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Operators on Grill $\mathcal{M}$-Space

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ABSTRACT: In this paper, we shall obtain a new topology from non topological space. We also discuss the various properties of such spaces.

Key Words: grill m-space, $\varphi_{\mathcal{G}}$- operator, ψ_{φ}- operator.

Contents

1. Introduction

The concept of grill is already in literature. Mathematicians like Choquet [5], Chattopadhyay, Njastad and Thrown [3,4] have considered the concept on topological space and using this concept they have developed the topics; Proximity spaces, Closure spaces, the Theory of Compactifications and similar other extension problems. The notion of grill topological space as like ideal topological space [6,7] was introduced by Roy and Mukherjee [9]. After that Al-Omari and Noiri [1] studied the field in detail. A new type of generalization of topological space has been introduced by Al-Omari and Noiri [2], and the space is called m-space. They studied this space in front of ideal.

In this paper we have considered m-space and grill on m-space, and introduced two operators. Ultimate we have obtained a topology, however m-space need not a topological space. We have also discussed the properties of the new topology.

2. Preliminaries

In this section we shall discuss some definitions and theorems:

Definition 2.1. *[2] A subfamily $\mathcal{M}$ of the power set $\wp(X)$ of a nonempty set X is called an m-structure on X if $\mathcal{M}$ satisfies the following conditions:*

1. *$\mathcal{M}$ contains ϕ and X,*
2. *$\mathcal{M}$ is closed under the finite intersection.*

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The pair $(X, \mathcal{M})$ *is called an* m*-space.*

Definition 2.2. *[2] A set* $A \in \wp(X)$ *is called an* m*-open set if* $A \in \mathcal{M}$*.* $B \in \wp(X)$ *is called an* m*-closed set if* $X \setminus B \in \mathcal{M}$*. We set* $mInt(A) = \cup\{U : U \subseteq A,\ U \in \mathcal{M}\}$ *and* $mCl(A) = \cap\{F : A \subseteq F,\ X \setminus F \in \mathcal{M}\}$*.*

Here we shall prove two theorems related to $mInt(A)$ and $mCl(A)$:

Theorem 2.3. *Let* $(X, \mathcal{M})$ *be an* m*-space. Then* $x \in mCl(A)$ *if and only if every* m*-open set* U_x *containing* x*,* $U_x \cap A \neq \phi$*.*

Proof: Let $x \in mCl(A)$. If possible supposed that $U_x \cap A = \phi$, where U_x is an m-open set containing x. Then $A \subseteq (X \setminus U_x)$ and $X \setminus U_x$ is an m-closed set containing A. Therefore $x \in (X \setminus U_x)$, a contradiction. Conversely supposed that $U_x \cap A \neq \phi$, for every m-open set U_x containing x. If possible suppose that $x \notin mCl(A)$, then there exists F subset of X which satisfy $A \subseteq F, X \setminus F \in \mathcal{M}$ and $x \notin F$. Therefore $x \in (X \setminus F)$. So for an m-open set $X \setminus F$ containing x, $A \cap (X \setminus F) = \phi$, a contradiction to the fact that $U_x \cap A \neq \phi$. □

Theorem 2.4. *Let* $(X, \mathcal{M})$ *be an* m*-space and* $A \subseteq X$*. Then* $mInt(A) = X \setminus mCl(X \setminus A)$*.*

Proof: Let $x \in mInt(A)$. Then there is an $U \in \mathcal{M}$, such that $x \in U \subseteq A$. Hence $x \notin (X \setminus U)$, i.e., $x \notin mCl(X \setminus U)$, since $X \setminus U$ is an m-closed set containing $X \setminus U$. So $x \notin mCl(X \setminus A)$(from Definition 2.2), and hence $x \in X \setminus mCl(X \setminus A)$.
Conversely suppose that $x \in X \setminus mCl(X \setminus A)$. So $x \notin mCl(X \setminus A)$, then there is an m-open set U_x containing x, such that $U_x \cap (X \setminus A) = \phi$. So $U_x \subseteq A$. Therefore $x \in mInt(A)$. Hence the result. □

A subcollection $\mathcal{G}$ (not containing the empty set) of $\wp(X)$ is called a grill [5] on X if $\mathcal{G}$ satisfies the following conditions:

1. $A \in \mathcal{G}$ and $A \subseteq B$ implies $B \in \mathcal{G}$;
2. $A, B \subseteq X$ and $A \cup B \in \mathcal{G}$ implies that $A \in \mathcal{G}$ or $B \in \mathcal{G}$.

An m-space $(X, \mathcal{M})$ with a grill $\mathcal{G}$ on X is called a grill m-space and is denoted as $(X, \mathcal{M}, \mathcal{G})$.

3. $\varphi_{\mathcal{G}}$-Operator

In this section we shall obtain a topology with the help of $\varphi_{\mathcal{G}}$-Operator.

Definition 3.1. *Let* $(X, \mathcal{M})$ *be an* m*-space and* $\mathcal{G}$ *be a grill on* X*. A mapping* $\varphi_{\mathcal{G}}$*:* $\wp(X) \to \wp(X)$ *is defined as follows:* $\varphi_{\mathcal{G}}(A) = \varphi(A) = \{x \in X : A \cap U \in \mathcal{G}$ *for all* $U \in \mathcal{M}(x)\}$ *for each* $A \in \wp(X)$*, where* $\mathcal{M}(x) = \{U \in \mathcal{M}\text{: } x \in U\}$*. The mapping* φ *is called the operator associated with the grill* $\mathcal{G}$ *and the* m*-structure* $\mathcal{M}$ *on* X*.*

Properties of $\varphi_{\mathcal{G}}$:
(1). Let $(X, \mathcal{M}, \mathcal{G})$ be a grill m-space. Then $\varphi(\phi) = \phi$.

Proof: Obvious from definition. □

Corollary 3.2. *Let $(X, \mathcal{M}, \mathcal{G})$ be a grill m-space. Then for $G \notin \mathcal{G}$, $\varphi(G) = \phi$.*

(2). Let $(X, \mathcal{M}, \mathcal{G})$ be a grill m-space. Then for $A \subseteq X$, $\varphi(A) \subseteq mCl(A)$.
Proof. Let $x \notin mCl(A)$, then from Theorem 2.3, $U \in \mathcal{M}(x)$ such that $U \cap A = \phi \notin \mathcal{G}$. Implies that $x \notin \varphi(A)$. Hence $\varphi(A) \subseteq mCl(A)$.
(3). Let $(X, \mathcal{M}, \mathcal{G})$ be a grill m-space. Then for $A \subseteq X$, $mCl[\varphi(A)] \subseteq \varphi(A)$.

Proof: Let $x \in mCl[\varphi(A)]$ and $U \in \mathcal{M}(x)$ then $U \cap \varphi(A) \neq \phi$. Let $y \in U \cap \varphi(A)$. Then $y \in U$ and $y \in \varphi(A)$. Therefore $U \cap A \in \mathcal{G}$, and hence $x \in \varphi(A)$. Thus $mCl[\varphi(A)] \subseteq \varphi(A)$. □

Corollary 3.3. *Let $(X, \mathcal{M}, \mathcal{G})$ be a grill m-space. Then for $A \subseteq X$, $\varphi(A)$ is an m-closed set.*

(4). Let $(X, \mathcal{M}, \mathcal{G})$ be a grill m- space. Then for $A \subseteq X$, $\varphi[\varphi(A)] \subseteq \varphi(A)$.

Proof: From (2), $\varphi[\varphi(A)] \subseteq mCl[\varphi(A)]$. Again from (3), $mCl[\varphi(A)] \subseteq \varphi(A)$. So, $\varphi[\varphi(A)] \subseteq \varphi(A)$. □

(5). Let$(X, \mathcal{M}, \mathcal{G})$ be a grill m- space. Then for $A, B \subseteq X$ and $A \subseteq B$, $\varphi(A) \subseteq \varphi(B)$.

Proof: Let $x \in \varphi(A)$. Then for all $U \in \mathcal{M}(x)$, $U \cap A \in \mathcal{G}$. Again it is obvious that $U \cap B \in \mathcal{G}$ (from definition of grill). Hence $x \in \varphi(B)$. □

(6). If $\mathcal{G}_1$ and $\mathcal{G}_2$ are two grills on m-space $(X, \mathcal{M})$ and $\mathcal{G}_1 \subseteq \mathcal{G}_2$, then $\varphi_{\mathcal{G}1}(A) \subseteq \varphi_{\mathcal{G}2}(A)$.

Proof: Obvious. □

(7). Let $(X, \mathcal{M}, \mathcal{G})$ be a grill m-space. Then for $A, B \subseteq X$, $\varphi(A \cup B) = \varphi(A) \cup \varphi(B)$.

Proof: From (5), $\varphi(A) \cup \varphi(B) \subseteq \varphi(A \cup B)$. For reverse inclusion, suppose that $x \notin \varphi(A) \cup \varphi(B)$. Then there are $U_1, U_2 \in \mathcal{M}(x)$ such that $U_1 \cap A \notin \mathcal{G}$, $U_2 \cap B \notin \mathcal{G}$ and hence $(U_1 \cap A) \cup (U_2 \cap B) \notin \mathcal{G}$. Now $U_1 \cap U_2 \in \mathcal{M}(x)$ and $(A \cup B) \cap (U_1 \cap U_2) \subseteq (U_1 \cap A) \cup (U_2 \cap B) \notin \mathcal{G}$, so, $x \notin \varphi(A \cup B)$. Therefore $\varphi(A \cup B) \subseteq \varphi(A) \cup \varphi(B)$. Hence the result. □

(8). Let $\mathcal{G}$ be a grill on m-space $(X, \mathcal{M})$. If $U \in \mathcal{M}$, then $U \cap \varphi(A) = U \cap \varphi(U \cap A)$, for any $A \subseteq X$.

Proof: From (5), $U \cap \varphi(U \cap A) \subseteq U \cap \varphi(A)$. For reverse inclusion, suppose $x \in U \cap \varphi(A)$ and $V \in \mathcal{M}(x)$. Then $U \cap V \in \mathcal{M}(x)$ and $x \in \varphi(A)$, implies $(U \cap V) \cap A \in \mathcal{G}$. So $(U \cap A) \cap V \in \mathcal{G}$. This implies that $x \in \varphi(U \cap A)$. Thus $x \in U \cap \varphi(U \cap A)$. □

(9). Let $\mathcal{G}$ be a grill on m-space $(X, \mathcal{M})$ and $A, B \subseteq X$. Then $[\varphi(A) \setminus \varphi(B)] = [\varphi(A \setminus B) \setminus \varphi(B)]$.

Proof: Here, $\varphi(A) = \varphi[(A \setminus B) \cup (A \cap B)] = [\varphi(A \setminus B) \cup \varphi(A \cap B)]$(from (7))$\subseteq [\varphi(A \setminus B) \cup \varphi(B)]$(from (5)). Thus $[\varphi(A) \setminus \varphi(B)] \subseteq [\varphi(A \setminus B) \setminus \varphi(B)]$. Again, $\varphi(A \setminus B) \subseteq \varphi(A)$(from (5)).This implies that $[\varphi(A \setminus B) \setminus \varphi(B)] \subseteq [\varphi(A) \setminus \varphi(B)]$. Hence $[\varphi(A) \setminus \varphi(B)] = [\varphi(A \setminus B) \setminus \varphi(B)]$. □

Corollary 3.4. *Let $\mathcal{G}$ be a grill on m-space $(X, \mathcal{M})$ and suppose $A, B \subseteq X$ with $B \notin \mathcal{G}$. Then $\varphi(A \cup B) = \varphi(A) = \varphi(A \setminus B)$.*

Proof: We know from (7), $\varphi(A \cup B) = \varphi(A) \cup \varphi(B) = \varphi(A)$ (from Corollary 3.2). Again from Property5, $\varphi(A \setminus B) \subseteq \varphi(A)$. Also from (5), $[\varphi(A) \setminus \varphi(B)] \subseteq \varphi(A \setminus B)$. This implies that $\varphi(A) \subseteq \varphi(A \setminus B)$, since $B \notin \mathcal{G}$. Thus $\varphi(A) = \varphi(A \setminus B)$. □

Let $\mathcal{G}$ be a grill on the m-space $(X, \mathcal{M})$. We define a map $CL : \wp(X) \rightarrow \wp(X)$ by $CL(A) = A \cup \varphi(B)$, for all $A \in \wp(X)$. Then we have:

Theorem 3.5. *The above map CL satisfies Kuratowski Closure axioms.*

Proof: From Property1, $CL(\phi) = \phi$, and obviously $A \subseteq CL(A)$. Now $CL(A \cup B) = (A \cup B) \cup \varphi(A \cup B) = (A \cup B) \cup \varphi(A) \cup \varphi(B)$(from Property7)$= CL(A) \cup CL(B)$. Again for any $A \subseteq X$, $CL[CL(A)] = CL[A \cup \varphi(A)] = [A \cup \varphi(A)] \cup \varphi[A \cup \varphi(A)] = A \cup \varphi(A) \cup \varphi[\varphi(A)]$(from Property7) $= A \cup \varphi(A)$ (from Property4) $= CL(A)$. □

If $\mathcal{G}$ is a grill on the m-space $(X, \mathcal{M})$, then from Kuratowski Closure operator CL, we get an unique topology on X which is given by following:

Theorem 3.6. *Let $(X, \mathcal{M}, \mathcal{G})$ be a grill m-space. Then $\tau_{\mathcal{M}\mathcal{G}} = \{V \subseteq X : CL(X \setminus V) = X \setminus V\}$ is a topology on X, where $CL(A) = A \cup \varphi(A)$.*

We denote the closure of A with respect to the topology $\tau_{\mathcal{M}\mathcal{G}}$ by $\tau_{\mathcal{M}\mathcal{G}}$-$cl(A)$
Properties of the topology $\tau_{\mathcal{M}\mathcal{G}}$:

Theorem 3.7. *(a). If $\mathcal{G}_1$ and $\mathcal{G}_1$ are two grills on X with $\mathcal{G}_1 \subseteq \mathcal{G}_2$, then $\tau_{\mathcal{M}\mathcal{G}2} \subseteq \tau_{\mathcal{M}\mathcal{G}1}$.*
(b). If $\mathcal{G}$ is a grill on a set X and $B \notin \mathcal{G}$, then B is closed in $(X, \tau_{\mathcal{M}\mathcal{G}})$.
(c). For any subset A of a m-space $(X, \mathcal{M})$ and any grill $\mathcal{G}$ on X, $\varphi(A)$ is $\tau_{\mathcal{M}\mathcal{G}}$-closed.

Proof: (a). Let $U \in \tau_{\mathcal{M}\mathcal{G}2}$. Then $\tau_{\mathcal{M}\mathcal{G}2}$-$cl(X \setminus U) = CL(X \setminus U)$. This implies that $(X \setminus U) = (X \setminus U) \cup \varphi_{\mathcal{G}2}(X \setminus U)$. Thus $\varphi_{\mathcal{G}2}(X \setminus U) \subseteq (X \setminus U)$. Implies that $\varphi_{\mathcal{G}1}(X \setminus U) \subseteq (X \setminus U)$(from Proprerty6). So $(X \setminus U) = \tau_{\mathcal{M}\mathcal{G}1}$-$cl(X \setminus U)$, and hence $U \in \tau_{\mathcal{M}\mathcal{G}1}$.
(b). It is obvious that, for $B \notin \mathcal{G}$, $\varphi(B) = \phi$. Then $\tau_{\mathcal{M}\mathcal{G}}$-$cl(B) = CL(B) = B \cup \varphi(B) = B$. Hence B is $\tau_{\mathcal{M}\mathcal{G}}$-closed.
(c). We have, $CL(\varphi(A)) = \varphi(A) \cup \varphi(\varphi(A)) = \varphi(A)$. Thus $\varphi(A)$ is $\tau_{\mathcal{M}\mathcal{G}}$- closed.
Here we find a simple open base for the topology $\tau_{\mathcal{M}\mathcal{G}}$ on X. □

Theorem 3.8. *Let $(X, \mathcal{M}, \mathcal{G})$ be a grill m- space. Then $\beta(\mathcal{M}, \mathcal{G}) = \{V \setminus A : V \in \mathcal{M}$ and $A \notin \mathcal{G}\}$ is an open base for $\tau_{\mathcal{M}\mathcal{G}}$.*

Proof: Let $U \in \tau_{\mathcal{M}\mathcal{G}}$ and $x \in U$. Then $(X \setminus U)$ is $\tau_{\mathcal{M}\mathcal{G}}$-closed so that $CL(X \setminus U) = (X \setminus U)$, and hence $\varphi(X \setminus U) \subseteq (X \setminus U)$. Then $x \notin \varphi(X \setminus U)$ and so there exists $V \in \mathcal{M}(x)$ such that $(X \setminus U) \cap V \notin \mathcal{G}$. Let $A = (X \setminus U) \cap V$, then $x \notin A$ and $A \notin \mathcal{G}$. Thus $x \in (V \setminus A) = V \setminus [(X \setminus U) \cap V] = V \setminus (V \setminus U) \subseteq U$, $V \setminus A \in \beta(\mathcal{M}, \mathcal{G})$. It now suffices to observe that $\beta(\mathcal{M}, \mathcal{G})$ is closed under finite intersections. Let $V_1 \setminus A$, $V_2 \setminus B \in \beta(\mathcal{M}, \mathcal{G})$, that is V_1, $V_2 \in \mathcal{M}$ and A, $B \notin \mathcal{G}$. Then $V_1 \cap V_2 \in \mathcal{M}$ and $A \cup B \notin \mathcal{G}$. Now, $(V_1 \setminus A) \cap (V_2 \setminus B) = (V_1 \cap V_2) \setminus (A \cup B) \in \beta(\mathcal{M}, \mathcal{G})$, proving ultimate that $\beta(\mathcal{M}, \mathcal{G})$ is an open base for $\tau_{\mathcal{M}\mathcal{G}}$. □

Corollary 3.9. *For any grill $\mathcal{G}$ on an m-space $(X, \mathcal{M})$, $\mathcal{M} \subseteq \beta(\mathcal{M}, \mathcal{G}) \subseteq \tau_{\mathcal{M}\mathcal{G}}$.*

4. ψ_φ-Operator

An important result in topological space (X, τ) is:
$Int(A) = X \setminus Cl(X \setminus A)$ [8]. This is the relation between interior and closure operators. Same relation also hold in m-space(Theorem 2.4). In this section we are interested to find out the similar result with the help of $\varphi_{\mathcal{G}}$ and ψ_φ operators.

Definition 4.1. *Let $(X, \mathcal{M}, \mathcal{G})$ be a grill m- space. An operator $\psi_\varphi : \wp(X) \to \mathcal{M}$ is defined as follows for every $A \in \wp(X)$, $\psi_\varphi(A) = \{x \in X$: there exists a $U \in \mathcal{M}(x)$ such that $U \setminus A \notin \mathcal{G}\}$ and observe that $\psi_\varphi(A) = X \setminus \varphi(X \setminus A)$.*

Several basic facts concerning the behavior of the operator ψ_φ are given bellow:

Theorem 4.2. *Let $(X, \mathcal{M}, \mathcal{G})$ be a grill m-space. Then following properties hold:*
(i). If $A \subseteq X$, then $\psi_\varphi(A)$ is $\mathcal{M}$-open in $(X, \mathcal{M})$.
(ii). If $A \subseteq B$, then $\psi_\varphi(A) \subseteq \psi_\varphi(B)$.
(iii). If $A, B \in \wp(X)$, then $\psi_\varphi(A \cap B) = \psi_\varphi(A) \cap \psi_\varphi(B)$.
(iv). If $U \in \tau_{\mathcal{M}\mathcal{G}}$, then $U \subseteq \psi_\varphi(U)$.
(v). If $A \subseteq X$, then $\psi_\varphi(A) \subseteq \psi_\varphi(\psi_\varphi(A))$.
(vi). Let $A \subseteq X$, then $\psi_\varphi(A) = \psi_\varphi(\psi_\varphi(A))$ if and only if $\varphi(X \setminus A) = \varphi[\varphi(X \setminus A)]$.
(vii). If $A \notin \mathcal{G}$, then $\psi_\varphi(A) = X \setminus \varphi(X)$.
(viii). If $A \subseteq X$, then $A \cap \psi_\varphi(A) = \tau_{\mathcal{M}\mathcal{G}}$-int$(A)$ (where $\tau_{\mathcal{M}\mathcal{G}}$-int$(A)$ denote the interior operator of $(X, \tau_{\mathcal{M}\mathcal{G}})$).
(ix). If $A \subseteq X$ and $G \notin \mathcal{G}$, then $\psi_\varphi(A \setminus G) = \psi_\varphi(A)$.
(x). If $A \subseteq X$ and $G \notin \mathcal{G}$, then $\psi_\varphi(A \cup G) = \psi_\varphi(A)$.
(xi). If $A, B \subseteq X$ and $(A \setminus B) \cup (B \setminus A) \notin \mathcal{G}$, then $\psi_\varphi(A) = \psi_\varphi(B)$.

Proof: (i). Obvious from definition.
(ii). Obvious from Property5.
(iii). It is obvious from (ii), $\psi_\varphi(A \cap B) \subseteq \psi_\varphi(A)$ and $\psi_\varphi(A \cap B) \subseteq \psi_\varphi(B)$. Hence $\psi_\varphi(A \cap B) \subseteq \psi_\varphi(A) \cap \psi_\varphi(B)$. Now, let $x \in \psi_\varphi(A) \cap \psi_\varphi(B)$. There exists $U, V \in \mathcal{M}(x)$ such that $U \setminus A \notin \mathcal{G}$ and $V \setminus B \notin \mathcal{G}$. Let $G = U \cap V \in \mathcal{M}(x)$ and we have $G \setminus A \notin \mathcal{G}$ and $G \setminus B \notin \mathcal{G}$(from definition of grill). Thus $[G \setminus (A \cap B)] =$

$[(G \setminus A) \cup (G \setminus B)] \notin \mathcal{G}$ (from definition of grill), and hence $x \in \psi_\varphi(A \cap B)$. We have shown that $\psi_\varphi(A) \cap \psi_\varphi(B) \subseteq \psi_\varphi(A \cap B)$. Hence the prove is completed.
(iv). If $U \in \tau_{\mathcal{M}\mathcal{G}}$, then $X \setminus U$ is $\tau_{\mathcal{M}\mathcal{G}}$-closed which implies $\varphi(X \setminus U) \subseteq (X \setminus U)$ and hence $U \subseteq [X \setminus \varphi(X \setminus U)] = \psi_\varphi(U)$.
(v). This follows from (i) and (iv).
(vi). This follows from the facts:
1. $\psi_\varphi(A) = X \setminus \varphi(X \setminus A)$.
2. $\psi_\varphi(\psi_\varphi(A)) = [X \setminus \varphi[X \setminus (X \setminus \varphi(X \setminus A))]] = [X \setminus \varphi[\varphi(X \setminus A)]]$.
(vii). We know from Corollary 3.4, $\varphi(X \setminus A) = \varphi(X)$ if $A \notin \mathcal{G}$. Then $\psi_\varphi(A) = X \setminus \varphi(X)$.
(viii). If $x \in A \cap \psi_\varphi(A)$, then $x \in A$ and there exists a $U \in \mathcal{M}(x)$ such that $U \setminus A \notin \mathcal{G}$. Then by Theorem 3.8, $[U \setminus (U \setminus A)]$ is a $\tau_{\mathcal{M}\mathcal{G}}$ -open neighbourhood of x and $x \in \tau_{\mathcal{M}\mathcal{G}}\text{-}int(A)$. Conversely suppose that $x \in \tau_{\mathcal{M}\mathcal{G}}\text{-}int(A)$, there exists a basic $\tau_{\mathcal{M}\mathcal{G}}$-open neighbourhood $V \setminus G$ of x where $V \in \mathcal{M}(x)$ and $G \notin \mathcal{G}$, such that $x \in V \setminus G \subseteq A$ which implies that $V \setminus A \subseteq G$ and hence $V \setminus A \notin \mathcal{G}$. Hence $x \in A \cap \psi_\varphi(A)$.
(ix). $\psi_\varphi(A \setminus G) = [X \setminus \varphi[X \setminus (A \setminus G)]] = [X \setminus \varphi[(X \setminus A) \cup G]] = [X \setminus \varphi(X \setminus A)]$(since $G \notin \mathcal{G}) = \psi_\varphi(A)$.
(x). $\psi_\varphi(A \cup G) = X \setminus \varphi[X \setminus (A \cup G)] = X \setminus \varphi[(X \setminus A) \setminus G)] = X \setminus \varphi(X \setminus A)$(from (ix))$= \psi_\varphi(A)$.
(xi). Assume $(A \setminus B) \cup (B \setminus A) \notin \mathcal{G}$. Let $A \setminus B = G_1$ and $B \setminus A = G_2$. Observe that $G_1, G_2 \notin \mathcal{G}$ (from definition of grill). Also observe that $B = (A \setminus G_1) \cup G_2$. Thus $\psi_\varphi(A) = \psi_\varphi(A \setminus G_1) = \psi_\varphi[(A \setminus G_1) \cup G_2] = \psi_\varphi(A)$(from (ix) and (x)). □

Corollary 4.3. *Let $(X, \mathcal{M}, \mathcal{G})$ be a grill m-space. Then $U \subseteq \psi_\varphi(U)$ for every $U \in \mathcal{M}$.*

Proof: This follows from the fact $\mathcal{M} \subseteq \tau_{\mathcal{M}\mathcal{G}}$. □

References

1. A. Al-Omari and T. Noiri, On $\psi_{\mathcal{G}}$ -operator in grill topological spaces, Accepted at Annals of Oradea University Mathematics Fascicola.
2. A. Al-Omari and T. Noiri, On $\psi_\star$-operator in ideal m-spaces, Bol. Soc. Paran. Mat. (3s) v. 30 1 (2012) 53-66.
3. K. C. Chattopadhyay, O. Njastad, and W. J. Thron, Merotopic spaces and extensions of closure spaces, Can. J. Math., 35(4) (1983), 613 - 629.
4. K. C. Chattopadhyay, and W. J. Thron, Extensions of closure spaces, Can. J. Math., 29(6)(1977), 1277 - 1286.
5. G. Choqet, Sur les notions de filter et grill, Comptes Rendus Acad. Sci. Paris., 224 (1947), 171-173.
6. T. R. Hamlett, and D. Jankovic, Ideals in topological spaces and the set operator ψ , Bull. U.M.I., (7), 4-B(1990), 863 - 874.
7. D. Jankovic, and T. R. Hamlett, New topologies from old via ideals, Amer. Math. Monthly, 97(1990) 295 - 310.
8. K. Kuratowski, Topology I, Warszawa, 1933.

9. B. Roy, and M. N. Mukherjee, On a typical topology induced by a grill, Soochow J. Math. 33(4) (2007), 771 - 786.

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