Bol. Soc. Paran. Mat.
©SPM –ISSN-2175-1188 ON LINE
SPM: www.spm.uem.br/bspm

(3s.) **v. 31** 1 (2013): 89–97.
ISSN-00378712 IN PRESS
doi:10.5269/bspm.v31i1.15249

On Inextensible Flows Of Tangent Developable of Biharmonic $\mathcal{B}-$Slant Helices according to Bishop Frames in the Special 3-Dimensional Kenmotsu Manifold

Vedat Asil, Talat Körpinar and Essin Turhan

ABSTRACT: In this paper, we study inextensible flows of tangent developable surfaces of biharmonic $\mathcal{B}-$slant helices in the special three-dimensional Kenmotsu manifold $\mathbb{K}$ with η-parallel ricci tensor. We express some interesting relations about inextensible flows of this surfaces.

Key Words: Biharmonic curve, Developable surface, Kenmotsu manifold, Bishop frame.

Contents

1. Introduction

Geometric flows have been extensively used in mesh processing. In particular, surface flows based on functional minimization (i.e., evolving a surface so as to progressively decrease an energy functional) is a common methodology in geometry processing with applications spanning surface diffusion.

On the other hand, a smooth map $\phi : N \longrightarrow M$ is said to be biharmonic if it is a critical point of the bienergy functional:

$$E_2(\phi) = \int_N \frac{1}{2} |\mathcal{T}(\phi)|^2 \, dv_h,$$

where $\mathcal{T}(\phi) := \mathrm{tr}\nabla^\phi d\phi$ is the tension field of ϕ

2000 *Mathematics Subject Classification:* 53C41, 53A10

The Euler–Lagrange equation of the bienergy is given by $\mathcal{T}_2(\phi) = 0$. Here the section $\mathcal{T}_2(\phi)$ is defined by

$$\mathcal{T}_2(\phi) = -\Delta_\phi \mathcal{T}(\phi) + \operatorname{tr} R\left(\mathcal{T}(\phi), d\phi\right) d\phi, \tag{1.1}$$

and called the bitension field of ϕ. Non-harmonic biharmonic maps are called proper biharmonic maps.

In this paper, we study inextensible flows of tangent developable surfaces of biharmonic $\mathcal{B}-$slant helices in the special three-dimensional Kenmotsu manifold $\mathbb{K}$ with η-parallel ricci tensor. We express some interesting relations about inextensible flows of this surfaces.

2. Preliminaries

Let $M^{2n+1}(\phi, \xi, \eta, g)$ be an almost contact Riemannian manifold with 1-form η, the associated vector field ξ, $(1,1)$-tensor field ϕ and the associated Riemannian metric g. It is well known that [2]

$$\begin{gathered}
\phi\xi = 0, \quad \eta(\xi) = 1, \quad \eta(\phi X) = 0, \\
\phi^2(X) = -X + \eta(X)\xi, \\
g(X, \xi) = \eta(X), \\
g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y),
\end{gathered}$$

for any vector fields X, Y on M.

We consider the three-dimensional manifold

$$\mathbb{K} = \left\{ \left(x^1, x^2, x^3\right) \in \mathbb{R}^3 : \left(x^1, x^2, x^3\right) \neq (0,0,0) \right\},$$

where $\left(x^1, x^2, x^3\right)$ are the standard coordinates in $\mathbb{R}^3$. The vector fields

$$\mathbf{e}_1 = x^3 \frac{\partial}{\partial x^1}, \quad \mathbf{e}_2 = x^3 \frac{\partial}{\partial x^2}, \quad \mathbf{e}_3 = -x^3 \frac{\partial}{\partial x^3} \tag{2.1}$$

are linearly independent at each point of $\mathbb{K}$, [2]. Let g be the Riemannian metric defined by

$$\begin{aligned}
g(\mathbf{e}_1, \mathbf{e}_1) &= g(\mathbf{e}_2, \mathbf{e}_2) = g(\mathbf{e}_3, \mathbf{e}_3) = 1, \\
g(\mathbf{e}_1, \mathbf{e}_2) &= g(\mathbf{e}_2, \mathbf{e}_3) = g(\mathbf{e}_1, \mathbf{e}_3) = 0.
\end{aligned}$$

The characterising properties of $\chi(\mathbb{K})$ are the following commutation relations:

$$[\mathbf{e}_1, \mathbf{e}_2] = 0, \quad [\mathbf{e}_1, \mathbf{e}_3] = \mathbf{e}_1, \quad [\mathbf{e}_2, \mathbf{e}_3] = \mathbf{e}_2.$$

Let η be the 1-form defined by

$$\eta(Z) = g(Z, \mathbf{e}_3) \text{ for any } Z \in \chi(M)$$

Let be the (1,1) tensor field defined by

$$\phi(\mathbf{e}_1) = -\mathbf{e}_2,\ \phi(\mathbf{e}_2) = \mathbf{e}_1,\ \phi(\mathbf{e}_3) = 0.$$

Then,

$$\eta(\mathbf{e}_3) = 1,$$
$$\phi^2(Z) = -Z + \eta(Z)\mathbf{e}_3,$$
$$g(\phi Z, \phi W) = g(Z, W) - \eta(Z)\eta(W),$$

for any $Z, W \in \chi(M)$. Thus for $\mathbf{e}_3 = \xi$, (ϕ, ξ, η, g) defines an almost contact metric structure on $\mathbb{K}$.

3. Biharmonic Curves in the Special Three-Dimensional Kenmotsu Manifold $\mathbb{K}$

Let $\{\mathbf{t}, \mathbf{n}, \mathbf{b}\}$ be the Frenet frame field along γ. Then, the Frenet frame satisfies the following Frenet–Serret equations:

$$\begin{aligned} \nabla_{\mathbf{t}}\mathbf{t} &= \kappa\mathbf{n}, \\ \nabla_{\mathbf{t}}\mathbf{n} &= -\kappa\mathbf{t} + \tau\mathbf{b}, \\ \nabla_{\mathbf{t}}\mathbf{b} &= -\tau\mathbf{n}, \end{aligned} \tag{3.1}$$

where $\kappa = |\mathcal{T}(\gamma)| = |\nabla_{\mathbf{t}}\mathbf{t}|$ is the curvature of γ and τ its torsion and

$$\begin{aligned} g(\mathbf{t}, \mathbf{t}) &= 1,\ g(\mathbf{n}, \mathbf{n}) = 1,\ g(\mathbf{b}, \mathbf{b}) = 1, \\ g(\mathbf{t}, \mathbf{n}) &= g(\mathbf{t}, \mathbf{b}) = g(\mathbf{n}, \mathbf{b}) = 0. \end{aligned} \tag{3.2}$$

In the rest of the paper, we suppose everywhere

$$\kappa \neq 0 \text{ and } \tau \neq 0. \tag{3.3}$$

The Bishop frame or parallel transport frame is an alternative approach to defining a moving frame that is well defined even when the curve has vanishing second derivative. The Bishop frame is expressed as

$$\begin{aligned} \nabla_{\mathbf{t}}\mathbf{t} &= k_1\mathbf{m}_1 + k_2\mathbf{m}_2, \\ \nabla_{\mathbf{t}}\mathbf{m}_1 &= -k_1\mathbf{t}, \\ \nabla_{\mathbf{t}}\mathbf{m}_2 &= -k_2\mathbf{t}, \end{aligned} \tag{3.4}$$

where

$$\begin{aligned} g(\mathbf{t}, \mathbf{t}) &= 1,\ g(\mathbf{m}_1, \mathbf{m}_1) = 1,\ g(\mathbf{m}_2, \mathbf{m}_2) = 1, \\ g(\mathbf{t}, \mathbf{m}_1) &= g(\mathbf{t}, \mathbf{m}_2) = g(\mathbf{m}_1, \mathbf{m}_2) = 0. \end{aligned} \tag{3.5}$$

Here, we shall call the set $\{\mathbf{t}, \mathbf{m}_1, \mathbf{m}_1\}$ as Bishop trihedra, k_1 and k_2 as Bishop curvatures, $\tau(s) = \zeta'(s)$ and $\kappa(s) = \sqrt{k_1^2 + k_2^2}$.

Bishop curvatures are defined by

$$\begin{aligned} k_1 &= \kappa(s)\cos\zeta(s), \\ k_2 &= \kappa(s)\sin\zeta(s). \end{aligned} \tag{3.6}$$

The relation matrix may be expressed as

$$\begin{aligned} &\mathbf{t} = \mathbf{t}, \\ &\mathbf{n} = \cos\zeta(s)\,\mathbf{m}_1 + \sin\zeta(s)\,\mathbf{m}_2, \\ &\mathbf{b} = -\sin\zeta(s)\,\mathbf{m}_1 + \cos\zeta(s)\,\mathbf{m}_2. \end{aligned}$$

On the other hand, using above equation we have

$$\begin{aligned} \mathbf{t} &= \mathbf{t}, \\ \mathbf{m}_1 &= \cos\zeta(s)\,\mathbf{n} - \sin\zeta(s)\,\mathbf{b}, \\ \mathbf{m}_2 &= \sin\zeta(s)\,\mathbf{n} + \cos\zeta(s)\,\mathbf{b}. \end{aligned}$$

With respect to the orthonormal basis $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ we can write

$$\begin{aligned} \mathbf{t} &= t^1\mathbf{e}_1 + t^2\mathbf{e}_2 + t^3\mathbf{e}_3, \\ \mathbf{m}_1 &= m_1^1\mathbf{e}_1 + m_1^2\mathbf{e}_2 + m_1^3\mathbf{e}_3, \\ \mathbf{m}_2 &= m_2^1\mathbf{e}_1 + m_2^2\mathbf{e}_2 + m_2^3\mathbf{e}_3. \end{aligned} \tag{3.7}$$

Lemma 3.1. $\gamma : I \longrightarrow \mathbb{K}$ *is a biharmonic curve with Bishop frame if and only if*

$$\begin{aligned} k_1^2 + k_2^2 &= \textit{constant} \neq 0, \\ k_1'' - \left[k_1^2 + k_2^2\right] k_1 &= -k_1, \\ k_2'' - \left[k_1^2 + k_2^2\right] k_2 &= -k_2. \end{aligned} \tag{3.8}$$

Definition 3.2. *A regular curve* $\gamma : I \longrightarrow \mathbb{K}$ *is called a slant helix provided the unit vector* $\mathbf{m}_1$ *of the curve* γ *has constant angle* θ *with unit vector field* u *along* γ *such that* $\nabla_{\mathbf{t}} u = 0$, *that is*

$$g(\mathbf{m}_1(s), u) = \cos\theta \textit{ for all } s \in I. \tag{3.9}$$

The condition is not altered by reparametrization, so without loss of generality we may assume that slant helices have unit speed. The slant helices can be identified by a simple condition on Bishop curvatures.

To separate a slant helix according to Bishop frame from that of Frenet-Serret frame, in the rest of the paper, we shall use notation for the curve defined above as $B-$slant helix.

Theorem 3.3. *(see [10]) Let* $\gamma : I \longrightarrow \mathbb{K}$ *be a unit speed curve with non-zero Bishop curvatures. Then* γ *is a* $B-$*slant helix if and only if*

$$\frac{k_1}{k_2} = \textit{constant}. \tag{3.10}$$

Theorem 3.4. *Let $\gamma : I \longrightarrow \mathbb{K}$be a unit speed biharmonic B-slant helix with non-zero Bishop curvatures. Then*

$$k_1 = constant \text{ and } k_2 = constant. \tag{3.11}$$

Proof: Using Theorem 3.3. we have above system. □

Theorem 3.5. *(see [10]) Let $\gamma : I \longrightarrow \mathbb{K}$ be a unit speed non-geodesic biharmonic B-slant helix. Then, the parametric equations of γ are*

$$\begin{aligned} x^1(s) &= \frac{a_1 \cos\theta e^{\sin\theta s}}{\mathbb{k}^2 + \sin^2\theta}(\sin\theta\cos(\mathbb{k}s + \ell) + \mathbb{k}\sin(\mathbb{k}s + \ell)) + a_2, \\ x^2(s) &= \frac{a_1 \cos\theta e^{\sin\theta s}}{\mathbb{k}^2 + \sin^2\theta}(-\mathbb{k}\cos(\mathbb{k}s + \ell) + \sin\theta\sin(\mathbb{k}s + \ell)) + a_3, \quad (3.12) \\ x^3(s) &= a_1 e^{\sin\theta s}, \end{aligned}$$

where a_1, a_2, a_3, ℓ are constants of integration and

$$\mathbb{k} = [\frac{k_1^2 + k_2^2 - \cos^4\theta - \cos^2\theta\sin^2\theta}{\cos^2\theta}]^{\frac{1}{2}}.$$

4. Inextensible Flows of Tangent Developable Surfaces according to Bishop Frame in the Special Three-Dimensional Kenmotsu Manifold $\mathbb{K}$

The tangent developable of γ is a ruled surface

$$\Pi(s,u) = \gamma(s) + u\gamma'(s). \tag{4.1}$$

Let ϖ be the standard unit normal vector field on a surface Π defined by

$$\varpi = \frac{\Pi_s \wedge \Pi_u}{|g(\Pi_s \wedge \Pi_u, \Pi_s \wedge \Pi_u)|^{\frac{1}{2}}}.$$

Then, the first fundamental form **I** and the second fundamental form **II** of a surface Π are defined by, respectively,

$$\begin{aligned} \mathbf{I} &= \mathbf{E}ds^2 + 2\mathbf{F}dsdu + \mathbf{G}dt^2, \\ \mathbf{II} &= \mathbf{e}ds^2 + 2\mathbf{f}dsdu + \mathbf{g}dt^2, \end{aligned}$$

where

$$\mathbf{E} = g(\Pi_s, \Pi_s), \quad \mathbf{F} = g(\Pi_s, \Pi_u), \quad \mathbf{G} = g(\Pi_u, \Pi_u),$$

$$\begin{aligned} \mathbf{e} &= -g(\Pi_s, \varpi_s), \\ \mathbf{f} &= -g(\Pi_s, \varpi_u), \\ \mathbf{g} &= -g(\Pi_u, \varpi_u). \end{aligned}$$

On the other hand, the Gaussian curvature $\mathbf{K}$ and the mean curvature $\mathbf{H}$ are

$$\mathbf{K} = \frac{\mathbf{eg} - \mathbf{f}^2}{\mathbf{EG} - \mathbf{F}^2},$$

$$\mathbf{H} = \frac{\mathbf{Eg} - \mathbf{2Ff} + \mathbf{Ge}}{2\left(\mathbf{EG} - \mathbf{F}^2\right)},$$

respectively.

Definition 4.1. *([9]) A surface evolution $\Pi(s,u,t)$ and its flow $\frac{\partial \Pi}{\partial t}$ are said to be inextensible if its first fundamental form $\{\mathbf{E},\mathbf{F},\mathbf{G}\}$ satisfies*

$$\frac{\partial \mathbf{E}}{\partial t} = \frac{\partial \mathbf{F}}{\partial t} = \frac{\partial \mathbf{G}}{\partial t} = 0. \tag{4.2}$$

This definition states that the surface $\Pi(s,u,t)$ is, for all time t, the isometric image of the original surface $\Pi(s,u,t_0)$ defined at some initial time t_0. For a developable surface, $\Pi(s,u,t)$ can be physically pictured as the parametrization of a waving flag. For a given surface that is rigid, there exists no nontrivial inextensible evolution.

Definition 4.2. *We can define the following one-parameter family of developable ruled surface*

$$\Pi(s,u,t) = \gamma(s,t) + u\gamma'(s,t). \tag{4.3}$$

Hence, we have the following theorem.

Theorem 4.3. *Let Π is the tangent developable surface associated with non-geodesic biharmonic B-slant helix. $\frac{\partial \Pi}{\partial t}$ is inextensible if and only if*

$$\begin{aligned}
&\frac{\partial}{\partial t}[\cos\theta\cos[(\frac{k_1^2+k_2^2-\cos^2\theta}{\cos^2\theta})^{\frac{1}{2}}s+\ell]+uk_1\sin\theta\cos[(\frac{k_1^2+k_2^2-\cos^2\theta}{\cos^2\theta})^{\frac{1}{2}}s+\ell]\\
&+uk_2\sin[(\frac{k_1^2+k_2^2-\cos^2\theta}{\cos^2\theta})^{\frac{1}{2}}s+\ell]]^2\\
&+\frac{\partial}{\partial t}[\cos\theta\sin[(\frac{k_1^2+k_2^2-\cos^2\theta}{\cos^2\theta})^{\frac{1}{2}}s+\ell]+uk_1\sin\theta\sin[(\frac{k_1^2+k_2^2-\cos^2\theta}{\cos^2\theta})^{\frac{1}{2}}s+\ell]\\
&-uk_2\cos[(\frac{k_1^2+k_2^2-\cos^2\theta}{\cos^2\theta})^{\frac{1}{2}}s+\ell]]^2\\
&+\frac{\partial}{\partial t}[-\sin\theta+uk_1\cos\theta]^2=0,
\end{aligned} \tag{4.4}$$

where ℓ is constant of integration and θ, k_1, k_2 are function of time t.

Proof: Assume that $\Pi(s,u,t)$ be a one-parameter family of ruled surface.

From (3.9), we have the following equation

$$\mathbf{m}_1 = \sin\theta\cos[(\frac{k_1^2+k_2^2-\cos^2\theta}{\cos^2\theta})^{\frac{1}{2}}s+\ell]\mathbf{e}_1+\sin\theta\sin[(\frac{k_1^2+k_2^2-\cos^2\theta}{\cos^2\theta})^{\frac{1}{2}}s+\ell]\mathbf{e}_2+\cos\theta\mathbf{e}_3, \tag{4.5}$$

where ℓ is constant of integration.

On the other hand, using Bishop formulas (3.4) and (2.1), we have

$$\mathbf{m}_2 = \sin[(\frac{k_1^2+k_2^2-\cos^2\theta}{\cos^2\theta})^{\frac{1}{2}}s+\ell]\mathbf{e}_1-\cos[(\frac{k_1^2+k_2^2-\cos^2\theta}{\cos^2\theta})^{\frac{1}{2}}s+\ell]\mathbf{e}_2. \tag{4.6}$$

Using above equation and (4.5), we get

$$\mathbf{t} = \cos\theta\cos[(\frac{k_1^2+k_2^2-\cos^2\theta}{\cos^2\theta})^{\frac{1}{2}}s+\ell]\mathbf{e}_1+\cos\theta\sin[(\frac{k_1^2+k_2^2-\cos^2\theta}{\cos^2\theta})^{\frac{1}{2}}s+\ell]\mathbf{e}_2-\sin\theta\mathbf{e}_3. \tag{4.7}$$

Furthermore, we have the natural frame $\{\Pi_s, \Pi_u\}$ given by

$$\begin{aligned}\Pi_s \;=\;& \mathbf{t}+uk_1\mathbf{m}_1+uk_2\mathbf{m}_2=[\cos\theta\cos[(\frac{k_1^2+k_2^2-\cos^2\theta}{\cos^2\theta})^{\frac{1}{2}}s+\ell]\\
&+uk_1\sin\theta\cos[(\frac{k_1^2+k_2^2-\cos^2\theta}{\cos^2\theta})^{\frac{1}{2}}s+\ell]+uk_2\sin[(\frac{k_1^2+k_2^2-\cos^2\theta}{\cos^2\theta})^{\frac{1}{2}}s+\ell]]\mathbf{e}_1\\
&+[\cos\theta\sin[(\frac{k_1^2+k_2^2-\cos^2\theta}{\cos^2\theta})^{\frac{1}{2}}s+\ell]+uk_1\sin\theta\sin[(\frac{k_1^2+k_2^2-\cos^2\theta}{\cos^2\theta})^{\frac{1}{2}}s+\ell]\\
&-uk_2\cos[(\frac{k_1^2+k_2^2-\cos^2\theta}{\cos^2\theta})^{\frac{1}{2}}s+\ell]]\mathbf{e}_2+[-\sin\theta+uk_1\cos\theta]\mathbf{e}_3\end{aligned}$$

and

$$\Pi_u = \cos\theta\cos[(\frac{k_1^2+k_2^2-\cos^2\theta}{\cos^2\theta})^{\frac{1}{2}}s+\ell]\mathbf{e}_1+\cos\theta\sin[(\frac{k_1^2+k_2^2-\cos^2\theta}{\cos^2\theta})^{\frac{1}{2}}s+\ell]\mathbf{e}_2-\sin\theta\mathbf{e}_3.$$

The components of the first fundamental form are

$$\begin{aligned}\mathbf{E} \;=\;& g(\Pi_s,\Pi_s)=[\cos\theta\cos[(\frac{k_1^2+k_2^2-\cos^2\theta}{\cos^2\theta})^{\frac{1}{2}}s+\ell]\\
&+uk_1\sin\theta\cos[(\frac{k_1^2+k_2^2-\cos^2\theta}{\cos^2\theta})^{\frac{1}{2}}s+\ell]+uk_2\sin[(\frac{k_1^2+k_2^2-\cos^2\theta}{\cos^2\theta})^{\frac{1}{2}}s+\ell]]^2\\
&+[\cos\theta\sin[(\frac{k_1^2+k_2^2-\cos^2\theta}{\cos^2\theta})^{\frac{1}{2}}s+\ell]+uk_1\sin\theta\sin[(\frac{k_1^2+k_2^2-\cos^2\theta}{\cos^2\theta})^{\frac{1}{2}}s+\ell]\\
&-uk_2\cos[(\frac{k_1^2+k_2^2-\cos^2\theta}{\cos^2\theta})^{\frac{1}{2}}s+\ell]]^2+[-\sin\theta+uk_1\cos\theta]^2,\end{aligned}$$

$$\begin{aligned}\mathbf{F} &= g(\Pi_s,\Pi_u)=1,\\
\mathbf{G} &= g(\Pi_u,\Pi_u)=1.\end{aligned}$$

Using second and third equation of above system, we have

$$\begin{aligned}\frac{\partial\mathbf{F}}{\partial t} &= 0,\\
\frac{\partial\mathbf{G}}{\partial t} &= 0.\end{aligned}$$

Hence, $\dfrac{\partial \Pi}{\partial t}$ is inextensible if and only if (4.4) is satisfied. This concludes the proof of theorem. □

Tangent developable of γ may be seen by the aid Mathematica program as follows:

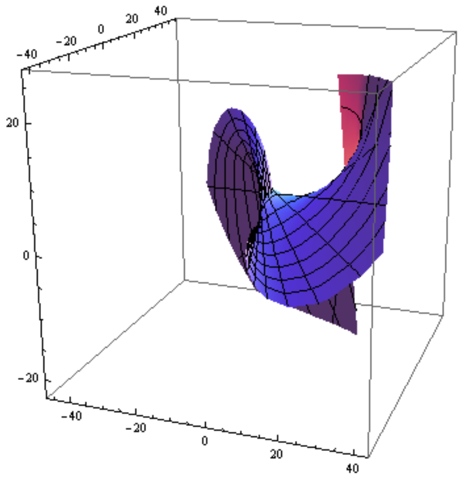

Figure 1.

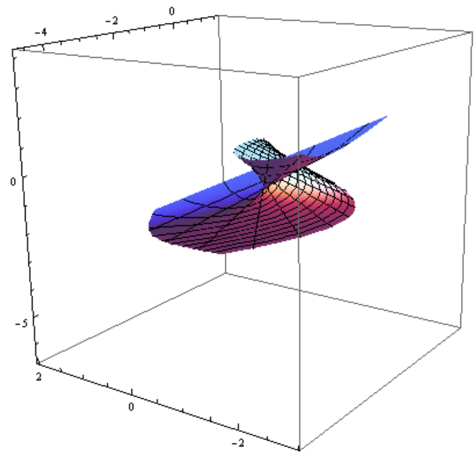

Figure 2.

References

1. K. Arslan, R. Ezentas, C. Murathan, T. Sasahara, *Biharmonic submanifolds 3-dimensional* (κ,μ)*-manifolds,* Internat. J. Math. Math. Sci. 22 (2005), 3575-3586.
2. D. E. Blair: *Contact Manifolds in Riemannian Geometry*, Lecture Notes in Mathematics, Springer-Verlag 509, Berlin-New York, 1976.
3. S. Degla, L. Todjihounde: *Biharmonic Reeb curves in Sasakian manifolds,* arXiv:1008.1903.
4. J. Eells and L. Lemaire: *A report on harmonic maps,* Bull. London Math. Soc. 10 (1978), 1–68.
5. J. Eells and J. H. Sampson: *Harmonic mappings of Riemannian manifolds,* Amer. J. Math. 86 (1964), 109–160.
6. T. Hasanis and T. Vlachos: *Hypersurfaces in* $\mathbb{E}^4$ *with harmonic mean curvature vector field,* Math. Nachr. 172 (1995), 145–169.
7. G. Y.Jiang: *2-harmonic isometric immersions between Riemannian manifolds,* Chinese Ann. Math. Ser. A 7(2) (1986), 130–144.
8. J.B. Jun, U.C. De and G. Pathak: *On Kenmotsu Manifolds*, J. Korean Math. Soc. 42, 3 (2005), 435-445.
9. DY. Kwon , FC. Park, DP Chi: *Inextensible flows of curves and developable surfaces*, Appl. Math. Lett. 18 (2005), 1156-1162.
10. T. Körpınar E. Turhan and V. Asil: *On biharmonic B-slant helices according to Bishop frame in the special 3-dimensional Kenmotsu Manifold* $\mathbb{K}$ *with* $\eta-$*parallel Ricci tensor,* (submitted)
11. B. O'Neill: *Semi-Riemannian Geometry*, Academic Press, New York (1983).
12. C. Özgür: *Contact metric manifolds with cyclic-parallel Ricci tensor,* Differ. Geom. Dyn. Syst. 4 (2002), 21-25.
13. P. Piu, A. Sanini: *One-parameter subgroups and minimal surfaces in the Heisenberg group,* Note di Matematica 18 (1) (1998), 143-153.
14. E. Turhan and T. Körpınar: *Characterize on the Heisenberg Group with left invariant Lorentzian metric*, Demonstratio Mathematica 42 (2) 2009, 423-428.

Vedat Asil
Fırat University,
Department of Mathematics
23119, Elazığ, Turkey
E-mail address: vasil@firat.edu.tr

and

Talat Körpinar
Fırat University,
Department of Mathematics
23119, Elazığ, Turkey
E-mail address: talatkorpinar@gmail.com

and

Essin Turhan
Fırat University,
Department of Mathematics
23119, Elazığ, Turkey
E-mail address: essin.turhan@gmail.com