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Note on contra $\delta\hat{g}$-continuous functions

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ABSTRACT: In this paper we introduce and investigate some classes of generalized functions called contra-$\delta\hat{g}$-continuous functions. We obtain several characterizations and some of their properties. Also we investigate its relationship with other types of functions. Finally we introduce two new spaces called $\delta\hat{g}$-Hausdorf spaces and $\delta\hat{g}$-normal spaces and obtain some new results.

Key Words: $\delta\hat{g}$-closed, $\delta\hat{g}$-continuous,$\delta\hat{g}$-irresolute,δg-continuous.

Contents

1. Introduction

Ganster and Reilly [5] introduced and studied the notion of LC-continuous functions. Dontchev [3] presented a new notion of continuous function called contra-continuity. This notion is a stronger form of LC-continuity. Dontchev and Noiri [4] introduced a weaker form of contra-continuity called contra-semi-continuity. The purpose of this present paper is to define a new class of generalised continuous functions called contra-$\delta\hat{g}$-continuous functions and investigate their relationships to other functions. We further introduce and study two new spaces called $\delta\hat{g}$-Hausdorf spaces and $\delta\hat{g}$-normal spaces and obtain some new results.

2. Preliminaries

Throughout this paper (X,τ) and, (Y,σ)and (Z,η) represent non-empty topological spaces on which no separation axioms are assumed unless or otherwise mentioned. For a subset A of X, cl(A), int(A) and A^cdenote the closure of A, the interior of A and the complement of A respectively. Let us recall the following definitions, which are useful in the sequel.

Definition 2.1. *[12] The δ-interior of a subset A of X is the union of all regular open set of X contained in A and is denoted by $int_\delta(A)$. The subset A is called δ-open if $A = int_\delta(A)$, i.e. a set is δ-open if it is the union of regular open sets.*

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The complement of a δ-open is called δ-closed. Alternatively, a set $A \subseteq (X,\tau)$ is called δ-closed if $A = cl_\delta(A)$, where $cl_\delta(A) = \{ x \in X: int(cl(U)) \cap A \neq \phi, U \in \tau$ and $x \in U\}$.

Definition 2.2. *A subset A of (X,τ) is called*

(i) *generalized closed (briefly g-closed) set [6] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open set in (X,τ).*

(ii) *semi-generalized closed (briefly sg-closed) set [2] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is a semi-open set in (X,τ).*

(iii) *$\hat{g}$-closed set [11] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is a semi-open set in (X,τ).*

(iv) *α-$\hat{g}$-closed (briefly $\alpha\hat{g}$-closed) set [1] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is a $\hat{g}$- open set in (X,τ).*

(v) *$\delta\hat{g}$-closed set [7] if $cl_\delta(A) \subseteq U$ whenever $A \subseteq U$ and U is a $\hat{g}$- open set in (X,τ).*

The complement of a g-closed (resp. sg-closed, $\hat{g}$-closed, $\alpha\hat{g}$-closed and $\delta\hat{g}$-closed) set is called g-open (resp. sg-open, $\hat{g}$-open, $\alpha\hat{g}$-open and $\delta\hat{g}$-open).

Definition 2.3. *A function $f : (X,\tau) \to (Y,\sigma)$ is called*

(i) *sg-continuous [6] if $f^{-1}(V)$ is sg-closed in (X,τ) for every closed set V of (Y,σ).*

(ii) *$\alpha\hat{g}$-continuous [9] if $f^{-1}(V)$ is $\alpha\hat{g}$-closed in (X,τ) for every closed set V of (Y,σ).*

(iii) *$\delta\hat{g}$-continuous [8] if $f^{-1}(V)$ is $\delta\hat{g}$-closed in (X,τ) for every closed set V of (Y,σ).*

(iv) *$\delta\hat{g}$-irresolute [8] if $f^{-1}(V)$ is $\delta\hat{g}$-closed in (X,τ) for every $\delta\hat{g}$ closed set V of (Y,σ).*

(v) *$\alpha\hat{g}$-closed [1] if the image of every closed set in (X,τ) is $\alpha\hat{g}$-closed in (Y,σ).*

(iii) *$\delta\hat{g}$-closed [8] if the image of every closed set in (X,τ) is $\delta\hat{g}$-closed in (Y,σ).*

(iii) *Weakly $\delta\hat{g}$-closed [8] (resp. weakly $\delta\hat{g}$-open) if the image of every δ-closed (resp. δ-open) set in (X,τ) is $\delta\hat{g}$-closed (resp. $\delta\hat{g}$-open) set in (Y,σ).*

Definition 2.4. *Recall that a function $f : (X,\tau) \to (Y,\sigma)$ is called*

(i) *contra-continuous [3] if $f^{-1}(V)$ is closed in (X,τ) for every open set V in (Y,σ).*

(ii) *contra-sg-continuous [4] if $f^{-1}(V)$ is sg-closed in (X,τ) for every open set V of (Y,σ).*

(iii) *contra-$\alpha\hat{g}$-continuous [9] if $f^{-1}(V)$ is $\alpha\hat{g}$-closed in (X,τ) for every open set V in (Y,σ).*

Definition 2.5. *[7] A space (X,τ) is called $\hat{T}_{3/4}$-space if every $\delta\hat{g}$-Closed set in it is δ-closed .*

Definition 2.6. *Ultra normal [10] if each pair of nonempty disjoint closed sets can be separated by disjoint clopen sets.*

3. Contra-$\delta\hat{g}$-continuous

We introduce the following definition.

Definition 3.1. *A function $f : (X,\tau) \to (Y,\sigma)$ is called contra-$\delta\hat{g}$-continuous if $f^{-1}(V)$ is $\delta\hat{g}$-closed in (X,τ) for every open set V in (Y,σ).*

Example 3.2. *Let $X = \{a, b, c\} = Y$ with topologies $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ and, $\sigma = \{\phi, \{a, b\}, Y\}$. Define $f : (X,\tau) \to (Y,\sigma)$ by $f(a) = c$, $f(b) = b$ and $f(c) = a$. Clearly f is contra-$\delta\hat{g}$-continuous function.*

Note 3.3. *The family of all $\delta\hat{g}$-open sets of (X,τ) is denoted by $\delta\hat{g}O(X)$. The set $\delta\hat{g}O(X,x) = \{V\in \delta\hat{g}O(X)/x \in V\}$ for $x \in X$.*

Remark 3.4. *For a subset A of (X,τ), $cl_\delta(A^c) = (int_\delta(A))^c$.*

Theorem 3.5. *$A \subseteq X$ is $\delta\hat{g}$-open if and only if $F \subseteq int_\delta(A)$ whenever F is $\hat{g}$-closed and $F \subseteq A$.*

Proof. *Necessity. Let A be an $\delta\hat{g}$-open set in (X,τ). Let F be $\hat{g}$-closed such that $F \subseteq A$. Then $A^c \subseteq F^c$ where F^c is $\hat{g}$-open. A^c is $\delta\hat{g}$-closed implies that $cl_\delta(A^c) \subseteq F^c$. By Remark 3.4, $(int_\delta(A))^c \subseteq F^c$. That is $F \subseteq int_\delta(A)$.*
Sufficiency. Suppose F is $\hat{g}$-closed and $F \subseteq A$ implies $F \subseteq int_\delta(A)$. Let $A^c \subseteq U$ where U is $\hat{g}$-open. Then $U^c \subseteq A$ where U^c is $\hat{g}$-closed. By hypothesis, $U^c \subseteq int_\delta(A)$. That is $(int_\delta(A))^c \subseteq U$. By Remark 3.4, $cl_\delta(A^c) \subseteq U$. This implies A^c is $\delta\hat{g}$-closed. Hence A is $\delta\hat{g}$-open.

Proposition 3.6. *The product of two $\delta\hat{g}$-open sets of two spaces is $\delta\hat{g}$-open set in the product space.*

Proof: *Let A and B be $\delta\hat{g}$-open sets of two spaces (X,τ) and (Y,σ) respectively and $V = A \times B \subseteq X \times Y$. Let $F \subseteq V$ be a $\hat{g}$-closed set in $X \times Y$, then there exists two $\hat{g}$-closed sets $F_1 \subseteq A$ and $F_2 \subseteq B$. So, $F_1 \subseteq int_\delta(A)$ and $F_2 \subseteq int_\delta(B)$. Hence $F_1 \times F_2 \subseteq A\times B$ and $F_1 \times F_2 \subseteq int_\delta(A)\times int_\delta(B) = int_\delta(A \times B)$. Therefore $A \times B$ is $\delta\hat{g}$-open subset of a space $X \times Y$.* □

Theorem 3.7. *Let $f : (X,\tau) \to (Y,\sigma)$ be a map. Then the following are equivalent.*

(i) *f is contra-$\delta\hat{g}$-continuous.*

(ii) *The inverse image of each closed set in (Y,σ) is $\delta\hat{g}$-open in (X,τ).*

(iii) *For each $x \in X$ and each $F \in C(Y,f(x))$, there exists $U \in \delta\hat{g}O(X,x)$ such that $f(U) \subset F$.*

Proof: (i)$\Rightarrow$ (ii), (ii)$\Rightarrow$ (i) and (ii)$\Rightarrow$ (iii) are obvious.
(iii)$\Rightarrow$ (ii) Let F be any closed set of (Y,σ) and $x \in f^{-1}(F)$. Then $f(x) \in F$ and there exists $U_x \in \delta\hat{g}O(X,x)$ such that $f(U_x) \subset F$. Hence we obtain $f^{-1}(F) = U\{U_x/x \in f^{-1}(F)\} \in \delta\hat{g}O(X)$. Thus the inverse of each closed set in (Y,σ) is $\delta\hat{g}$-open in (X,τ). □

Remark 3.8. *The concept of $\delta\hat{g}$-continuity and contra-$\delta\hat{g}$-continuity are independent as shown in the following example.*

Example 3.9. *Let $X = \{a,b,c\} = Y$ with topologies $\tau = \{\phi,\{b\},\{a,b\},X\}$, $\sigma = \{\phi,\{b\},Y\}$ respectively. Let $f : (X,\tau) \to (Y,\sigma)$ be the identity function. Clearly f is $\delta\hat{g}$-continuous function, but f is not contra-$\delta\hat{g}$-continuous because $f^{-1}(\{b\}) = \{b\}$ is not $\delta\hat{g}$-closed in (X,τ) where $\{b\}$ is open in (Y,σ).*

Example 3.10. *Let $X = \{a,b,c\} = Y$ with topologies $\tau = \{\phi,\{a\},\{c\},\{a,c\},X\}$, $\sigma = \{\phi,\{a,c\},Y\}$ respectively. Define a function $f : (X,\tau) \to (Y,\sigma)$ by $f(a) = c$, $f(b) = a$, $f(c) = b$. Clearly f is contra-$\delta\hat{g}$-continuous function, but f is not $\delta\hat{g}$-continuous because $f^{-1}(\{b\}) = \{c\}$ is not $\delta\hat{g}$-closed in (X,τ) where $\{b\}$ is closed in (Y,σ).*

Remark 3.11. *A function $f : (X,\tau) \to (Y,\sigma)$ is $\delta\hat{g}$-continuous if for each $x \in X$ and each open set V of Y containing $f(x)$, there exists $U \in \delta\hat{g}O(X,x)$ such that $f(U) \subset V$.*

Theorem 3.12. *If a function $f : (X,\tau) \to (Y,\sigma)$ is contra-$\delta\hat{g}$-continuous and (Y,σ) is regular then f is $\delta\hat{g}$-continuous.*

Proof: Let x be an arbitrary point of (X,τ) and V be an open set of (Y,σ) containing $f(x)$. Since (Y,σ) is regular, there exists an open set W of (Y,σ) containing $f(x)$ such that cl(W)$\subset$V. Since f is contra-$\delta\hat{g}$-continuous, by theorem 3.7, there exists U$\in\delta\hat{g}O(X,x)$ such that $f(U) \subset cl$(W). Then $f(U) \subset cl(W) \subset V$. Hence by Remark 3.11, f is $\delta\hat{g}$-continuous. □

Theorem 3.13. *Every contra-$\delta\hat{g}$-continuous function is contra-$\alpha\hat{g}$-continuous.*

Proof: Let V be open set in (Y,σ). Since f is contra-$\delta\hat{g}$-continuous function, f^{-1}(V) is $\delta\hat{g}$-closed in (X,τ). Every $\delta\hat{g}$-closed set is $\alpha\hat{g}$-closed. Hence f^{-1}(V) is $\alpha\hat{g}$-closed in (X,τ). Thus f is contra-$\alpha\hat{g}$-continuous. □

Remark 3.14. *The converse of theorem* 3.13 *need not be true as shown in the following example.*

Example 3.15. *Let* $X = \{a, b, c\} = Y$ *with topologies* $\tau = \{\phi, \{a\}, \{a, b\}, \{a, c\}, X\}$ *and* $\sigma = \{\phi, \{b\}, \{b, c\}, Y\}$. *Let* $f : (X, \tau) \to (Y, \sigma)$ *be the identity function. Clearly* f *is contra-*$\alpha\hat{g}$*-continuous. But* f *is not contra-*$\delta\hat{g}$*-continuous because* $f^{-1}(\{b\}) = \{b\}$ *is not* $\delta\hat{g}$*-closed in* (X, τ) *where* $\{b\}$ *is open in* (Y, σ).

Remark 3.16. *The concept of contra-continuous and contra-*$\delta\hat{g}$*-continuous are independent as shown in the following examples.*

Example 3.17. *Let* $X = \{a, b, c\} = Y$ *with topologies* $\tau = \{\phi, \{a\}, \{a, b\}, X\}$ *and* $\sigma = \{\phi, \{b\}, \{a, b\}, \{b, c\}, Y\}$. *Define a function* $f : (X, \tau) \to (Y, \sigma)$ *by* $f(a) = a$, $f(b) = c$, $f(c) = b$. *Then clearly* f *is contra-*$\delta\hat{g}$*-continuous but* f *is not contra-continuous because* $f^{-1}(\{a, b\}) = \{a, c\}$ *is not closed in* (X, τ) *but* $\{a, b\}$ *is open in* (Y, σ).

Example 3.18. *Let* $X = \{a, b, c\} = Y$ *with topologies* $\tau = \{\phi, \{c\}, \{a, c\}, \{b, c\}, X\}$ *and* $\sigma = \{\phi, \{a\}, \{b\}, \{a, b\}, Y\}$. *Let* $f : (X, \tau) \to (Y, \sigma)$ *be the identity function. Then clearly* f *is contra-continuous. But* f *is not contra-*$\delta\hat{g}$*-continuous because* $f^{-1}(\{b\}) = \{b\}$ *is not* $\delta\hat{g}$*-closed in* (X, τ) *where* $\{b\}$ *is open in* (Y, σ).

Remark 3.19. *The concept of contra-*$\delta\hat{g}$*-continuous and contra-sg-continuous are independent of each other as shown in the following examples.*

Example 3.20. *Let* $X = \{a, b, c\} = Y$ *with topologies* $\tau = \{\phi, \{a\}, \{c\}, \{a, c\}, X\}$ *and* $\sigma = \{\phi, \{a\}, Y\}$. *Let* $f : (X, \tau) \to (Y, \sigma)$ *be the identity function.* f *is not contra-*$\delta\hat{g}$*-continuous because* $f^{-1}(\{a\}) = \{a\}$ *is not* $\delta\hat{g}$*-closed in* (X, τ) *where* $\{a\}$ *is open in* (Y, σ). *However* f *is contra-sg-continuous*

Example 3.21. *Let* $X = \{a, b, c\} = Y$ *with topologies* $\tau = \{\phi, \{a\}, \{a, c\}, X\}$ *and* $\sigma = \{\phi, \{a, b\}, Y\}$. *Let* $f : (X, \tau) \to (Y, \sigma)$ *be the identity function. Then clearly* f *is contra-*$\delta\hat{g}$*-continuous but* f *is not contra-sgcontinuous because* $f^{-1}(\{a, b\}) = \{a, b\}$ *is not sg-closed in* (X, τ) *where* $\{a, b\}$ *is open in* (Y, σ).

Remark 3.22. *The composition of two contra-*$\delta\hat{g}$*-continuous functions need not be contra-*$\delta\hat{g}$*-continuous as the following example shows.*

Example 3.23. *Let* $X = \{a, b, c\} = Y = Z$, $\tau = \{\phi, \{c\}, \{a, b\}, X\}$ *and* $\sigma = \{\phi, \{c\}, Y\}$, $\eta = \{\phi, \{a, c\}, Z\}$. *Let* $f : (X, \tau) \to (Y, \sigma)$ *and* $g : (Y, \sigma) \to (Z, \eta)$ *be two identity functions. Then both* f *and* g *are contra-*$\delta\hat{g}$*-continuous but* $g \circ f : (X, \tau) \to (Z, \eta)$ *is not contra-*$\delta\hat{g}$*continuous because* $(g \circ f)^{-1}(\{a, c\}) = \{a, c\}$ *is not* $\delta\hat{g}$*-closed in* (X, τ) *where* $\{a, c\}$ *is open in* (Z, η).

Theorem 3.24. *If* $f : (X, \tau) \to (Y, \sigma)$ *is contra-*$\delta\hat{g}$*-continuous function and* $g : (Y, \sigma) \to (Z, \eta)$ *is a continuous function. Then* $g \circ f : (X, \tau) \to (Z, \eta)$ *is contra-*$\delta\hat{g}$*-continuous.*

Proof: Let V be open in (Z, η). Since g is continuous, $g^{-1}(V)$ is open in (Y, σ). Since f is contra-$\delta\hat{g}$-continuous, $f^{-1}(g^{-1}(V))$ is $\delta\hat{g}$-closed in (X, τ). That is $(g \circ f)^{-1}(V)$ is $\delta\hat{g}$-closed in (X, τ). Hence $(g \circ f)$ is contra-$\delta\hat{g}$-continuous. $\square$

Theorem 3.25. *If $f:(X,\tau)\to(Y,\sigma)$ is $\delta\hat{g}$-irresolute and $g:(Y,\sigma)\to(Z,\eta)$ is contra-$\delta\hat{g}$-continuous function. Then $g\circ f:(X,\tau)\to(Z,\eta)$ is contra-$\delta\hat{g}$-continuous.*

Proof: Let V be open in (Z,η). Since g is contra-$\delta\hat{g}$-continuous, $g^{-1}(V)$ is $\delta\hat{g}$-closed in (Y,σ). Since f is $\delta\hat{g}$-irresolute, $f^{-1}(g^{-1}(V))$ is $\delta\hat{g}$-closed in (X,τ). That is $(g\circ f)^{-1}(V)$ is $\delta\hat{g}$-closed in (X,τ). Hence $(g\circ f)$ is contra-$\delta\hat{g}$-continuous. □

4. Applications

Theorem 4.1. *Let $f:(X,\tau)\to(Y,\sigma)$ be function and $g:X\to X\times Y$ be the graph function, given by $g(x)=(x,f(x))$ for every $x\in X$. Then f is contra-$\delta\hat{g}$-continuous iff g is contra-$\delta\hat{g}$-continuous.*

Proof: Necessity. Let $x\in X$ and let V be a closed subset of $X\times Y$ such that $x\in g^{-1}(V))$. That is $g(x)=(x,f(x))\in V$. Then V $\cap(\{\text{x}\}\times Y)$ is closed in $\{x\}\times Y$ containing $g(x)$. Also $\{x\}\times Y$ is homeomorphic to Y, hence $\{y\in Y/(x,y)\in V\}$ is a closed subset of Y. Since f is contra-$\delta\hat{g}$-continuous, $\cup\{f^{-1}(y)/(x,y)\in V\}$ is an $\delta\hat{g}$-open subset of X. Further $x\in\cup\{f^{-1}(y)/(x,y)\in V\}\subseteq g^{-1}(V)$. Hence $g^{-1}(V)$ is $\delta\hat{g}$-open. Thus g is contra-$\delta\hat{g}$-continuous.
Sufficiency. Let U be a closed subset of Y. Then $X\times U$ is a closed subset of $X\times Y$. Since g is contra-$\delta\hat{g}$-continuous, $g^{-1}(X\times U)$ is an $\delta\hat{g}$-open subset of X. Also $g^{-1}(X\times U)=f^{-1}(U)$. Hence f is contra-$\delta\hat{g}$-continuous. □

Theorem 4.2. *Let $f:(X,\tau)\to(Y,\sigma)$ be surjective $\delta\hat{g}$-irresolute and weakly-$\delta\hat{g}$-closed function where (X,τ) is $\hat{T}_{3/4}$-space and $g:(Y,\sigma)\to(Z,\eta)$ be function. Then $g\circ f:(X,\tau)\to(Z,\eta)$ is contra-$\delta\hat{g}$-continuous iff g is contra-$\delta\hat{g}$-continuous.*

Proof: Let V be open in (Z,η) and g be contra-$\delta\hat{g}$-continuous function. then $g^{-1}(V)$ is $\delta\hat{g}$-closed in (Y,σ). Since f is $\delta\hat{g}$-irresolute, $f^{-1}(g^{-1}(V))$ is $\delta\hat{g}$-closed in (X,τ). That is $(g\circ f)^{-1}(V)$ is $\delta\hat{g}$-closed in (X,τ). Hence $(g\circ f)$ is contra-$\delta\hat{g}$-continuous. Conversely, let $g\circ f:(X,\tau)\to(Z,\eta)$ be contra-$\delta\hat{g}$-continuous function. Let U be an open set in (Z,η). Then $(g\circ f)^{-1}(U)$ is $\delta\hat{g}$-closed in (X,τ). That is $f^{-1}(g^{-1}(U))$ is $\delta\hat{g}$-closed in (X,τ). Since (X,τ) is $\hat{T}_{3/4}$-space, $f^{-1}(g^{-1}(U))$ is δ-closed in (X,τ). Also since f is weakly-$\delta\hat{g}$-closed, $f(f^{-1}(g^{-1}(U)))$ is $\delta\hat{g}$-closed in (Y,σ). Since f is surjective, $g^{-1}(U)$ is $\delta\hat{g}$-closed in (Y,σ). Hence g is contra-$\delta\hat{g}$-continuous. □

We introduce the following definition.

Definition 4.3. *A topological space (X,τ) is said to be $\delta\hat{g}$-Hausdorff space if for each pair of distinct points x and y in X there exists $U\in\delta\hat{g}O(X,x)$ and $V\in\delta\hat{g}O(X,y)$ such that $U\cap V=\phi$.*

Example 4.4. *Let X ={a, b, c}, $\tau=\{\phi,\{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\},X\}$. Let x and y be two distinct points of X, there exists an $\delta\hat{g}$-open neighbourhood of x and y respectively such that $\{x\}\cap\{y\}=\phi$. Hence (X,τ) is $\delta\hat{g}$-Hausdroff space.*

Theorem 4.5. *If X is a topological space and for each pair of distinct points x_1 and x_2 in X , there exists a function f of X into Uryshon topological space Y such that $f(x_1) \neq f(x_2)$ and f is contra-$\delta\hat{g}$-continuous at x_1 and x_2, then X is $\delta\hat{g}$-Hausdroff space.*

Proof: Let x_1 and x_2 be any distinct points in X. Then by hypothesis, there is a Uryshon space Y and a function $f : X \to Y$ such that $f(x_1) \neq f(x_2)$ and f is contra-$\delta\hat{g}$-continuous at x_1 and x_2. Let $y_i = f(x_i)$ for $i = 1, 2$ then $y_1 \neq y_2$. Since Y is Uryshon, there exists open sets U_{y1} and U_{y2} containing y_1 and y_2 respectively in Y such that cl(U_{y1})∩cl(U_{y2})=ϕ. Since f is contra-$\delta\hat{g}$-continuous at x_1 and x_2, there exists an $\delta\hat{g}$-open sets V_{x1} and V_{x2} containing x_1 and x_2 respectively in X such that $f(V_{xi}) \subset cl(U_{yi})$ for $i = 1, 2$. Therefore we get $V_{x1} \cap V_{x2} = \phi$. Hence X is $\delta\hat{g}$-Hausdroff. □

Corollary 4.6. *If f is contra-$\delta\hat{g}$-continuous injection of a topological space X into a Uryshon space Y then Y is $\delta\hat{g}$-Hausdroff.*

Proof: Let x_1 and x_2 be distinct points in X. Then by hypothesis, f is a contra-$\delta\hat{g}$-continuous function of X into a Uryshon space Y such that $f(x_1) \neq f(x_2)$ because f is injective. Hence by theorem 4.5, X is $\delta\hat{g}$-Hausdroff. □

Theorem 4.7. *Let $f_1 : X_1 \to Y$ and $f_2 : X_2 \to Y$ be two contra-$\delta\hat{g}$-continuous functions. If Y is a Uryshon space then $\{(x_1, x_2)/f_1(x_1) = f_2(x_2)\}$ is $\delta\hat{g}$-closed in the product space $X_1 \times X_2$.*

Proof: Let A denote the set $\{(x_1, x_2)/f_1(x_1) = f_2(x_2)\}$. We have to prove that A is $\delta\hat{g}$-closed in the product space $(X_1 \times X_2) - A$ is $\delta\hat{g}$-open. Let $(x_1, x_2) \notin A$. Then$f_1(x_1) \neq f_2(x_2)$. Since Y is Uryshon space, there exists open sets V_1 and V_2 containing $f_1(x_1)$ and $f_2(x_2)$ respectively such that $cl(V_1) \cap cl(V_2) = \phi$. Since f_1 and f_2 are contra-$\delta\hat{g}$-continuous, $f_1^{-1}(cl(V_1)$and $f_2^{-1}(cl(V_2))$ are $\delta\hat{g}$-open sets containing x_1 in X_1 and x_2 in X_2 respectively. Hence by Proposition 3.6, $f_1^{-1}(cl(V_1))\times$ $f_2^{-1}(cl(V_2))$ is $\delta\hat{g}$-open in $X_1{\times}X_2$. Further,$(x_1, x_2) \in f_1^{-1}(cl(V_1)) \times f_2^{-1}(cl(V_2)) \subset (X_1 \times X_2) - A$. This implies that $(X_1 \times X_2) - A$ is $\delta\hat{g}$-open in $(X_1 \times X_2)$. Hence A is $\delta\hat{g}$-closed in $X_1 \times X_2$. □

Definition 4.8. *A topological space (X, τ) is said to be $\delta\hat{g}$-normal if each pair of nonempty disjoint closed sets in (X, τ) can be separated by disjoint $\delta\hat{g}$-open sets in (X, τ).*

Example 4.9. *Let $X = \{a, b, c\}$ and $\tau = \{\phi, \{b\}, \{a, c\}, X\}$. Then {b} and {a,c} are nonempty disjoint closed sets in (X, τ). There exists two $\delta\hat{g}$-open sets {b} and {a,c} such that {b} ⊆{b}, {a,c}⊆{a,c} and {b}∩{a,c} =ϕ. Thus (X, τ) is an $\delta\hat{g}$-normal space.*

Theorem 4.10. *If $f : X \to Y$ is a contra-$\delta\hat{g}$-continuous, closed, injection and Y is Ultra normal, then X is a $\delta\hat{g}$-normal.*

Proof: Let U and V be disjoint closed subsets of X. Since f is closed and injective, $f(U)$ and $f(V)$ are disjoint closed subsets of Y. Since Y is Ultra-normal, there exists disjoint closed sets A and B such that $f(U) \subset A$ and $f(V) \subset B$. Hence $U \subseteq f^{-1}(A)$ and $V \subseteq f^{-1}(B)$. Since f is contra-$\delta\hat{g}$-continuous and injective, $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint $\delta\hat{g}$-open sets in X. Hence X is $\delta\hat{g}$-normal. □

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