



Note on contra $\delta\hat{g}$ -continuous functions

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ABSTRACT: In this paper we introduce and investigate some classes of generalized functions called contra- $\delta\hat{g}$ -continuous functions. We obtain several characterizations and some of their properties. Also we investigate its relationship with other types of functions. Finally we introduce two new spaces called $\delta\hat{g}$ -Hausdorff spaces and $\delta\hat{g}$ -normal spaces and obtain some new results.

Key Words: $\delta\hat{g}$ -closed, $\delta\hat{g}$ -continuous, $\delta\hat{g}$ -irresolute, δg -continuous.

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1. Introduction

Ganster and Reilly [5] introduced and studied the notion of LC-continuous functions. Dontchev [3] presented a new notion of continuous function called contra-continuity. This notion is a stronger form of LC-continuity. Dontchev and Noiri [4] introduced a weaker form of contra-continuity called contra-semi-continuity. The purpose of this present paper is to define a new class of generalised continuous functions called contra- $\delta\hat{g}$ -continuous functions and investigate their relationships to other functions. We further introduce and study two new spaces called $\delta\hat{g}$ -Hausdorff spaces and $\delta\hat{g}$ -normal spaces and obtain some new results.

2. Preliminaries

Throughout this paper (X, τ) and (Y, σ) and (Z, η) represent non-empty topological spaces on which no separation axioms are assumed unless or otherwise mentioned. For a subset A of X , $\text{cl}(A)$, $\text{int}(A)$ and A^c denote the closure of A , the interior of A and the complement of A respectively. Let us recall the following definitions, which are useful in the sequel.

Definition 2.1. [12] *The δ -interior of a subset A of X is the union of all regular open set of X contained in A and is denoted by $\text{int}_\delta(A)$. The subset A is called δ -open if $A = \text{int}_\delta(A)$, i.e. a set is δ -open if it is the union of regular open sets.*

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The complement of a δ -open is called δ -closed. Alternatively, a set $A \subseteq (X, \tau)$ is called δ -closed if $A = cl_\delta(A)$, where $cl_\delta(A) = \{x \in X: int(cl(U)) \cap A \neq \emptyset, U \in \tau \text{ and } x \in U\}$.

Definition 2.2. A subset A of (X, τ) is called

- (i) generalized closed (briefly g -closed) set [6] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open set in (X, τ) .
- (ii) semi-generalized closed (briefly sg -closed) set [2] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is a semi-open set in (X, τ) .
- (iii) $\hat{g}$ -closed set [11] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is a semi-open set in (X, τ) .
- (iv) $\alpha\hat{g}$ -closed (briefly $\alpha\hat{g}$ -closed) set [1] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is a $\hat{g}$ -open set in (X, τ) .
- (v) $\delta\hat{g}$ -closed set [7] if $cl_\delta(A) \subseteq U$ whenever $A \subseteq U$ and U is a $\hat{g}$ -open set in (X, τ) .

The complement of a g -closed (resp. sg -closed, $\hat{g}$ -closed, $\alpha\hat{g}$ -closed and $\delta\hat{g}$ -closed) set is called g -open (resp. sg -open, $\hat{g}$ -open, $\alpha\hat{g}$ -open and $\delta\hat{g}$ -open).

Definition 2.3. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called

- (i) sg -continuous [6] if $f^{-1}(V)$ is sg -closed in (X, τ) for every closed set V of (Y, σ) .
- (ii) $\alpha\hat{g}$ -continuous [9] if $f^{-1}(V)$ is $\alpha\hat{g}$ -closed in (X, τ) for every closed set V of (Y, σ) .
- (iii) $\delta\hat{g}$ -continuous [8] if $f^{-1}(V)$ is $\delta\hat{g}$ -closed in (X, τ) for every closed set V of (Y, σ) .
- (iv) $\delta\hat{g}$ -irresolute [8] if $f^{-1}(V)$ is $\delta\hat{g}$ -closed in (X, τ) for every $\delta\hat{g}$ closed set V of (Y, σ) .
- (v) $\alpha\hat{g}$ -closed [1] if the image of every closed set in (X, τ) is $\alpha\hat{g}$ -closed in (Y, σ) .
- (iii) $\delta\hat{g}$ -closed [8] if the image of every closed set in (X, τ) is $\delta\hat{g}$ -closed in (Y, σ) .
- (iii) Weakly $\delta\hat{g}$ -closed [8] (resp. weakly $\delta\hat{g}$ -open) if the image of every δ -closed (resp. δ -open) set in (X, τ) is $\delta\hat{g}$ -closed (resp. $\delta\hat{g}$ -open) set in (Y, σ) .

Definition 2.4. Recall that a function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called

- (i) contra-continuous [3] if $f^{-1}(V)$ is closed in (X, τ) for every open set V in (Y, σ) .

- (ii) contra-sg-continuous [4] if $f^{-1}(V)$ is sg-closed in (X, τ) for every open set V of (Y, σ) .
- (iii) contra- $\alpha\hat{g}$ -continuous [9] if $f^{-1}(V)$ is $\alpha\hat{g}$ -closed in (X, τ) for every open set V in (Y, σ) .

Definition 2.5. [7] A space (X, τ) is called $\hat{T}_{3/4}$ -space if every $\delta\hat{g}$ -Closed set in it is δ -closed.

Definition 2.6. Ultra normal [10] if each pair of nonempty disjoint closed sets can be separated by disjoint clopen sets.

3. Contra- $\delta\hat{g}$ -continuous

We introduce the following definition.

Definition 3.1. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called contra- $\delta\hat{g}$ -continuous if $f^{-1}(V)$ is $\delta\hat{g}$ -closed in (X, τ) for every open set V in (Y, σ) .

Example 3.2. Let $X = \{a, b, c\} = Y$ with topologies $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ and, $\sigma = \{\phi, \{a, b\}, Y\}$. Define $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = c$, $f(b) = b$ and $f(c) = a$. Clearly f is contra- $\delta\hat{g}$ -continuous function.

Note 3.3. The family of all $\delta\hat{g}$ -open sets of (X, τ) is denoted by $\delta\hat{g}O(X)$. The set $\delta\hat{g}O(X, x) = \{V \in \delta\hat{g}O(X) / x \in V\}$ for $x \in X$.

Remark 3.4. For a subset A of (X, τ) , $cl_\delta(A^c) = (int_\delta(A))^c$.

Theorem 3.5. $A \subseteq X$ is $\delta\hat{g}$ -open if and only if $F \subseteq int_\delta(A)$ whenever F is $\hat{g}$ -closed and $F \subseteq A$.

Proof. Necessity. Let A be an $\delta\hat{g}$ -open set in (X, τ) . Let F be $\hat{g}$ -closed such that $F \subseteq A$. Then $A^c \subseteq F^c$ where F^c is $\hat{g}$ -open. A^c is $\delta\hat{g}$ -closed implies that $cl_\delta(A^c) \subseteq F^c$. By Remark 3.4, $(int_\delta(A))^c \subseteq F^c$. That is $F \subseteq int_\delta(A)$.

Sufficiency. Suppose F is $\hat{g}$ -closed and $F \subseteq A$ implies $F \subseteq int_\delta(A)$. Let $A^c \subseteq U$ where U is $\hat{g}$ -open. Then $U^c \subseteq A$ where U^c is $\hat{g}$ -closed. By hypothesis, $U^c \subseteq int_\delta(A)$. That is $(int_\delta(A))^c \subseteq U$. By Remark 3.4, $cl_\delta(A^c) \subseteq U$. This implies A^c is $\delta\hat{g}$ -closed. Hence A is $\delta\hat{g}$ -open.

Proposition 3.6. The product of two $\delta\hat{g}$ -open sets of two spaces is $\delta\hat{g}$ -open set in the product space.

Proof: Let A and B be $\delta\hat{g}$ -open sets of two spaces (X, τ) and (Y, σ) respectively and $V = A \times B \subseteq X \times Y$. Let $F \subseteq V$ be a $\hat{g}$ -closed set in $X \times Y$, then there exists two $\hat{g}$ -closed sets $F_1 \subseteq A$ and $F_2 \subseteq B$. So, $F_1 \subseteq int_\delta(A)$ and $F_2 \subseteq int_\delta(B)$. Hence $F_1 \times F_2 \subseteq A \times B$ and $F_1 \times F_2 \subseteq int_\delta(A) \times int_\delta(B) = int_\delta(A \times B)$. Therefore $A \times B$ is $\delta\hat{g}$ -open subset of a space $X \times Y$. $\square$

Theorem 3.7. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a map. Then the following are equivalent.

- (i) f is contra- $\delta\hat{g}$ -continuous.
- (ii) The inverse image of each closed set in (Y, σ) is $\delta\hat{g}$ -open in (X, τ) .
- (iii) For each $x \in X$ and each $F \in C(Y, f(x))$, there exists $U \in \delta\hat{g}O(X, x)$ such that $f(U) \subset F$.

Proof: (i) $\Rightarrow$ (ii), (ii) $\Rightarrow$ (i) and (ii) $\Rightarrow$ (iii) are obvious.

(iii) $\Rightarrow$ (ii) Let F be any closed set of (Y, σ) and $x \in f^{-1}(F)$. Then $f(x) \in F$ and there exists $U_x \in \delta\hat{g}O(X, x)$ such that $f(U_x) \subset F$. Hence we obtain $f^{-1}(F) = U\{U_x/x \in f^{-1}(F)\} \in \delta\hat{g}O(X)$. Thus the inverse of each closed set in (Y, σ) is $\delta\hat{g}$ -open in (X, τ) . $\square$

Remark 3.8. The concept of $\delta\hat{g}$ -continuity and contra- $\delta\hat{g}$ -continuity are independent as shown in the following example.

Example 3.9. Let $X = \{a, b, c\} = Y$ with topologies $\tau = \{\phi, \{b\}, \{a, b\}, X\}$, $\sigma = \{\phi, \{b\}, Y\}$ respectively. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity function. Clearly f is $\delta\hat{g}$ -continuous function, but f is not contra- $\delta\hat{g}$ -continuous because $f^{-1}(\{b\}) = \{b\}$ is not $\delta\hat{g}$ -closed in (X, τ) where $\{b\}$ is open in (Y, σ) .

Example 3.10. Let $X = \{a, b, c\} = Y$ with topologies $\tau = \{\phi, \{a\}, \{c\}, \{a, c\}, X\}$, $\sigma = \{\phi, \{a, c\}, Y\}$ respectively. Define a function $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = c$, $f(b) = a$, $f(c) = b$. Clearly f is contra- $\delta\hat{g}$ -continuous function, but f is not $\delta\hat{g}$ -continuous because $f^{-1}(\{b\}) = \{c\}$ is not $\delta\hat{g}$ -closed in (X, τ) where $\{b\}$ is closed in (Y, σ) .

Remark 3.11. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is $\delta\hat{g}$ -continuous if for each $x \in X$ and each open set V of Y containing $f(x)$, there exists $U \in \delta\hat{g}O(X, x)$ such that $f(U) \subset V$.

Theorem 3.12. If a function $f : (X, \tau) \rightarrow (Y, \sigma)$ is contra- $\delta\hat{g}$ -continuous and (Y, σ) is regular then f is $\delta\hat{g}$ -continuous.

Proof: Let x be an arbitrary point of (X, τ) and V be an open set of (Y, σ) containing $f(x)$. Since (Y, σ) is regular, there exists an open set W of (Y, σ) containing $f(x)$ such that $cl(W) \subset V$. Since f is contra- $\delta\hat{g}$ -continuous, by theorem 3.7, there exists $U \in \delta\hat{g}O(X, x)$ such that $f(U) \subset cl(W)$. Then $f(U) \subset cl(W) \subset V$. Hence by Remark 3.11, f is $\delta\hat{g}$ -continuous. $\square$

Theorem 3.13. Every contra- $\delta\hat{g}$ -continuous function is contra- $\alpha\hat{g}$ -continuous.

Proof: Let V be open set in (Y, σ) . Since f is contra- $\delta\hat{g}$ -continuous function, $f^{-1}(V)$ is $\delta\hat{g}$ -closed in (X, τ) . Every $\delta\hat{g}$ -closed set is $\alpha\hat{g}$ -closed. Hence $f^{-1}(V)$ is $\alpha\hat{g}$ -closed in (X, τ) . Thus f is contra- $\alpha\hat{g}$ -continuous. $\square$

Remark 3.14. The converse of theorem 3.13 need not be true as shown in the following example.

Example 3.15. Let $X = \{a, b, c\} = Y$ with topologies $\tau = \{\phi, \{a\}, \{a, b\}, \{a, c\}, X\}$ and $\sigma = \{\phi, \{b\}, \{b, c\}, Y\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity function. Clearly f is contra- $\alpha\hat{g}$ -continuous. But f is not contra- $\delta\hat{g}$ -continuous because $f^{-1}(\{b\}) = \{b\}$ is not $\delta\hat{g}$ -closed in (X, τ) where $\{b\}$ is open in (Y, σ) .

Remark 3.16. The concept of contra-continuous and contra- $\delta\hat{g}$ -continuous are independent as shown in the following examples.

Example 3.17. Let $X = \{a, b, c\} = Y$ with topologies $\tau = \{\phi, \{a\}, \{a, b\}, X\}$ and $\sigma = \{\phi, \{b\}, \{a, b\}, \{b, c\}, Y\}$. Define a function $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = a$, $f(b) = c$, $f(c) = b$. Then clearly f is contra- $\delta\hat{g}$ -continuous but f is not contra-continuous because $f^{-1}(\{a, b\}) = \{a, c\}$ is not closed in (X, τ) but $\{a, b\}$ is open in (Y, σ) .

Example 3.18. Let $X = \{a, b, c\} = Y$ with topologies $\tau = \{\phi, \{c\}, \{a, c\}, \{b, c\}, X\}$ and $\sigma = \{\phi, \{a\}, \{b\}, \{a, b\}, Y\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity function. Then clearly f is contra-continuous. But f is not contra- $\delta\hat{g}$ -continuous because $f^{-1}(\{b\}) = \{b\}$ is not $\delta\hat{g}$ -closed in (X, τ) where $\{b\}$ is open in (Y, σ) .

Remark 3.19. The concept of contra- $\delta\hat{g}$ -continuous and contra-sg-continuous are independent of each other as shown in the following examples.

Example 3.20. Let $X = \{a, b, c\} = Y$ with topologies $\tau = \{\phi, \{a\}, \{c\}, \{a, c\}, X\}$ and $\sigma = \{\phi, \{a\}, Y\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity function. f is not contra- $\delta\hat{g}$ -continuous because $f^{-1}(\{a\}) = \{a\}$ is not $\delta\hat{g}$ -closed in (X, τ) where $\{a\}$ is open in (Y, σ) . However f is contra-sg-continuous.

Example 3.21. Let $X = \{a, b, c\} = Y$ with topologies $\tau = \{\phi, \{a\}, \{a, c\}, X\}$ and $\sigma = \{\phi, \{a, b\}, Y\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity function. Then clearly f is contra- $\delta\hat{g}$ -continuous but f is not contra-sg-continuous because $f^{-1}(\{a, b\}) = \{a, b\}$ is not sg-closed in (X, τ) where $\{a, b\}$ is open in (Y, σ) .

Remark 3.22. The composition of two contra- $\delta\hat{g}$ -continuous functions need not be contra- $\delta\hat{g}$ -continuous as the following example shows.

Example 3.23. Let $X = \{a, b, c\} = Y = Z$, $\tau = \{\phi, \{c\}, \{a, b\}, X\}$ and $\sigma = \{\phi, \{c\}, Y\}$, $\eta = \{\phi, \{a, c\}, Z\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \eta)$ be two identity functions. Then both f and g are contra- $\delta\hat{g}$ -continuous but $g \circ f : (X, \tau) \rightarrow (Z, \eta)$ is not contra- $\delta\hat{g}$ -continuous because $(g \circ f)^{-1}(\{a, c\}) = \{a, c\}$ is not $\delta\hat{g}$ -closed in (X, τ) where $\{a, c\}$ is open in (Z, η) .

Theorem 3.24. If $f : (X, \tau) \rightarrow (Y, \sigma)$ is contra- $\delta\hat{g}$ -continuous function and $g : (Y, \sigma) \rightarrow (Z, \eta)$ is a continuous function. Then $g \circ f : (X, \tau) \rightarrow (Z, \eta)$ is contra- $\delta\hat{g}$ -continuous.

Proof: Let V be open in (Z, η) . Since g is continuous, $g^{-1}(V)$ is open in (Y, σ) . Since f is contra- $\delta\hat{g}$ -continuous, $f^{-1}(g^{-1}(V))$ is $\delta\hat{g}$ -closed in (X, τ) . That is $(g \circ f)^{-1}(V)$ is $\delta\hat{g}$ -closed in (X, τ) . Hence $(g \circ f)$ is contra- $\delta\hat{g}$ -continuous. $\square$

Theorem 3.25. *If $f : (X, \tau) \rightarrow (Y, \sigma)$ is $\delta\hat{g}$ -irresolute and $g : (Y, \sigma) \rightarrow (Z, \eta)$ is contra- $\delta\hat{g}$ -continuous function. Then $g \circ f : (X, \tau) \rightarrow (Z, \eta)$ is contra- $\delta\hat{g}$ -continuous.*

Proof: Let V be open in (Z, η) . Since g is contra- $\delta\hat{g}$ -continuous, $g^{-1}(V)$ is $\delta\hat{g}$ -closed in (Y, σ) . Since f is $\delta\hat{g}$ -irresolute, $f^{-1}(g^{-1}(V))$ is $\delta\hat{g}$ -closed in (X, τ) . That is $(g \circ f)^{-1}(V)$ is $\delta\hat{g}$ -closed in (X, τ) . Hence $(g \circ f)$ is contra- $\delta\hat{g}$ -continuous. $\square$

4. Applications

Theorem 4.1. *Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be function and $g : X \rightarrow X \times Y$ be the graph function, given by $g(x) = (x, f(x))$ for every $x \in X$. Then f is contra- $\delta\hat{g}$ -continuous iff g is contra- $\delta\hat{g}$ -continuous.*

Proof: Necessity. Let $x \in X$ and let V be a closed subset of $X \times Y$ such that $x \in g^{-1}(V)$. That is $g(x) = (x, f(x)) \in V$. Then $V \cap (\{x\} \times Y)$ is closed in $\{x\} \times Y$ containing $g(x)$. Also $\{x\} \times Y$ is homeomorphic to Y , hence $\{y \in Y / (x, y) \in V\}$ is a closed subset of Y . Since f is contra- $\delta\hat{g}$ -continuous, $\cup\{f^{-1}(y) / (x, y) \in V\}$ is an $\delta\hat{g}$ -open subset of X . Further $x \in \cup\{f^{-1}(y) / (x, y) \in V\} \subseteq g^{-1}(V)$. Hence $g^{-1}(V)$ is $\delta\hat{g}$ -open. Thus g is contra- $\delta\hat{g}$ -continuous.

Sufficiency. Let U be a closed subset of Y . Then $X \times U$ is a closed subset of $X \times Y$. Since g is contra- $\delta\hat{g}$ -continuous, $g^{-1}(X \times U)$ is an $\delta\hat{g}$ -open subset of X . Also $g^{-1}(X \times U) = f^{-1}(U)$. Hence f is contra- $\delta\hat{g}$ -continuous. $\square$

Theorem 4.2. *Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be surjective $\delta\hat{g}$ -irresolute and weakly- $\delta\hat{g}$ -closed function where (X, τ) is $\hat{T}_{3/4}$ -space and $g : (Y, \sigma) \rightarrow (Z, \eta)$ be function. Then $g \circ f : (X, \tau) \rightarrow (Z, \eta)$ is contra- $\delta\hat{g}$ -continuous iff g is contra- $\delta\hat{g}$ -continuous.*

Proof: Let V be open in (Z, η) and g be contra- $\delta\hat{g}$ -continuous function. then $g^{-1}(V)$ is $\delta\hat{g}$ -closed in (Y, σ) . Since f is $\delta\hat{g}$ -irresolute, $f^{-1}(g^{-1}(V))$ is $\delta\hat{g}$ -closed in (X, τ) . That is $(g \circ f)^{-1}(V)$ is $\delta\hat{g}$ -closed in (X, τ) . Hence $(g \circ f)$ is contra- $\delta\hat{g}$ -continuous. Conversely, let $g \circ f : (X, \tau) \rightarrow (Z, \eta)$ be contra- $\delta\hat{g}$ -continuous function. Let U be an open set in (Z, η) . Then $(g \circ f)^{-1}(U)$ is $\delta\hat{g}$ -closed in (X, τ) . That is $f^{-1}(g^{-1}(U))$ is $\delta\hat{g}$ -closed in (X, τ) . Since (X, τ) is $\hat{T}_{3/4}$ -space, $f^{-1}(g^{-1}(U))$ is δ -closed in (X, τ) . Also since f is weakly- $\delta\hat{g}$ -closed, $f(f^{-1}(g^{-1}(U)))$ is $\delta\hat{g}$ -closed in (Y, σ) . Since f is surjective, $g^{-1}(U)$ is $\delta\hat{g}$ -closed in (Y, σ) . Hence g is contra- $\delta\hat{g}$ -continuous. $\square$

We introduce the following definition.

Definition 4.3. *A topological space (X, τ) is said to be $\delta\hat{g}$ -Hausdorff space if for each pair of distinct points x and y in X there exists $U \in \delta\hat{g}O(X, x)$ and $V \in \delta\hat{g}O(X, y)$ such that $U \cap V = \emptyset$.*

Example 4.4. *Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$. Let x and y be two distinct points of X , there exists an $\delta\hat{g}$ -open neighbourhood of x and y respectively such that $\{x\} \cap \{y\} = \emptyset$. Hence (X, τ) is $\delta\hat{g}$ -Hausdorff space.*

Theorem 4.5. *If X is a topological space and for each pair of distinct points x_1 and x_2 in X , there exists a function f of X into Uryshon topological space Y such that $f(x_1) \neq f(x_2)$ and f is contra- $\delta\hat{g}$ -continuous at x_1 and x_2 , then X is $\delta\hat{g}$ -Hausdroff space.*

Proof: Let x_1 and x_2 be any distinct points in X . Then by hypothesis, there is a Uryshon space Y and a function $f : X \rightarrow Y$ such that $f(x_1) \neq f(x_2)$ and f is contra- $\delta\hat{g}$ -continuous at x_1 and x_2 . Let $y_i = f(x_i)$ for $i = 1, 2$ then $y_1 \neq y_2$. Since Y is Uryshon, there exists open sets U_{y_1} and U_{y_2} containing y_1 and y_2 respectively in Y such that $\text{cl}(U_{y_1}) \cap \text{cl}(U_{y_2}) = \emptyset$. Since f is contra- $\delta\hat{g}$ -continuous at x_1 and x_2 , there exists $\delta\hat{g}$ -open sets V_{x_1} and V_{x_2} containing x_1 and x_2 respectively in X such that $f(V_{x_i}) \subset \text{cl}(U_{y_i})$ for $i = 1, 2$. Therefore we get $V_{x_1} \cap V_{x_2} = \emptyset$. Hence X is $\delta\hat{g}$ -Hausdroff. $\square$

Corollary 4.6. *If f is contra- $\delta\hat{g}$ -continuous injection of a topological space X into a Uryshon space Y then X is $\delta\hat{g}$ -Hausdroff.*

Proof: Let x_1 and x_2 be distinct points in X . Then by hypothesis, f is a contra- $\delta\hat{g}$ -continuous function of X into a Uryshon space Y such that $f(x_1) \neq f(x_2)$ because f is injective. Hence by theorem 4.5, X is $\delta\hat{g}$ -Hausdroff. $\square$

Theorem 4.7. *Let $f_1 : X_1 \rightarrow Y$ and $f_2 : X_2 \rightarrow Y$ be two contra- $\delta\hat{g}$ -continuous functions. If Y is a Uryshon space then $\{(x_1, x_2) / f_1(x_1) = f_2(x_2)\}$ is $\delta\hat{g}$ -closed in the product space $X_1 \times X_2$.*

Proof: Let A denote the set $\{(x_1, x_2) / f_1(x_1) = f_2(x_2)\}$. We have to prove that A is $\delta\hat{g}$ -closed in the product space $(X_1 \times X_2) - A$ is $\delta\hat{g}$ -open. Let $(x_1, x_2) \notin A$. Then $f_1(x_1) \neq f_2(x_2)$. Since Y is Uryshon space, there exists open sets V_1 and V_2 containing $f_1(x_1)$ and $f_2(x_2)$ respectively such that $\text{cl}(V_1) \cap \text{cl}(V_2) = \emptyset$. Since f_1 and f_2 are contra- $\delta\hat{g}$ -continuous, $f_1^{-1}(\text{cl}(V_1))$ and $f_2^{-1}(\text{cl}(V_2))$ are $\delta\hat{g}$ -open sets containing x_1 in X_1 and x_2 in X_2 respectively. Hence by Proposition 3.6, $f_1^{-1}(\text{cl}(V_1)) \times f_2^{-1}(\text{cl}(V_2))$ is $\delta\hat{g}$ -open in $X_1 \times X_2$. Further, $(x_1, x_2) \in f_1^{-1}(\text{cl}(V_1)) \times f_2^{-1}(\text{cl}(V_2)) \subset (X_1 \times X_2) - A$. This implies that $(X_1 \times X_2) - A$ is $\delta\hat{g}$ -open in $(X_1 \times X_2)$. Hence A is $\delta\hat{g}$ -closed in $X_1 \times X_2$. $\square$

Definition 4.8. *A topological space (X, τ) is said to be $\delta\hat{g}$ -normal if each pair of nonempty disjoint closed sets in (X, τ) can be separated by disjoint $\delta\hat{g}$ -open sets in (X, τ) .*

Example 4.9. *Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, \{b\}, \{a, c\}, X\}$. Then $\{b\}$ and $\{a, c\}$ are nonempty disjoint closed sets in (X, τ) . There exists two $\delta\hat{g}$ -open sets $\{b\}$ and $\{a, c\}$ such that $\{b\} \subseteq \{b\}$, $\{a, c\} \subseteq \{a, c\}$ and $\{b\} \cap \{a, c\} = \emptyset$. Thus (X, τ) is an $\delta\hat{g}$ -normal space.*

Theorem 4.10. *If $f : X \rightarrow Y$ is a contra- $\delta\hat{g}$ -continuous, closed, injection and Y is Ultra normal, then X is a $\delta\hat{g}$ -normal.*

Proof: Let U and V be disjoint closed subsets of X . Since f is closed and injective, $f(U)$ and $f(V)$ are disjoint closed subsets of Y . Since Y is Ultra-normal, there exists disjoint closed sets A and B such that $f(U) \subset A$ and $f(V) \subset B$. Hence $U \subseteq f^{-1}(A)$ and $V \subseteq f^{-1}(B)$. Since f is contra- $\delta\hat{g}$ -continuous and injective, $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint $\delta\hat{g}$ -open sets in X . Hence X is $\delta\hat{g}$ -normal. $\square$

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