10,54D15

2. preliminaries

We list some definitions which are useful in the following sections. The interior and the closure of a subset A of (X, τ) are denoted by $Int(A)$ and $Cl(A)$, respectively. Throughout the present paper (X, τ) and (Y, σ)(or X and Y) represent non-empty topological spaces on which no separation axiom is defined, unless otherwise mentioned.

Definition 2.1 *A subset A of a topological space $(X, \tau))$ is called*

(i) an ω-closed set [6] $(= \widehat{g} - closed\ )$ if $Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in (X, τ),

*(ii) a *g-closed set [9] if $Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is ω-open in (X, τ) ,*

*(iii)a $^{\#}$g-semi-closed set[10](briefly $^{\#}$gs-closed) if $sCl(A) \subseteq U$ whenever $A \subseteq U$ and U is *g-open in (X, τ) and*

(iv) a $\widetilde{g}_\alpha$-closed [3] if $\alpha Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $^{\#}$gs-open in X.

*The complement of *g-closed (resp ω-closed,$^{\#}$gs-closed,$\widetilde{g}_\alpha$-closed)set is said to be *g-open(resp ω-open,$^{\#}$gs-open,$\widetilde{g}_\alpha$-open)respectively.*

Definition 2.2 *A function $f : (X, \tau) \to (Y, \sigma)$ is called*

(i) strongly continuous[4] if $f^{-1}(V)$ is both open and closed in (X, τ) for every subset set V of (Y, σ)

(ii) $\widetilde{g}_\alpha$-continuous[7] if $f^{-1}(V)$ is $\widetilde{g}_\alpha$-closed for every closed set V in (Y, σ).

Definition 2.3 *Let $f : (X, \tau) \to (Y, \sigma)$ be a map. Then f is said to be*

(i)semi-irresolute[2] if $f^{-1}(V)$ is semi-open in (X, τ) for each semi-open set V of (Y, σ),

(ii) $^{\#}$g-semi-irresolute [10] (briefly $^{\#}$gs-irresolute) if $f^{-1}(V)$ is $^{\#}$gs-closed in (X, τ) for each $^{\#}$gs-closed set V of (Y, σ) and

(iii) α-irrsolute[1] if $f^{-1}(V)$ is α-open in (X, τ) for each α-open set V of (Y, σ)

Definition 2.4 *A map $f : (X, \tau) \to (Y, \sigma)$ is said to be*

(i) $^{\#}$g-semi-closed [10] (briefly $^{\#}$gs-closed) if f(V) is $^{\#}$gs-closed in (Y, σ) for every closed set V of (X, τ),

(ii) pre-$^{\#}$gs-open (resp. pre-$^{\#}$gs-closed) [10] if f(V) is $^{\#}$gs-open (resp.$^{\#}$gs-closed) in (Y, σ) for every $^{\#}$gs-open (resp. $^{\#}$gs-closed) set V of (X, τ),

Remark 2.5 *The class of $\widetilde{g}_\alpha$-closed sets forms a topology.The class of all $\widetilde{g}_\alpha$-open sets and the class of all $\widetilde{g}_\alpha$-closed sets of (X, τ) are denoted by $\widetilde{G}_\alpha O(X)$ and $\widetilde{G}_\alpha C(X)$ respectively.*

3. $\widetilde{g}_\alpha$-irresolute maps

Definition 3.1 *A map $f:(X,\tau)\to(Y,\sigma)$ is called a $\widetilde{g}_\alpha$-irresolute map if the inverse image of every $\widetilde{g}_\alpha$ -closed set in (Y,σ) is $\widetilde{g}_\alpha$ -closed in (X,τ).*

Remark 3.2 *The following examples show that the notion of semi-irresolute functions and $\widetilde{g}_\alpha$ -irresolute functions are independent.*

Example 3.3 *Let $X=Y=\{a,b,c\},\tau=\{\phi,X,\{a\},\{b\},\{a,b\}\}$, $\sigma=\{\phi,Y,\{a\}\}$ $\widetilde{G}_\alpha Cl(X)=\{\phi,X,\{c\},\{a,c\},\{b,c\}\}$, $\widetilde{G}_\alpha Cl(Y)=\{\phi,Y,\{b\},\{c\},\{b,c\}\}$. The function f is the identity function.Then f is semi-irresolute but not $\widetilde{g_\alpha}$ -irresolute.Since for the $\widetilde{g}_\alpha$ -closed set $\{b\}$ in (Y,σ) $f^{-1}(\{b\})=\{b\}$ which is not $\widetilde{g}_\alpha$ -closed in (X,τ)*

Example 3.4 *Let $X=Y=\{a,b,c\},\tau=\{\phi,X,\{a,b\}\}$, $\sigma=\{\phi,Y,\{a\},\{b\},\{a,b\}\}$ $\widetilde{G}_\alpha Cl(X)=\{\phi,X,\{c\},\{a,c\},\{b,c\}\}$, $\widetilde{G}_\alpha Cl(Y)=\{\phi,Y,\{c\},\{a,c\},\{b,c\}\}$. The function f is the identity function.Then f is $\widetilde{g}_\alpha$ -irresolute but not semi-irresolute.Since for the semi-closed set $\{b\}$ in (Y,σ) $f^{-1}(\{b\})=\{b\}$ which is not semi-closed in (X,τ)*

Proposition 3.5 *A map $f:(X,\tau)\to(Y,\sigma)$ is $\widetilde{g}_\alpha$ -irresolute if and only if the inverse image of every $\widetilde{g}_\alpha$ -open set in (Y,σ) is $\widetilde{g}_\alpha$ -open in (X,τ).*

Proof. Let f be $\widetilde{g}_\alpha$-irresolute.Let U be any $\widetilde{g}_\alpha$-open set in (Y,σ) then $f^{-1}(U^c)$ is $\widetilde{g}_\alpha$-closed in (X,τ). Since $f^{-1}(U^c)=(f^{-1}(U))^c$. So $f^{-1}(U)$ is $\widetilde{g}_\alpha$-open in (X,τ). Conversely let U be $\widetilde{g}_\alpha$-closed set in (Y,σ) then $f^{-1}(U^c)$ is $\widetilde{g}_\alpha$-open in (X,τ). Since $f^{-1}(U^c)=(f^{-1}(U))^c$.So f is $\widetilde{g}_\alpha$-irresolute.

Proposition 3.6 *If the map $f:(X,\tau)\to(Y,\sigma)$ is $\widetilde{g}_\alpha$-irresolute, then it is $\widetilde{g}_\alpha$ -continuous but not conversely.*

Proof. Let U be any open set in (Y,σ).Since any open set is $\widetilde{g}_\alpha$-open and f is $\widetilde{g}_\alpha$-irresolute $f^{-1}(U)$ is $\widetilde{g}_\alpha$-open in (X,τ).Therefore f is $\widetilde{g}_\alpha$-continuous.

Remark 3.7 *The converse of Proposition 3.6 need not be true as seen from the following example.*

Example 3.8 *Let $X=Y=\{a,b,c\},\tau=\{\phi,X,\{a\},\{a,b\}\},\sigma=\{\phi,Y,\{a,b\}\}$ $\widetilde{G}_\alpha Cl(X)=\{\phi,X,\{b\},\{c\},\{b,c\}\}$, $\widetilde{G}_\alpha Cl(Y)=\{\phi,Y,\{c\},\{a,c\},\{b,c\}\}$.The function f is the identity function. f is $\widetilde{g}_\alpha$-continuous but not $\widetilde{g}_\alpha$-irresolute.Since for the $\widetilde{g}_\alpha$-closed set $\{a,c\}$ in (Y,σ) $f^{-1}(\{a,c\})=\{a,c\}$ which is not $\widetilde{g}_\alpha$-closed in (X,τ)*

Proposition 3.9 *If $f:(X,\tau)\to(Y,\sigma)$ is $\widetilde{g}_\alpha$-irresolute and H is a $\widetilde{g}_\alpha$ -closed subset of (X,τ), then the restriction $f_{/H}:(H,\tau_H)\to(Y,\sigma)$ is $\widetilde{g}_\alpha$ -irresolute.*

Proof. Let F be any $\widetilde{g}_\alpha$-closed subset of (Y,σ). Since f is $\widetilde{g}_\alpha$-irresolute, $f^{-1}(F)$ is $\widetilde{g}_\alpha$ -closed in (X,τ).Let $f^{-1}(F)\cap H=A$. Then A is $\widetilde{g}_\alpha$-closed in (H,τ_H) as in the proof of Proposition 5.1[7] But $(f_{/H)})^{-1}(F)=f^{-1}(F)\cap H=A$ and so $f_{/H}$ is also $\widetilde{g}_\alpha$ -irresolute.

Proposition 3.10 *If $f:(X,\tau)\to(Y,\sigma)$ is bijective, pre-$^{\#}$gs-open and α-irresolute, then f is $\widetilde{g}_\alpha$-irresolute.*

Proof. Let A be $\widetilde{g}_\alpha$ -closed set in (Y,σ). Let U be any $^{\#}$gs-open set in (X,τ) such that $f^{-1}(A)\subseteq U$. Then $A\subseteq f(U)$. Since A is $\widetilde{g}_\alpha$-closed and f(U) is $^{\#}$gs-open in (Y,σ), $\alpha Cl(A)\subseteq f(U)$ holds and hence $f^{-1}(\alpha Cl(A))\subseteq U$. Since f is α-irresolte and $\alpha Cl(A)$ is α-closed in (Y,σ),$\alpha Cl(f^{-1}(\alpha Cl(A)))\subseteq U$ and so $\alpha Cl(f^{-1}(A))\subseteq U$(since $\alpha Cl(f^{-1}\alpha Cl(A))=f^{-1}(\alpha Cl(A))$). Therefore, $f^{-1}(A)$ is $\widetilde{g}_\alpha$-closed in (X,τ) and hence f is $\widetilde{g}_\alpha$-irresolute.

Proposition 3.11 *If $f:(X,\tau)\to(Y,\sigma)$ is bijective,pre α- closed and $^{\#}$gs-irresolute, then the inverse map $f^{-1}:(Y,\sigma)\to(X,\tau)$ is $\widetilde{g}_\alpha$-irresolute.*

Proof. Let A be $\widetilde{g}_\alpha$-closed in (X,τ). Let $(f^{-1})^{-1}(A)=f(A)\subseteq U$ where U is $^{\#}$gs-open in (Y,σ). Then $A\subseteq f^{-1}(U)$ holds. Since $f^{-1}(U)$ is $^{\#}$gs-open in (X,τ) and A is $\widetilde{g}_\alpha$-closed in (X,τ), $\alpha Cl(A)\subseteq f^{-1}(U)$ and hence $f(\alpha Cl(A))\subseteq U$. Since f is pre-$\alpha$-closed and $\alpha Cl(A)$ is α-closed in (X,τ),$f(\alpha Cl(A))$ is α-closed in (Y,σ) and so $f(\alpha Cl(A))$ is $\widetilde{g}_\alpha$-closed in (Y,σ). Therefore, $\alpha Cl(f(\alpha Cl(A)))\subseteq U$ and hence $\alpha Cl(f(A))\subseteq U$.Thus, f(A) is $\widetilde{g}_\alpha$ -closed in (Y,σ) and so f^{-1} is $\widetilde{g}_\alpha$-irresolute.

Definition 3.12 *A space (X,τ) is called a $^{\#}T_{\widetilde{g}_\alpha}$-space if every $\widetilde{g}_\alpha$-closed set in it is closed in (X,τ).*

Proposition 3.13 *Let (X,τ) be any topological space and (Y,σ) be a $^{\#}T_{\widetilde{g}_\alpha}$ space and $f:(X,\tau)\to(Y,\sigma)$ be a map. Then the following are equivalent:*

(i) f is $\widetilde{g}_\alpha$-irresolute.

(ii) f is $\widetilde{g}_\alpha$-continuous.

Proof.

$(i)\Rightarrow(ii)$.Let f be $\widetilde{g}_\alpha$-irresolute. Let U be a closed set in (Y,σ).Since Y is a $^{\#}T_{\widetilde{g}_\alpha}$-space U is $\widetilde{g}_\alpha$-closed in Y.Since f is $\widetilde{g}_\alpha$-irresolute $f^{-1}(U)$ is $\widetilde{g}_\alpha$-closed in (X,τ).Thus f is $\widetilde{g}_\alpha$-continuous.

$(ii)\Rightarrow(ii)$ Let F be a $\widetilde{g}_\alpha$-closed set in (Y,σ).Since Y is a $^{\#}T_{\widetilde{g}_\alpha}$-space F is closed in Y.By hypothsis $f^{-1}(F)$ is $\widetilde{g}_\alpha$-closed in $(X\tau)$.Therefore f is $\widetilde{g}_\alpha$-irresolute.

4. Strongly $\widetilde{g}_\alpha$-continuous mappings

Definition 4.1 *A map $f:(X,\tau)\to(Y,\sigma)$ is called strongly $\widetilde{g}_\alpha$ -continuous if the inverse image of every $\widetilde{g}_\alpha$ -open set in (Y,σ) is open in (X,τ).*

Proposition 4.2 *If $f:(X,\tau)\to(Y,\sigma)$ is strongly $\widetilde{g}_\alpha$-continuous, then it is continuous but not conversely.*

Proof. Let U be any open set in (Y,σ). Since every open set is $\widetilde{g}_\alpha$-open (by Theorem 3.23[3]) U is $\widetilde{g}_\alpha$-open in (Y,σ).Then $f^{-1}(U)$ is open in (X,τ).Hence f is continuous.

Remark 4.3 *The converse of Proposition 4.2 need not be true as seen from the following example.*

Example 4.4 *Let* $X = Y = \{a, b, c\}, \tau = \{\phi, X, \{a\}\}, \sigma = \{\phi, Y, \{c\}\}$ $\widetilde{G}_\alpha O(Y) = \{\phi, Y, \{c\}, \{a, c\}, \{b, c\}\}$*.The function f is defined as* $f(a) = c, f(b) = a, f(c) = b$*.The function f is continuous but f is not strongly* $\widetilde{g}_\alpha$ *-continuous. Since for the* $\widetilde{g}_\alpha$*-open set* $\{b, c\}$ *in* (Y, σ) $f^{-1}(\{b, c\}) = \{a, c\}$ *which is not open in* (X, τ)

Proposition 4.5 *Let* (X, τ) *be any topological space and* (Y, σ) *be a* $^{\#}T_{\widetilde{g_\alpha}}$*-space and* $f : (X, \tau) \to (Y, \sigma)$ *be a map. Then the following are equivalent:*

(i)f is strongly $\widetilde{g}_\alpha$*-continuous.*

(ii) f is continuous.

Proof.$(i) \Rightarrow (ii)$: Follows from Proposition 4.2.
$(ii) \Rightarrow (i)$: Let U be any $\widetilde{g}_\alpha$ -open set in (Y, σ). Since (Y, σ) is a $^{\#}T_{\widetilde{g}_\alpha}$-space, U is open in (Y, σ) and since f is continuous, we have $f^{-1}(U)$ is open in (X, τ). Therefore, f is strongly $\widetilde{g}_\alpha$-continuous.

Proposition 4.6 *If* $f : (X, \tau) \to (Y, \sigma)$ *is strongly continuous, then it is strongly* $\widetilde{g}_\alpha$*-continuous but not conversely.*

Proof. Let U be a $\widetilde{g}_\alpha$-open set in (Y, σ).Then $f^{-1}(U)$ is both open and closed in (X, τ).Hence f is strongly $\widetilde{g}_\alpha$-continuous.

Remark 4.7 *The converse of Proposition 4.6 need not be true as seen from the following example.*

Example 4.8 *Let* $X = Y = \{a, b, c\}, \tau = \{\phi, X, \{a\}, \{b\}\{a, b\}\}, \sigma = \{\phi, Y, \{a, b\}\}$ $\widetilde{G}_\alpha O(Y) = \{\phi, Y, \{a\}, \{b\}, \{a, b\}\}$*.The function f is the identity function.The function f is strongly* $\widetilde{g}_\alpha$ *-continuous but not strongly coninuous. Since for the subset* $\{a, b\}$ *in* (Y, σ) $f^{-1}(\{a, b\}) = \{a, b\}$ *which is not closed in* (X, τ)

Proposition 4.9 *Let* $f : (X, \tau) \to (Y, \sigma)$ *be a map and both* (X, τ) *and* (Y, σ) *are* $^{\#}T_{\widetilde{g}_\alpha}$*-spaces. Then the following are equivalent:*

(i)f is strongly $\widetilde{g}_\alpha$*-continuous*

(ii)f is continuous

(iii)f is $\widetilde{g}_\alpha$*-irresolute and*

(iv)f is $\widetilde{g}_\alpha$*-continuous.*

Proof. Follows from Proposition 4.2, 4.5 and 4.6.

Proposition 4.10 *A map* $f : (X, \tau) \to (Y, \sigma)$ *is strongly* $\widetilde{g}_\alpha$ *-continuous if and only if the inverse image of every* $\widetilde{g}_\alpha$*-closed set in* (Y, σ) *is closed in* (X, σ)*.*

Proof. Proof is Similar to Proposition 4.5[7].

Proposition 4.11 *If $f : (X,\tau) \to (Y,\sigma)$ and $g : (Y,\sigma) \to (Z,\eta)$ are strongly $\widetilde{g}_\alpha$-continuous, then their composition $gof : (X,\tau) \to (Z,\eta)$ is also strongly $\widetilde{g_\alpha}$ -continuous.*

Proof. Let U be a $\widetilde{g}_\alpha$-open set in (Z,η). Since g is strongly $\widetilde{g}_\alpha$-continuous,$g^{-1}(U)$ is open in (Y,σ). Since $g^{-1}(U)$ is open, it is $\widetilde{g}_\alpha$-open in (Y,σ). As f is also strongly $\widetilde{g}_\alpha$-continuous, $f^{-1}(g^{-1}(U)) = (gof)^{-1}(U)$ is open in (X,τ) and so gof is strongly $\widetilde{g}_\alpha$-continuous.

Proposition 4.12 *Let $f : (X,\tau) \to (Y,\sigma)$ and $g : (Y,\sigma) \to (Z,\eta)$ be any two functions. Then their composition $gof : (X,\tau) \to (Z,\eta)$ is*

(i)strongly $\widetilde{g}_\alpha$-continuous if g is strongly $\widetilde{g}_\alpha$-continuous and f is continuous

(ii)$\widetilde{g}_\alpha$-irresolute if g is strongly $\widetilde{g}_\alpha$ -continuous and f is $\widetilde{g}_\alpha$-continuous (or f is $\widetilde{g}_\alpha$ -irresolute)

(iii) continuous if g is $\widetilde{g}_\alpha$-continuous and f is strongly $\widetilde{g}_\alpha$-continuous.

Proof.

(i)Let U be a $\widetilde{g}_\alpha$-open set in (Z,η). Since g is strongly $\widetilde{g}_\alpha$-continuous,$g^{-1}(U)$ is open in (Y,σ). Since f is continuous, $f^{-1}(g^{-1}(U)) = (gof)^{-1}(U)$ is open in (X,τ) and so gof is strongly $\widetilde{g}_\alpha$-continuous.

(ii)Let U be a $\widetilde{g}_\alpha$-open set in (Z,η). Since g is strongly $\widetilde{g}_\alpha$-continuous,$g^{-1}(U)$ is open in (Y,σ). As f is $\widetilde{g}_\alpha$-continuous, $f^{-1}(g^{-1}(U)) = (gof)^{-1}(U)$ is $\widetilde{g}_\alpha$-open in (X,τ) and so gof is $\widetilde{g}_\alpha$-irresolute.

(iii)Let U be an open set in (Z,η). Since g is $\widetilde{g}_\alpha$ -continuous,$g^{-1}(U)$ is $\widetilde{g}_\alpha$-open in (Y,σ). As f is also strongly $\widetilde{g}_\alpha$-continuous, $f^{-1}(g^{-1}(U)) = (gof)^{-1}(U)$ is open in (X,τ) and so gof is-continuous.

Proposition 4.13 *If $f : (X,\tau) \to (Y,\sigma)$ is strongly $\widetilde{g}_\alpha$-continuous and H is any subset of (X,τ), then the restriction $f_{/H} : (H,\tau_H) \to (Y,\sigma)$ is also strongly $\widetilde{g}_\alpha$-continuous.*

Proof. Let U be any $\widetilde{g}_\alpha$-open subset of (Y,σ).Since f is strongly $\widetilde{g}_\alpha$-continuous $f^{-1}(U)$ is open in (X,τ).Let $f^{-1}(U) \cap H = A$.Then A is $\widetilde{g}_\alpha$-open in $(H,\tau_{/H})$.Since $f_{/H}^{-1}(U) = f^{-1}(U) \cap H = A$ and so $f_{/H}$ is strongly $\widetilde{g}_\alpha$ -continuous.

Definition 4.14 *A map $f : (X,\tau) \to (Y,\sigma)$ is called perfectly $\widetilde{g}_\alpha$-continuous if the inverse image of every $\widetilde{g}_\alpha$-open set in (Y,σ) is both open and closed in (X,τ).*

Proposition 4.15 *If $f : (X,\tau) \to (Y,\sigma)$ is perfectly $\widetilde{g}_\alpha$-continuous, then it is strongly $\widetilde{g}_\alpha$-continuous but not conversely.*

Proof. Since $f : (X, \tau) \to (Y, \sigma)$ is perfectly $\widetilde{g}_\alpha$-continuous, $f^{-1}(U)$ is both open and closed in (X, τ) for every $\widetilde{g}_\alpha$-open set U in (Y, σ). Therefore, f is strongly $\widetilde{g}_\alpha$-continuous.

Remark 4.16 *The converse of Proposition 4.15 need not be true as seen from the following example.*

Example 4.17 *Let* $X = Y = \{a, b, c\}, \tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}, \sigma = \{\phi, Y, \{a, b\}\}$
$\widetilde{G}_\alpha O(X) = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$.
$\widetilde{G}_\alpha O(Y) = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$. *The function f is the identity function.f is strongly* $\widetilde{g}_\alpha$ *-continuous but not perfectly* $\widetilde{g}_\alpha$*-continuous. Since for the* $\widetilde{g}_\alpha$*-open set* $\{a\}$ *in* (Y, σ) $f^{-1}(\{a\}) = \{a\}$ *which is not closed in* (X, τ)

Proposition 4.18 *If* $f : (X, \tau) \to (Y, \sigma)$ *is strongly continuous, then it is perfectly* $\widetilde{g}_\alpha$*-continuous but not conversely.*

Proof. Since $f : (X, \tau) \to (Y, \sigma)$ is strongly continuous, $f^{-1}(U)$ is both open and closed in (X, τ), for every $\widetilde{g}_\alpha$-open set U in (Y, σ). Therefore, f is perfectly $\widetilde{g}_\alpha$-continuous.

Remark 4.19 *The converse of Proposition 4.18 need not be true as seen from the following example.*

Example 4.20 *Let* $X = Y = \{a, b, c\}, \tau = \{\phi, X, \{a\}, \{b, c\}\} = \sigma$ $\widetilde{G}_\alpha O(X) = \{\phi, X, \{a\}, \{b, c\} = \widetilde{G}_\alpha O(Y)$. *The function f is the identity function.f is perfectly* $\widetilde{g}_\alpha$ *-continuous but not strongly-continuous. Since for the subset* $\{a, b\}$ *in* (Y, σ) $f^{-1}(\{a, b\}) = \{a, b\}$ *which is not closed or open in* (X, τ)

Proposition 4.21 *Let* (X, τ) *be a discrete topological space,* (Y, σ) *be any topological space and* $f : (X, \tau) \to (Y, \sigma)$ *be a map. Then the following statements are equivalent:*

(i)f is perfectly $\widetilde{g}_\alpha$*-continuous.*

(ii)f is strongly $\widetilde{g}_\alpha$*-continuous.*

Proof.$(i) \Rightarrow (ii)$ Let U be any $\widetilde{g}_\alpha$-open set in (Y, σ) then $f^{-1}(U)$ is both and closed in (X, τ).
$(ii) \Rightarrow (i)$ Let U be any $\widetilde{g}_\alpha$ -open set in (Y, σ). By hypothesis $f^{-1}(U)$ is open in (X, τ). Since (X, τ) is a discrete space, $f^{-1}(U)$ is also closed in (X, τ).$f^{-1}(U)$ is both open and closed in (X, τ) and so f is perfectly $\widetilde{g}_\alpha$-continuous

Proposition 4.22 *A map* $f : (X, \tau) \to (Y, \sigma)$ *is perfectly* $\widetilde{g}_\alpha$*-continuous if and only if the inverse image of every* $\widetilde{g}_\alpha$*-closed set in* (Y, σ) *is both open and closed in* (X, τ).

Proof. Let U be any $\widetilde{g}_{\alpha}$- closed set in (Y, σ) ,since f is perfectly $\widetilde{g}_{\alpha}$-continuous then $f^{-1}(U^c)$ is both open and closed in (Y, σ).$f^{-1}(U^c) = (f^{-1}(U))^c$. Conversely let U be any $\widetilde{g}_{\alpha}$-closed set in (Y, σ).Since $f^{-1}(U^c) = (f^{-1}(U))^c$ is both and closed in (X, τ).

Proposition 4.23 *If $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \eta)$ are perfectly $\widetilde{g}_{\alpha}$-continuous, then their composition $gof : (X, \tau) \to (Z, \eta)$ is also perfectly $\widetilde{g}_{\alpha}$-continuous.*

Proof. Let U be any $\widetilde{g}_{\alpha}$-open set in (Z, η) then $g^{-1}(U)$ is both open and closed in (Y, σ).Since any open (closed) set is $\widetilde{g}_{\alpha}$-open($\widetilde{g}_{\alpha}$-closed)set in (X, τ) and f is perfefctly $\widetilde{g}_{\alpha}$-continuous. $(gof)^{-1}(U) = f^{-1}(g^{-1}(U)$ is both and closed in (X, τ). Hence gof is perfectly continuous.

Proposition 4.24 *Let $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \eta)$ be any two functions. Then their composition $gof : (X, \tau) \to (Z, \eta)$ is*

(i)$\widetilde{g}_{\alpha}$-continuous if g is strongly continuous and f is $\widetilde{g}_{\alpha}$-continuous

(ii)$\widetilde{g}_{\alpha}$-irresolute if g is perfectly $\widetilde{g}_{\alpha}$ -continuous and f is $\widetilde{g}_{\alpha}$-continuous (or f is $\widetilde{g_{\alpha}}$ -irresolute)

(iii)strongly $\widetilde{g_{\alpha}}$ -continuous if g is perfectly $\widetilde{g_{\alpha}}$ -continuous and f is continuous (or f is strongly -continuous).

(iv) perfectly $\widetilde{g_{\alpha}}$ -continuous if g is strongly continuous and f is perfectly $\widetilde{g_{\alpha}}$ -continuous.

Proof.

(i)Let U be any open set in (Z, η) then $g^{-1}(U)$ is both open and closed in (Y, σ).Hence $f^{-1}(g^{-1}(U))$ is $\widetilde{g}_{\alpha}$-open in (X, τ).Thus gof is $\widetilde{g}_{\alpha}$-continuous.

(ii)Let U be any $\widetilde{g}_{\alpha}$-open set in (Z, η) then $g^{-1}(U)$ is open or closed in (Y, σ).Since f is $\widetilde{g}_{\alpha}$-continuos $f^{-1}(g^{-1}(U))$ is $\widetilde{g}_{\alpha}$-closed or $\widetilde{g}_{\alpha}$-open in (X, τ).Thus gof is $\widetilde{g}_{\alpha}$-irresolute.

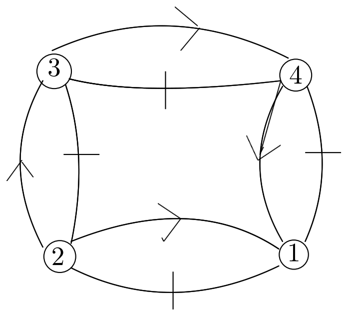
(iii)Let U be any $\widetilde{g}_{\alpha}$-open set in (Z, η).Then $g^{-1}(U)$ is both open and closed in (Y, σ) and hence $f^{-1}(g^{-1}(U))$ is open and closed in (X, τ) and hence gof is strongly $\widetilde{g}_{\alpha}$-continuous.

(iv)Let U be any $\widetilde{g}_{\alpha}$-open set in (Z, η) then $g^{-1}(U)$ is open and closed in (Y, σ) which is $\widetilde{g}_{\alpha}$-open in (Y, σ) then $f^{-1}(g^{-1}(U))$ is both open and closed in $(X\tau)$.Hence gof is perfectly $\widetilde{g}_{\alpha}$-continuous.

Proposition 4.25 *If $f : (X, \tau) \to (Y, \sigma)$ is perfectly $\widetilde{g}_{\alpha}$ -continuous and H is any subset of (X, τ), then the restriction $f_{/H} : (H, \tau_H) \to (Y, \sigma)$ is also perfectly $\widetilde{g}_{\alpha}$-continuous.*

Proof. Let A be any $\widetilde{g}_\alpha$-closed set in (Y,σ).Since f is perfectly $\widetilde{g}_\alpha$-continuous $f^{-1}(A)$ is both open and closed in (X,τ).Let $f^{-1}(A)\cap H = B$. Then B is both open and closed in (H,τ_H). But $(f_{/H})^{-1}(A) = f^{-1}(A)\cap H = B$.So $f_{/H}$ is also perfectly $\widetilde{g}_\alpha$-continuous.

Remark 4.26 *From the above discussions we have the following diagram where $A \to B$ represents A implies B but not coversely.*
(1)Continuity (2) strongly coninuity
(3)perfectly $\widetilde{g}_\alpha$-continuity (4)strongly $\widetilde{g}_\alpha$-continuity.



5. Applications

In this section we introduce $\widetilde{g}_\alpha$-compact and $\widetilde{g}_\alpha$-conected spaces and study their basic properties

Definition 5.1 *A collection $\{A_i : i \in \Lambda\}$ of $\widetilde{g}_\alpha$-open sets in a topological space (X,τ) is called a $\widetilde{g}_\alpha$-open cover of a subset A in (X,τ) if $A \subseteq \bigcup_{i\in\Lambda} A_i$.*

Definition 5.2 *A topological space (X,τ) is called $\widetilde{g}_\alpha$-compact if every $\widetilde{g}_\alpha$-open cover of (X,τ) has a finite $\widetilde{g}_\alpha$ -subcover.*

Definition 5.3 *A subset A of a topological spaces (X,τ) is called $\widetilde{g}_\alpha$-compact relative to (X,τ), if for every collection $\{A_i : i \in \Lambda\}$ of $\widetilde{g}_\alpha$-open subsets of (X,τ) such that $A \subseteq \bigcup_{i\in\Lambda} A_i$, there exists a finite subset Λ_0 of Λ such that $A \subseteq \bigcup_{i\in\Lambda_0} A_i$.*

Definition 5.4 *A subset A of a topological space (X,τ) is called $\widetilde{g}_\alpha$-compact if A is $\widetilde{g}_\alpha$-compact as a subspace of (X,τ).*

Proposition 5.5 *A $\widetilde{g}_\alpha$-closed subset of a $\widetilde{g}_\alpha$-compact space is $\widetilde{g}_\alpha$-compact relative to (X,τ).*

Proof. Let A be a $\widetilde{g}_\alpha$-closed subset of a $\widetilde{g}_\alpha$-compact spaces (X,τ). Then A^c is $\widetilde{g}_\alpha$-open in (X,τ). Let E be a $\widetilde{g}_\alpha$-open cover of A in (X,τ). Therefore, E along with A^c form a $\widetilde{g}_\alpha$-open cover of (X,τ). Since (X,τ) is $\widetilde{g}_\alpha$-compact, it has a finite sub cover, say $\{V_1, V_2, , V_n\}$. If this subcover contains A^c, we discard it. Otherwise leave the subcover as it is. Thus, we obtained a finite subcover of A and so A is $\widetilde{g}_\alpha$ -compact relative to (X,τ).

Proposition 5.6 *A $\widetilde{g}_\alpha$-continuous image of a $\widetilde{g}_\alpha$-compact space is compact.*

Proof. Let $f:(X,\tau)\to(Y,\sigma)$ be a $\widetilde{g}_\alpha$-continuous onto map, where (X,τ) is a $\widetilde{g}_\alpha$-compact space. Let $\{A_i : i\in\Lambda\}$ be an open cover of (Y,σ). Then $\{f^{-1}(A_i) : i\in\Lambda\}$ is a $\widetilde{g}_\alpha$-open cover of (X,τ). Since (X,τ) is $\widetilde{g}_\alpha$-compact, it has a finite subcover, say $\{f^{-1}(A_1), f^{-1}(A_2), ..f^{-1}(A_n)\}$. Since f is onto,$\{A_1, A_2, A_n\}$ is an open cover of (Y,σ) and so (Y,σ) is compact.

Proposition 5.7 *If a map $f:(X,\tau)\to(Y,\sigma)$ is $\widetilde{g}_\alpha$-irresolute and a subset B is $\widetilde{g}_\alpha$-compact relative to (X,τ), then the image f(B) is $\widetilde{g}_\alpha$-compact relative to (Y,σ).*

Proof. Let $\{A_i : i\in\Lambda\}$ be any collection of $\widetilde{g}_\alpha$-open sets of (Y,σ) such that $f(B)\subseteq\bigcup_{i\in\Lambda}A_i$. Then $B\subseteq\bigcup_{i\in\Lambda}f^{-1}(A_i)$. By hypothesis, there exists a finite subset of Λ_0 of Λ such that $B\subseteq_{i\in\Lambda_0} f^{-1}(Ai)$. Therefore $f(B)\subseteq\bigcup_{i\in\Lambda}A_i$ and so f(B) is $\widetilde{g}_\alpha$-compact relative to (Y,σ).

Proposition 5.8 *If $f:(X,\tau)\to(Y,\sigma)$ is a strongly $\widetilde{g}_\alpha$-continuous onto map where (X,τ) is a compact space, then (Y,σ) is $\widetilde{g}_\alpha$ -compact.*

Proof. Let $\{A_i : i\in\Lambda\}$ be a $\widetilde{g}_\alpha$-open cover of (Y,σ). Then $\{f^{-1}(A_i) : i\in\Lambda\}$ is an open cover of (X,τ), since f is strongly $\widetilde{g}_\alpha$-continuous. Since (X,τ) is compact, it has a finite subcover say, $\{f^{-1}(A_1), f^{-1}(A_2) f^{-1}(A_n)\}$ and since f is onto,$\{A_1, A_2, .A_n\}$ is a finite subcover of (Y,σ) and therefore (Y,σ) is $\widetilde{g}_\alpha$-compact.

Corollary 5.9 *If a map $f:(X,\tau)\to(Y,\sigma)$ is a perfectly $\widetilde{g}_\alpha$-continuous onto map, where (X,τ) is compact, then (Y,σ) is $\widetilde{g}_\alpha$-compact.*

Proof. Since every perfectly $\widetilde{g}_\alpha$-continuous function is strongly $\widetilde{g}_\alpha$-continuous and therefore the result follows from Proposition 5.8.

Definition 5.10 *A topological space (X,τ) is called a $\widetilde{g}_\alpha$-connected space if (X,τ) cannot be written as a disjoint union of two non-empty $\widetilde{g}_\alpha$ -open sets. A subset of (X,τ) is $\widetilde{g}_\alpha$-connected if it is $\widetilde{g}_\alpha$-connected as subspace of (X,τ).*

Theorem 5.11 *For a topological space (X,τ), the following are equivalent:*

(i) *(X,τ) is $\widetilde{g}_\alpha$-connected.*

(ii) *The only subsets of (X,τ) which are both $\widetilde{g}_\alpha$-open and $\widetilde{g}_\alpha$-closed are the empty set and X.*

(iii) *Each $\widetilde{g}_\alpha$-continuous map of (X,τ) into a discrete space (Y,σ) with at least two points is a constant map.*

Proof.$(i) \Rightarrow (ii)$: Let U be a $\widetilde{g}_\alpha$-open and $\widetilde{g}_\alpha$-closed subset of (X, τ). Then U^c is both $\widetilde{g}_\alpha$-open and $\widetilde{g}_\alpha$-closed in (X, τ). Since (X, τ) is the disjoint union of the $\widetilde{g}_\alpha$-open sets U and U^c, by assumption one of these must be empty. i.e.,$U = \phi$ or $U = X$.
$(ii) \Rightarrow (i)$: Suppose that $X = A \cup B$ where A and B are disjoint non-empty $\widetilde{g}_\alpha$-open subsets of (X, τ). Then A is both $\widetilde{g}_\alpha$-open and $\widetilde{g}_\alpha$ -closed subset of (X, τ) and therefore by assumption, $A = \phi$ or $A = X$. Thus, (X, τ) is $\widetilde{g}_\alpha$-connected.
$(ii) \Rightarrow (iii)$: Let $f : (X, \tau) \to (Y, \sigma)$ be a $\widetilde{g}_\alpha$-continuous map. Then (X, τ) is covered by $\widetilde{g}_\alpha$-open and $\widetilde{g}_\alpha$-closed covering $\{f^{-1}(y) : y \in Y\}$. By assumption $f^{-1}(y) = \phi$ or $f^{-1}(y) = X$ for each $y \in Y$. If $f^{-1}(y) = \phi$ for each $y \in Y$, then f fails to be a map. Therefore, there exists atleast one point say, $y_1 \in Y$ such that $f^{-1}(y_1) \neq \phi$ and hence $f(y_1) = X$, which shows that f is a constant map.
$(iii) \Rightarrow (ii)$: Let U be both $\widetilde{g}_\alpha$-open and $\widetilde{g}_\alpha$-closed in (X, τ). Suppose that $U \neq \phi$. Define $f : (X, \tau) \to (Y, \sigma)$ by $f(U) = \{y_1\}$ and $f(U^c) = \{y_2\}$ for some distinct points y_1 and y_2 in (Y, σ), then f is a $\widetilde{g}_\alpha$-continuous map. Therefore, by assumption f is a constant map. Therefore, $y_1 = y_2$ and so $U = X$.

Proposition 5.12 *Every $\widetilde{g}_\alpha$-connected space is connected but not conversely.*

Proof. Let (X, τ) be a $\widetilde{g}_\alpha$-connected space. Suppose that (X, τ) is not connected.Then $X = A \cup B$ where A and B are disjoint nonempty open sets in (X, τ). By Proposition , A and B are $\widetilde{g}_\alpha$-open and $X = A \cup B$, where A and B are disjoint nonempty and $\widetilde{g}_\alpha$-open sets in (X, τ). This contradicts the fact that (X, τ) is $\widetilde{g}_\alpha$-connected and so (X, τ) is connected.

Example 5.13 *Let $X = \{a, b, c\}$ and $\tau = \{\phi, X\}$. Then (X, τ) is a connected space but not a $\widetilde{g}_\alpha$-connected space, because $X = \{a\} \cup \{b, c\}$, where $\{a\}$ and $\{b, c\}$ are $\widetilde{g}_\alpha$-open sets in (X, τ).*

Proposition 5.14 *If (X, τ) is a $^{\#}T_{\widetilde{g}_\alpha}$-space and connected, then (X, τ) is $\widetilde{g}_\alpha$-connected.*

Proof. Suppose X is not $\widetilde{g}_\alpha$-connected let A and B are two non empty disjoint $\widetilde{g}_\alpha$-open subsets of X such that $X = A \cup B$.Since X is a $^{\#}T_{\widetilde{g}_\alpha}$-space A and B are open which is a contradiction to our assumption that X is connected. Hence X is $\widetilde{g}_\alpha$-connected.

Proposition 5.15 *If $f : (X, \tau) \to (Y, \sigma)$ is a $\widetilde{g}_\alpha$-continuous surjection and (X, τ) is $\widetilde{g}_\alpha$-connected, then (Y, σ) is connected.*

Proof. Suppose that $Y = A \cup B$, where A and B are disjoint nonempty $\widetilde{g}_\alpha$-open sets of (Y, σ). Since f is a $\widetilde{g}_\alpha$-continuous and onto, $X = f^{-1}(A) \cup f^{-1}(B)$ where, $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint nonempty $\widetilde{g}_\alpha$-open sets in (X, τ). This contradicts the fact that (X, τ) is $\widetilde{g}_\alpha$-connected and so (Y, σ) is connected.

Proposition 5.16 *If $f : (X, \tau) \to (Y, \sigma)$ is a $\widetilde{g}_\alpha$-irresolute surjection and (X, τ) is $\widetilde{g}_\alpha$-connected, then (Y, σ) is $\widetilde{g}_\alpha$-connected.*

Proof. Similar to Proposition 5.15.

Proposition 5.17 *If* $f : (X, \tau) \to (Y, \sigma)$ *is strongly* $\widetilde{g}_\alpha$*-continuous onto map,where* (X, τ) *is a connected space, then* (Y, σ) *is* $\widetilde{g}_\alpha$ *-connected.*

Proof. Similar to Proposition 5.15.

Proposition 5.18 *If* $f : (X, \tau) \to (Y, \sigma)$ *is a* $\widetilde{g}_\alpha$*-continuous map, then f(H) is a connected subset of* (Y, σ) *for every* $\widetilde{g}_\alpha$*-closed and* $\widetilde{g}_\alpha$*-connected subset H of* (X, τ).

Proof. The restriction f_H of f to H is $\widetilde{g}_\alpha$-continuous [7]. By Proposition 5.15, the image of the $\widetilde{g}_\alpha$-connected space (H, τ_H) under $f_H : (H, \tau_H) \to (f(H), \sigma_{f(H)})$ is connected. Therefore $(f(H), \sigma_{f(H)})$ is connected.Thus, f(H) is a connected subset of (Y, σ).

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