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Topology of Grill Filter Space and Continuity

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abstract: This paper will discuss about a new topology, obtained from a grill and a filter on the same set. The Characterizations and open base of the new topology are also aim of this paper. The generalized continuity is also a part of this paper.

Key Words: grill-filter space, Ω-operator, ψ_Ω-operator, $\tau^{\psi}_{\mathcal{FG}}$-topology, $\mathcal{F}$-continuity.

Contents

1. Introduction

The notion of grill [7] and filter [21] is already in literature from 1947 and 1937 respectively. The topics - Proximity spaces, Closure spaces, the Theory of Compactifications and similar other extension problems [5,6,7,20] have been enriched by the study of grill. The filter is an important part in topological space for the discussion of the separation axioms, compactness, continuity etc. Recently mathematicians: Roy and Mukherjee [19], Noiri and Al-Omiri [1,2,3] have used grill on topological space as like ideal topological space [4,8,10,11,17,22] and have obtained many new topologies. Further Noiri and Al-Omiri and Modak et al [13,14,15,16] have considered ideal or grill on generalized spaces and discussed different types of topological space.

In this paper, we shall use grill and filter in different aspect something different from traditional uses of the same. Actually we shall define a space with grill and filter together on a set. From this space we define a topology via two operators. We also give a standard form of base, and characterize the topology. We also discuss a new type of generalized continuity on the new topological space. At last we shall obtain the relations of this continuity with usual continuity.

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2. Preliminaries

In this section we shall give some definitions and prove some results, which are the preliminaries for the paper. At first we shall give the formal definition of filter. A subcollection $\mathcal{F}$ (not containing the empty set) of $\wp(X)$ is called a filter [21] on X if $\mathcal{F}$ satisfies the following conditions:

1. $A \in \mathcal{F}$ and $A \subseteq B$ implies $B \in \mathcal{F}$;
2. $A,\ B \in \mathcal{F}$ implies $A \cap B \in \mathcal{F}$.

In this paper we shall try to obtain a topology with the help of filter, for this we shall discuss following:

Definition 2.1. *A set $A \in \wp(X)$ is called an $\mathcal{F}$-open set if $A \in \mathcal{F}$. $B \in \wp(X)$ is called a $\mathcal{F}$-closed set if $X \setminus B \in \mathcal{F}$. We set $\mathcal{F}$-$Int(A) = \cup\{U : U \subseteq A,\ U \in \mathcal{F}\}$ and $\mathcal{F}$-$Cl(A) = \cap\{F : A \subseteq F, X \setminus F \in \mathcal{F}\}$.*

Here we shall prove some theorems related to $\mathcal{F}$-Int and $\mathcal{F}$-Cl:

Theorem 2.1. *Let $\mathcal{F}$ be a filter on X and $A \subseteq X$. Then $x \in \mathcal{F}$-$Cl(A)$ if and only if every $\mathcal{F}$-open set U_x containing x, $U_x \cap A \neq \phi$.*

Proof: Let $x \in \mathcal{F}$-$Cl(A)$. Supposed that $U_x \cap A = \phi$, where U_x is an $\mathcal{F}$-open set containing x. Then $A \subseteq (X \setminus U_x)$ and $X \setminus U_x$ is a $\mathcal{F}$-closed set containing A. Therefore $x \notin (X \setminus U_x)$, and this is a contradiction. Conversely supposed that $U_x \cap A \neq \phi$, for every $\mathcal{F}$-open set U_x containing x. If possible suppose that $x \notin \mathcal{F}$-$Cl(A)$, then there exists F subset of X which satisfy $A \subseteq F, X \setminus F \in \mathcal{F}$ and $x \notin F$. Therefore $x \in (X \setminus F)$. So $A \cap (X \setminus F) = \phi$ for an $\mathcal{F}$-open set $X \setminus F$ containing x. It is a contradiction. □

Theorem 2.2. *Let $\mathcal{F}$ be a filter on X and $A \subseteq X$. Then $\mathcal{F}$-$Int(A) = X \setminus \mathcal{F}$-$Cl(X \setminus A)$.*

Proof: Let $x \in \mathcal{F}$-$Int(A)$. Then there is an $U \in \mathcal{F}$, such that $x \in U \subseteq A$. Hence $x \notin (X \setminus U)$, i.e., $x \notin \mathcal{F}$-$Cl(X \setminus U)$, since $X \setminus U$ is a $\mathcal{F}$-closed set containing $X \setminus A$. So $x \notin \mathcal{F}$-$Cl(X \setminus A)$(from Definition 2.1), and hence $x \in X \setminus \mathcal{F}$-$Cl(X \setminus A)$.
Conversely suppose that $x \in X \setminus \mathcal{F}$-$Cl(X \setminus A)$. So $x \notin \mathcal{F}$-$Cl(X \setminus A)$, then there is an $\mathcal{F}$-open set U_x containing x, such that $U_x \cap (X \setminus A) = \phi$. So $U_x \subseteq A$. Therefore $x \in \mathcal{F}$-$Int(A)$(from Definition 2.1). Hence the result. □

Theorem 2.3. *Let $\mathcal{F}$ be a filter on X and $A \subseteq X$. Then for $G \in \mathcal{F}$, $G \cap \mathcal{F}$-$Cl(A) \subseteq \mathcal{F}$-$Cl(G \cap A)$.*

Proof: Let $x \in G \cap \mathcal{F}$-$Cl(A)$. Then $x \in G$ and $x \in \mathcal{F}$-$Cl(A)$. Implies that $x \in G$ and for every $\mathcal{F}$-open set U_x containing x, $U_x \cap A \neq \phi$. Again $G \cap U_x$ is an $\mathcal{F}$-open set containing x, then $(G \cap U_x) \cap A \neq \phi$. Hence $x \in \mathcal{F}\text{-}Cl(G \cap A)$. Therefore $G \cap \mathcal{F}\text{-}Cl(A) \subseteq \mathcal{F}\text{-}Cl(G \cap A)$. □

Following is the concepts of grill [7]:
A subcollection $\mathcal{G}$ (not containing the empty set) of $\wp(X)$ is called a grill [7] on X if $\mathcal{G}$ satisfies the following conditions:

1. $A \in \mathcal{G}$ and $A \subseteq B$ implies $B \in \mathcal{G}$;
2. $A, B \subseteq X$ and $A \cup B \in \mathcal{G}$ implies that $A \in \mathcal{G}$ or $B \in \mathcal{G}$.

Let $\mathcal{F}$ and $\mathcal{G}$ be the filter and grill respectively on the same set X. Then $(X, \mathcal{F}, \mathcal{G})$ is denoted as grill-filter space.
One of the operator on grill-filter space is:

Definition 2.2. *[14]. Let $(X, \mathcal{F}, \mathcal{G})$ be a grill-filter space. A mapping $\Omega_{\mathcal{G}} \colon \wp(X) \to \wp(X)$ is defined as follows $\Omega_{\mathcal{G}}(A) = \Omega(A) = \{x \in X : A \cap U \in \mathcal{G}$ for all $U \in \mathcal{F}(x)\}$ for each $A \in \wp(X)$, where $\mathcal{F}(x) = \{U \in \mathcal{F} : x \in U\}$.*

Here we shall mention a property on Ω-operator, although so many properties have been discussed in [14].

Theorem 2.4. *Let $(X, \mathcal{F}, \mathcal{G})$ be a grill-filter space and $A, B \subseteq X$. Then $\Omega(A \cap B) \subseteq \Omega(A) \cap \Omega(B)$.*

Proof: Since $A \cap B \subseteq A$ and $A \cap B \subseteq B$, then $\Omega(A \cap B) \subseteq \Omega(A)$ [14] and $\Omega(A \cap B) \subseteq \Omega(B)$ [14]. Hence $\Omega(A \cap B) \subseteq \Omega(A) \cap \Omega(B)$. □

Following example shows that the reverse inclusion of the above theorem does not hold in general, however the relation, $\Omega(A \cup B) = \Omega(A) \cup \Omega(B)$ [14] hold.

Example 2.1. *Let $X = \{a, b, c, d\}$, $\mathcal{F} = \{X, \{a, b, c\}\}$ and $\mathcal{G} = \{\{a\}, \{b\}, \{a, c\}, \{a, b\}, \{a, d\}, \{a, b, c\}, \{c, b, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c\}, \{b, d\}, X\}$. Consider $A = \{a, d\}$, and $B = \{b, d\}$. Then $\Omega(\{a, d\}) = \{a, b, c, d\}$ and $\Omega(\{b, d\}) = \{a, b, c, d\}$, and hence $\Omega(A) \cap \Omega(B) = \{a, b, c, d\}$. But $\Omega(A \cap B) = \Omega(\{d\}) = \phi$. So, $\Omega(A) \cap \Omega(B)$ is not a subset of $\Omega(A \cap B)$.*

New topology from grill-filter space is:

Remark 2.1. *[14]. Let $(X, \mathcal{F}, \mathcal{G})$ be a grill-filter space. We define a map $CL : \wp(X) \to \wp(X)$ by $CL(A) = A \cup \Omega(A)$, for all $A \in \wp(X)$. Then the map $'CL'$ is a Kuratowski closure operator. We will denote $\tau_{\mathcal{F}\mathcal{G}}$, the topology, generated by CL, that is $\tau_{\mathcal{F}\mathcal{G}} = \{V \subseteq X : CL(X \setminus V) = X \setminus V\}$.*

In this paper we shall denote interior and closure operator of $(X, \tau_{\mathcal{F}\mathcal{G}})$ by $Int_{\mathcal{F}\mathcal{G}}$ and $Cl_{\mathcal{F}\mathcal{G}}$ respectively. Again $(X, \tau_{\mathcal{F}\mathcal{G}}, \mathcal{F}, \mathcal{G})$ will be denoted as grill-filter topological space.
Following is the representation of an open base for the topology $\tau_{\mathcal{F}\mathcal{G}}$:

Theorem 2.5. *[14]. Let $(X, \mathcal{F}, \mathcal{G})$ be a grill-filter space. Then $\beta(\mathcal{F}, \mathcal{G}) = \{V \setminus G$: $V \in \mathcal{F}$, $G \notin \mathcal{G}\}$ is an open base for the topology $\tau_{\mathcal{F}\mathcal{G}}$.*

Another operator on $(X, \mathcal{F}, \mathcal{G})$ is defined as follows:

Definition 2.3. *[14]. Let $(X, \mathcal{F}, \mathcal{G})$ be a grill-filter space. An operator $\psi_\Omega : \wp(X) \to \mathcal{F}$ is defined as follows for every $A \in \wp(X)$, $\psi_\Omega(A) = \{x \in X$: there exists $U \in \mathcal{F}(x)$ such that $U \setminus A \notin \mathcal{G}\}$ and observe that $\psi_\Omega(A) = X \setminus \Omega(X \setminus A)$.*

Now we shall prove some characterizations:

Theorem 2.6. *Let $(X, \mathcal{F}, \mathcal{G})$ be a grill-filter space. Then $\mathcal{F} \subseteq \mathcal{G}$ if and only if $\Omega(X) = X$.*

Proof: Suppose that $\mathcal{F} \subseteq \mathcal{G}$. It is obvious that $\Omega(X) \subseteq X$. For reverse inclusion, let $x \in X$ but $x \notin \Omega(X)$. Then there exists $U \in \mathcal{F}(x)$, $U \cap X \notin \mathcal{G}$. Then $U \notin \mathcal{G}$, a contradiction to the fact that $\mathcal{F} \subseteq \mathcal{G}$. Hence $\Omega(X) = X$.
Conversely suppose that $\Omega(X) = X$. Let $\phi \neq V \in \mathcal{F}$, then $V \cap X \neq \phi$. Since $\Omega(X) = X$, therefore $V \cap X \in \mathcal{G}$. Implies that $V \in \mathcal{G}$, and hence $\mathcal{F} \subseteq \mathcal{G}$. □

Corollary 2.1. *Let $(X, \mathcal{F}, \mathcal{G})$ be a grill-filter space and $A \in \mathcal{F}$. Then $\mathcal{F} \subseteq \mathcal{G}$ if and only if $\Omega(A) = \mathcal{F}\text{-}Cl(A)$.*

Proof: Suppose that $\mathcal{F} \subseteq \mathcal{G}$. It is obvious that $\Omega(A) \subseteq \mathcal{F}\text{-}Cl(A)$ [14]. For reverse inclusion, let $\alpha \in \mathcal{F}\text{-}Cl(A)$, then for every $U_\alpha \in \mathcal{F}(\alpha)$, $U_\alpha \cap A \neq \phi$ (from Theorem 2.1). Implies that $U_\alpha \cap A \in \mathcal{F} \subseteq \mathcal{G}$. So $\alpha \in \Omega(A)$, and hence $\Omega(A) = \mathcal{F}\text{-}Cl(A)$.
Converse part is obvious from Theorem 2.6. □

Joint result of the Theorem 2.6 and the Corollary 2.1 is:

Theorem 2.7. *Let $(X, \mathcal{F}, \mathcal{G})$ be a grill-filter space. Then following properties are equivalent:*

1. *$\mathcal{F} \subseteq \mathcal{G}$;*
2. *$X = \Omega(X)$;*
3. *If $A \in \mathcal{F}$, then $\Omega(A) = \mathcal{F}\text{-}Cl(A)$.*

Theorem 2.8. *Let $(X, \mathcal{F}, \mathcal{G})$ be a grill-filter space with $\mathcal{F} \subseteq \mathcal{G}$. Then for $\mathcal{F}$-closed subset A, $\psi_\Omega(A) \setminus A = \phi$.*

Proof: $\psi_\Omega(A) \setminus A = [X \setminus \Omega(X \setminus A] \setminus A = [X \setminus \mathcal{F}\text{-}Cl(X \setminus A)] \setminus A$(from Theorem 2.7) $= \mathcal{F}\text{-}Int(A) \setminus A$(from Theorem 2.2) $= \phi$. □

Theorem 2.9. *Let $(X, \mathcal{F}, \mathcal{G})$ be a grill-filter space and $A \subseteq X$. Then $\mathcal{F} \subseteq \mathcal{G}$ if and only if $\Omega[\psi_\Omega(A)] = \mathcal{F}\text{-}Cl[\psi_\Omega(A)]$.*

Proof: Proof is obvious from the fact that $\psi_\Omega(A)$ is an $\mathcal{F}$-open set [14] and the Theorem 2.7. □

Following theorem is an important part for the next section.

Theorem 2.10. *Let $(X, \mathcal{F}, \mathcal{G})$ be a grill-filter space, where $\mathcal{F} \subseteq \mathcal{G}$. Then for $A \subseteq X$, $\psi_\Omega(A) \subseteq \Omega(A)$.*

Proof: Suppose $x \in \psi_\Omega(A)$ but $x \notin \Omega(A)$. Then there exists $U_x \in \mathcal{F}(x)$ such that $U_x \cap A \notin \mathcal{G}$. Since $x \in \psi_\Omega(A)$, therefore there exists $V_x \in \mathcal{F}(x)$ such that $V_x \setminus A \notin \mathcal{G}$. Now $U_x \cap V_x \in \mathcal{F}(x)$ and $(U_x \cap V_x) \cap A \notin \mathcal{G}$(from definition of grill). Again $(U_x \cap V_x) \setminus A \notin \mathcal{G}$(from definition of grill). Write $U_x \cap V_x = [(U_x \cap V_x) \cap A] \cup [(U_x \cap V_x) \setminus A] \notin \mathcal{G}$. That is, $U_x \cap V_x \notin \mathcal{F}$, a contradiction. Hence $\psi_\Omega(A) \subseteq \Omega(A)$. □

Here we shall define some generalized open sets which are already in literature.

Definition 2.4. *[12]. A set A in a topological space (X, τ) is called semi-open if $A \subseteq cl(int(A))$. The set of all semi-open sets in a topological space (X, τ) is denoted as $SO(X, \tau)$.*

Definition 2.5. *[18]. A set A in a topological space (X, τ) is called α-set if $A \subseteq int(cl(int(A)))$. The set of all α-sets in a topological space (X, τ) is denoted as τ^α.*

Definition 2.6. *[9]. A topological space (X, τ) is said to be resolvable if there is a subset D of X such that both D and $X \setminus D$ are dense in (X, τ), otherwise it is said to be irresolvable.*

The space of reals with usual topology provides an example of a resolvable space while any topological space with an isolated point furnishes for an irresolvable one.

3. ψ_Ω-C set

This section deals with a new type of set and its properties:

Definition 3.1. *Let $(X, \mathcal{F}, \mathcal{G})$ be a grill-filter space. A subset A of X is called a ψ_Ω-C set if $A \subseteq \mathcal{F}$-$Cl[\psi_\Omega(A)]$.*

The collection of all ψ_Ω-C sets in $(X, \mathcal{F}, \mathcal{G})$ is denoted as $\psi_\Omega(X, \mathcal{F})$.

Properties of $\psi_\Omega(X, \mathcal{F})$:
It is obvious that $\mathcal{F} \subseteq \tau_{\mathcal{F}\mathcal{G}}$ [14]$\subseteq \psi_\Omega(X, \mathcal{F})$. But reverse inclusion does not hold in general, which will be discussed afterwards.

Theorem 3.1. *Let $\{A_\alpha : \alpha \in \Delta\}$ be a collection of nonempty ψ_Ω-C sets in a grill-filter space $(X, \mathcal{F}, \mathcal{G})$, then $\cup_{\alpha \in \Delta} A_\alpha \in \psi_\Omega(X, \mathcal{F})$.*

Proof: For each $\alpha \in \Delta$, $A_\alpha \subseteq \mathcal{F}\text{-}Cl[\psi_\Omega(A_\alpha)] \subseteq \mathcal{F}\text{-}Cl[\psi_\Omega(\cup_{\alpha\in\Delta}A_\alpha)]$ [14]. This implies that $\cup_{\alpha\in\Delta}A_\alpha \subseteq \mathcal{F}\text{-}Cl[\psi_\Omega(\cup_{\alpha\in\Delta}A_\alpha)]$. Thus $\cup_{\alpha\in\Delta}A_\alpha \in \psi_\Omega(X,\mathcal{F})$. □

Intersection of two ψ_Ω-C sets may not be a ψ_Ω-C set in general, which will be discussed by the following:

Observation 3.1. *Suppose intersection of two ψ_Ω-C sets is ψ_Ω-C set. Then for $A, B \in \psi_\Omega(X,\mathcal{F})$, Then $A\cap B \subseteq \mathcal{F}\text{-}Cl[\psi_\Omega(A\cap B)] = \Omega[\psi_\Omega(A\cap B)]$(from Theorem 2.7)$\subseteq \Omega[\Omega(A\cap B)]$(by Theorem 2.10) $\subseteq \Omega(A\cap B)$ [14], when $\mathcal{F}\subseteq\mathcal{G}$.*

Example 3.1. *If intersection of two ψ_Ω-C sets is also a ψ_Ω-C set. Then from above observation, $A\cap B\subseteq\Omega(A\cap B)$. But from Example 2.1, $A\cap B$ is not a subset of $\Omega(A\cap B)$. Hence intersection of two ψ_Ω-C sets may not be a ψ_Ω-C set again.*

Observation 3.2. *If possible supposed that every open set of $(X,\tau_{\mathcal{F}\mathcal{G}},\mathcal{F},\mathcal{G})$ is also a member of $\psi_\Omega(X,\mathcal{F})$. Then intersection of two ψ_Ω-C sets is again a ψ_Ω-C set, which is a contradiction to the Example 3.1. Hence the reverse inclusion of $\tau_{\mathcal{F}\mathcal{G}}\subseteq\psi_\Omega(X,\mathcal{F})$ does not hold. Again the empty set, $\phi\in\wp(X)$, $\phi\notin\mathcal{F}$ but $\phi\in\tau_{\mathcal{F}\mathcal{G}}$. Hence the reverse inclusion of $\mathcal{F}\subseteq\tau_{\mathcal{F}\mathcal{G}}$ fails to hold.*

However following hold:

Theorem 3.2. *Let $(X,\mathcal{F},\mathcal{G})$ be a grill-filter space and $A\in\psi_\Omega(X,\mathcal{F})$. If $U\in\mathcal{F}$, then $U\cap A\in\psi_\Omega(X,\mathcal{F})$.*

Proof: Let $U\in\mathcal{F}$ and $A\in\psi_\Omega(X,\mathcal{F})$. Then $U\cap A\subseteq U\cap\mathcal{F}\text{-}Cl[\psi_\Omega(A)]$(since $A\in\psi_\Omega(X,\mathcal{F})$)$\subseteq\mathcal{F}\text{-}Cl[U\cap\psi_\Omega(A)]$(using Theorem 2.3)$\subseteq\mathcal{F}\text{-}Cl[\psi_\Omega(U)\cap\psi_\Omega(A)]$ [14] $=\mathcal{F}\text{-}Cl[\psi_\Omega(U\cap A)]$ [14]. Hence the result. □

4. $\tau^{\psi}_{\mathcal{F}\mathcal{G}}$ -topology

In this section we shall introduce a new type of set whose collection form a topology. Although, the collection used in section3 does not form a topology.

Definition 4.1. *Let $(X,\mathcal{F},\mathcal{G})$ be a grill-filter space. A subset A of X is called a ψ_Ω- set if $A\subseteq\mathcal{F}\text{-}Int[\mathcal{F}\text{-}Cl(\psi_\Omega(A))]$.*
The collection of all ψ_Ω sets in $(X,\mathcal{F},\mathcal{G})$ is denoted by $\tau^{\psi}_{\mathcal{F}\mathcal{G}}$. This collection lying in between $\mathcal{F}$ and $\psi_\Omega(X,\mathcal{F})$ i.e., $\mathcal{F}\subseteq\tau^{\psi}_{\mathcal{F}\mathcal{G}}\subseteq\psi_\Omega(X,\mathcal{F})$.

Properties of $\tau^{\psi}_{\mathcal{F}\mathcal{G}}$:

Theorem 4.1. *Let $(X,\mathcal{F},\mathcal{G})$ be a grill-filter space, then $\tau^{\psi}_{\mathcal{F}\mathcal{G}}=\{A\subseteq X:\ A\subseteq\mathcal{F}\text{-}Int[\mathcal{F}\text{-}Cl(\psi_\Omega(A))]\}$ forms a topology on X, where $\mathcal{F}\subseteq\mathcal{G}$.*

Proof: (i). $\psi_\Omega(\phi)=X\setminus\Omega(X\setminus\phi)=\phi$(from Theorem 2.7). So, $\phi\in\tau^{\psi}_{\mathcal{F}\mathcal{G}}$. Now $\psi_\Omega(X)=X\setminus\Omega(X\setminus X)=X\setminus\phi$(from Definition 2.1) $=X$. Hence $X\subseteq\mathcal{F}\text{-}Int[\mathcal{F}\text{-}Cl(\psi_\varphi(X))]$. Therefore $X\in\tau^{\psi}_{\mathcal{F}\mathcal{G}}$.

(ii). Let $A_i \in \tau^{\psi}_{\mathcal{FG}}$ for all i. Now we are to show that $\cup_i A_i \in \tau^{\psi}_{\mathcal{FG}}$. Since $A_i \subseteq \cup_i A_i$, $\psi_\Omega(A_i) \subseteq \psi_\Omega(\cup_i A_i)$ [14]. Thus $\mathcal{F}$-$Int[\mathcal{F}$-$Cl(\psi_\Omega(A_i))] \subseteq \mathcal{F}$-$Int[\mathcal{F}$-$Cl(\psi_\Omega(\cup_i A_i))]$(since $\psi_\Omega(A_i)$ is an $\mathcal{F}$-open set [14]). So $A_i \subseteq \mathcal{F}$-$Int[\mathcal{F}$-$Cl(\psi_\Omega(A_i))] \subseteq \mathcal{F}$-$Int[\mathcal{F}$-$Cl(\psi_\Omega(\cup_i A_i))]$ for all i. Therefore $\cup_i A_i \in \tau^{\psi}_{\mathcal{FG}}$.

(iii). Let $A_1, A_2 \in \tau^{\psi}_{\mathcal{FG}}$. We are to show that $A_1 \cap A_2 \in \tau^{\psi}_{\mathcal{FG}}$. If $A_1 \cap A_2 = \phi$, we are done. Let $A_1 \cap A_2 \neq \phi$. Let $x \in A_1 \cap A_2$. Now $A_1 \subseteq \mathcal{F}$-$Int[\mathcal{F}$-$Cl(\psi_\Omega(A_1))]$ and $A_2 \subseteq \mathcal{F}$-$Int[\mathcal{F}$-$Cl(\psi_\Omega(A_2))]$, implies that $x \in \mathcal{F}$-$Int[\mathcal{F}$-$Cl(\psi_\Omega(A_1))] \cap \mathcal{F}$-$Int[\mathcal{F}$-$Cl(\psi_\Omega(A_2))]$. So $x \in \mathcal{F}$-$Int[\mathcal{F}$-$Cl(\psi_\Omega(A_1)) \cap \mathcal{F}$-$Cl(\psi_\Omega(A_2))]$ (from Definition 2.1). Therefore there exists an $\mathcal{F}$-open set V_x containing x such that $V_x \subseteq \mathcal{F}$-$Cl(\psi_\Omega(A_1)) \cap \mathcal{F}$-$Cl(\psi_\Omega(A_2))$. Let U_x be any $\mathcal{F}$-open set containing x. Then $\phi \neq V_x \cap U_x \subseteq \mathcal{F}$-$Cl(\psi_\Omega(A_1))$ and $V_x \cap U_x \subseteq \mathcal{F}$-$Cl(\psi_\Omega(A_2))$. Let $y \in V_x \cap U_x$. Consider any $\mathcal{F}$-open set G_y containing y. Without loss of generality we may suppose that $G_y \subseteq V_x \cap U_x$. So $G_y \cap (\psi_\Omega(A_1)) \neq \phi$. From the definition of $\psi_\Omega(A_1)$, there exists a $U \in \mathcal{F}(x)$ such that $U \subseteq G_y$ and $U \setminus A_1 \notin \mathcal{G}$. Again $U \subseteq \mathcal{F}$-$Cl(\psi_\Omega(A_2))$, so there exists a nonempty $\mathcal{F}$-open set $U^{/} \subseteq U$ such that $U^{/} \setminus A_2 \notin \mathcal{G}$. Now $U^{/} \setminus (A_1 \cap A_2) = (U^{/} \setminus A_1) \cup (U^{/} \setminus A_2) \subseteq (U \setminus A_1) \cup (U^{/} \setminus A_2) \notin \mathcal{G}$(from definition of grill). Hence from definition of ψ_Ω, $U^{/} \subseteq \psi_\Omega(A_1 \cap A_2)$. Since $U^{/} \subseteq G_y$, $G_y \cap \psi_\Omega(A_1 \cap A_2) \neq \phi$, therefore $y \in \mathcal{F}$-$Cl(\psi_\Omega(A_1 \cap A_2))$. Since y was any point of $V_x \cap U_x$, it follows that $V_x \cap U_x \subseteq \mathcal{F}$-$Cl(\psi_\Omega(A_1 \cap A_2))$, implies that $x \in \mathcal{F}$-$Int[\mathcal{F}$-$Cl(\psi_\Omega(A_1 \cap A_2))]$. Thus $A_1 \cap A_2 \subseteq \mathcal{F}$-$Int[\mathcal{F}$-$Cl(\psi_\Omega(A_1 \cap A_2))]$. Hence $A_1 \cap A_2 \in \tau^{\psi}_{\mathcal{FG}}$.

From (i), (ii) and (iii) $\tau^{\psi}_{\mathcal{FG}}$ forms a topology. □

Proposition 4.1. *Let $(X, \mathcal{F}, \mathcal{G})$ be a grill-filter space with $\mathcal{F} \subseteq \mathcal{G}$. Then $\psi_\Omega(A) \neq \phi$ if and only if A contains a nonempty $\tau_{\mathcal{FG}}$-interior.*

Proof: Let $\psi_\Omega(A) \neq \phi$. Then from definition of $\psi_\Omega(A)$, there exists a nonempty set $U \in \mathcal{F}$ such that $U \setminus A = P$, where $P \notin \mathcal{G}$. Now $U \setminus P \subseteq A$. By the Theorem 2.5, $U \setminus P \in \tau_{\mathcal{FG}}$ and A contains a nonempty $\tau_{\mathcal{FG}}$-interior.

Conversely suppose that A contains a nonempty $\tau_{\mathcal{FG}}$-interior. Hence there exists a $U \in \mathcal{F}$ and $P \notin \mathcal{G}$ such that $U \setminus P \subseteq A$. So $U \setminus A \subseteq P$. Let $H = U \setminus A \subseteq P$, then $H \notin \mathcal{G}$. Thus $\psi_\Omega(A) \neq \phi$. □

Two topologies $\tau^{\psi}_{\mathcal{FG}}$ and $\tau_{\mathcal{FG}}$ have been obtained from $(X, \mathcal{F}, \mathcal{G})$ space. Now we shall discuss the resolvability of $\tau^{\psi}_{\mathcal{FG}}$ vis-a-vis resolvability of $\tau_{\mathcal{FG}}$.

Theorem 4.2. *If $\mathcal{F} \subseteq \mathcal{G}$ in $(X, \mathcal{F}, \mathcal{G})$ then $\mathcal{D}(X, \tau_{\mathcal{FG}}) = \mathcal{D}(X, \tau^{\psi}_{\mathcal{FG}})$($D(X, \tau)$ denotes the collection of all dense subsets in the topological space (X, τ)).*

Proof: Since $\tau_{\mathcal{FG}} \subseteq \tau^{\psi}_{\mathcal{FG}}$ then

$$\mathcal{D}(X, \tau^{\psi}_{\mathcal{FG}}) \subseteq \mathcal{D}(X, \tau_{\mathcal{FG}}) \text{———}(i).$$

Next let $D \in \mathcal{D}(X, \tau_{\mathcal{FG}})$. We are to show that $D \in \mathcal{D}(X, \tau^{\psi}_{\mathcal{FG}})$. Let $\phi \neq A \in \tau^{\psi}_{\mathcal{FG}}$, so $\psi_\Omega(A) \neq \phi$. By Proposition 4.1, A has a nonempty $\tau_{\mathcal{FG}}$-interior. Thus $Int_{\mathcal{FG}}(A) \neq \phi$. Now $Int_{\mathcal{FG}}(A) \cap D \subseteq A \cap D$, where $Int_{\mathcal{FG}}(A) \cap D \neq \phi$, since $D \in (X, \tau_{\mathcal{FG}})$. Thus $A \cap D \neq \phi$ so that $D \in \mathcal{D}(X, \tau^{\psi}_{\mathcal{FG}})$. Therefore

$$\mathcal{D}(X, \tau_{\mathcal{FG}}) \subseteq \mathcal{D}(X, \tau^{\psi}_{\mathcal{FG}}) \text{ —————(ii)}.$$

From (i) and (ii) we have $\mathcal{D}(X, \tau_{\mathcal{FG}}) = \mathcal{D}(X, \tau^{\psi}_{\mathcal{FG}})$. □

Theorem 4.3. *Let $(X, \mathcal{F}, \mathcal{G})$ be a grill-filter space, with $\mathcal{F} \subseteq \mathcal{G}$. Then $(X, \tau^{\psi}_{\mathcal{FG}})$ is resolvable if and only if $(X, \tau_{\mathcal{FG}})$ is resolvable.*

Proof: Since $\mathcal{D}(X, \tau_{\mathcal{FG}}) = \mathcal{D}(X, \tau^{\psi}_{\mathcal{FG}})$, it follows from definition of resolvability that $(X, \tau^{\psi}_{\mathcal{FG}})$ is resolvable if and only if $(X, \tau_{\mathcal{FG}})$ is resolvable. □

Now we shall give an representation of α-topology of $\tau_{\mathcal{FG}}$ with the help of ψ_{Ω}-operator in the following way:

Theorem 4.4. *Let $x \in X$. Then $\{x\} \in \psi_{\Omega}(X, \mathcal{F})$ if and only if $\{x\}$ is open in $(X, \tau_{\mathcal{FG}})$.*

Proof: Let $\{x\} \in \psi_{\Omega}(X, \mathcal{F})$ then $\psi_{\Omega}(\{x\}) \neq \phi$. By Proposition 4.1, $\{x\}$ contain a nonempty $\tau_{\mathcal{FG}}$-interior. Therefore $\{x\}$ is open in $(X, \tau_{\mathcal{FG}})$. Conversely suppose that $\{x\}$ is open in $(X, \tau_{\mathcal{FG}})$, implies that $\{x\} \subseteq \psi_{\Omega}(\{x\})$ [14]. Therefore $\{x\} \subseteq \mathcal{F}$-$Cl(\psi_{\Omega}(\{x\}))$, that is $\{x\} \in \psi_{\Omega}(X, \mathcal{F})$. □

Theorem 4.5. *Let $x \in X$. Then $\{x\} \in \psi_{\Omega}(X, \mathcal{F})$ if and only if $\{x\} \in \tau^{\psi}_{\mathcal{FG}}$.*

Proof: Let $\{x\} \in \psi_{\Omega}(X, \mathcal{F})$. Therefore $\{x\}$ is open in $(X, \tau_{\mathcal{FG}})$ (by above theorem). So $\{x\} \subseteq \psi_{\Omega}(\{x\})$ [14] implies that $\{x\} \subseteq \mathcal{F}$-$Int[\mathcal{F}$-$Cl(\psi_{\Omega}(\{x\}))]$, since $\psi_{\Omega}(\{x\})$ is an $\mathcal{F}$-open set. Thus $\{x\} \in \tau^{\psi}_{\mathcal{FG}}$. Conversely suppose that $\{x\} \in \tau^{\psi}_{\mathcal{FG}}$, then $\{x\} \subseteq \mathcal{F}$-$Int[\mathcal{F}$-$Cl(\psi_{\Omega}(\{x\}))]$, implying that $\{x\} \subseteq \mathcal{F}$-$Cl(\psi_{\Omega}(\{x\}))$, hence $\{x\} \in \psi_{\Omega}(X, \mathcal{F})$. □

From the above two theorems we get the following corollary:

Corollary 4.1. *$\tau^{\psi}_{\mathcal{FG}}$ is exactly the collection such that A belongs to $\tau^{\psi}_{\mathcal{FG}}$ and B belongs to $\psi_{\Omega}(X, \mathcal{F})$ implies $A \cap B$ belongs to $\psi_{\Omega}(X, \mathcal{F})$, where $\mathcal{F} \subseteq \mathcal{G}$.*

Proof: Let $A \in \tau^{\psi}_{\mathcal{FG}}$ and $B \in \psi_{\Omega}(X, \mathcal{F})$. Now we are to show that $A \cap B \in \psi_{\Omega}(X, \mathcal{F})$. If $A \cap B = \phi$, we are done. Let $A \cap B \neq \phi$. Let $x \in A \cap B$. This implies that $x \in \mathcal{F}$-$Int[\mathcal{F}$-$Cl(\psi_{\Omega}(A))]$, therefore $x \in \mathcal{F}$-$Cl(\psi_{\Omega}(A))$. So for every $\mathcal{F}$-open set U_x containing x, $U_x \cap \psi_{\Omega}(A) \neq \phi$. Again $x \in B \subseteq \mathcal{F}$-$Cl(\psi_{\Omega}(B))$, then for every $\mathcal{F}$-open set V_x containing x, $V_x \cap \psi_{\Omega}(B) \neq \phi$. Therefore for $\mathcal{F}$-open set $W_x = U_x \cap V_x$ containing x, $W_x \cap \psi_{\Omega}(A) \neq \phi$ and $W_x \cap \psi_{\Omega}(B) \neq \phi$. Again $W_x \cap \psi_{\Omega}(A) \subseteq W_x$ and $W_x \cap \psi_{\Omega}(B) \subseteq W_x$. Therefore $W_x \cap \psi_{\Omega}(A) \cap \psi_{\Omega}(B) \neq \phi$. So $x \in \mathcal{F}$-$Cl[\psi_{\Omega}(A) \cap \psi_{\Omega}(B)]$, that is $x \in \mathcal{F}$-$Cl[\psi_{\Omega}(A \cap B)]$, therefore $A \cap B \in \psi_{\Omega}(X, \mathcal{F})$. Next we consider a subset A of X such that $A \cap B \in \psi_{\Omega}(X, \mathcal{F})$ for each $B \in \psi_{\Omega}(X, \mathcal{F})$. We have to show that $A \in \tau^{\psi}_{\mathcal{FG}}$, that is $A \subseteq \mathcal{F}$-$Int[\mathcal{F}$-$Cl(\psi_{\Omega}(A))]$, that is $A \subseteq \mathcal{F}$-$Int[\Omega(\psi_{\Omega}(A))]$(by Theorem 2.7). If possible suppose that $x \in A$

but $x \notin \mathcal{F}\text{-}Int[\Omega(\psi_\Omega(A))]$. Therefore $x \in A \cap [X \setminus \mathcal{F}\text{-}Int(\Omega(\psi_\Omega(A)))] = A \cap \mathcal{F}\text{-}Cl[X \setminus \Omega(\psi_\Omega(A))]$(from Theorem 2.2) $= A \cap \mathcal{F}\text{-}ClC$, where $C = X \setminus \Omega(\psi_\Omega(A))$. It is obvious that C is an nonempty $\mathcal{F}$-open set in $(X, \mathcal{F})$, since $\Omega(\psi_\Omega(A))$ is a $\mathcal{F}$-closed set [14]. Since $x \in \mathcal{F}\text{-}ClC$ then for all $\mathcal{F}$-open set V_x containing x, $V_x \cap C \neq \phi$. Therefore $V_x \cap \psi_\Omega(C) \neq \phi$, since $C \subseteq \psi_\Omega(C)$ [14]. This implies that

$$x \in \mathcal{F}\text{-}Cl(\psi_\Omega(C)) \subseteq \mathcal{F}\text{-}Cl[\psi_\Omega(\{x\} \cup C)] \text{———————-(i).}$$

$$\text{Again } C \subseteq \mathcal{F}\text{-}Cl(\psi_\Omega(C)) \subseteq \mathcal{F}\text{-}Cl[\psi_\Omega(\{x\} \cup C)] \text{——————-(ii).}$$

From (i) and (ii) $\{x\} \cup C \subseteq \mathcal{F}\text{-}Cl[\psi_\Omega(\{x\} \cup C)]$. Therefore $\{x\} \cup C \in \psi_\Omega(X, \mathcal{F})$. Now by hypothesis $A \cap (\{x\} \cup C)$ is a ψ_Ω-C set. We show that $A \cap (\{x\} \cup C) = \{x\}$. If possible suppose that $y \in X$ and $x \neq y$ such that $y \in A \cap (\{x\} \cup C)$. So $y \in A$ and $y \in C$. Now $A = A \cap X$ and $X \in \psi_\Omega(X, \mathcal{F})$, again by hypothesis $A \in \psi_\Omega(X, \mathcal{F})$. Since $y \in A$, $y \in \mathcal{F}\text{-}Cl(\psi_\Omega(A))$, a contradiction to the fact that $y \in C = [X \setminus \Omega(\psi_\Omega(A))] = [X \setminus \mathcal{F}\text{-}Cl(\psi_\Omega(A))]$. Thus $A \cap (\{x\} \cup C) = \{x\}$. Since $\{x\} \in \psi_\Omega(X, \mathcal{F})$, then $\{x\} \in \tau^{\psi}_{\mathcal{F}\mathcal{G}}$(by Theorem 4.5). So $\{x\} \subseteq \mathcal{F}\text{-}Int[\mathcal{F}\text{-}Cl(\psi_\Omega(\{x\}))] = \mathcal{F}\text{-}Int[\mathcal{F}\text{-}Cl(\psi_\Omega(A \cap (\{x\} \cup C)))] \subseteq \mathcal{F}\text{-}Int(\mathcal{F}\text{-}Cl(\psi_\Omega(A)))$. But $x \in \mathcal{F}\text{-}Int(\mathcal{F}\text{-}Cl(\psi_\Omega(A)))$, a contradiction to the fact that $x \notin \mathcal{F}\text{-}Int[\Omega(\psi_\Omega(A))]$. Therefore we get $A \subseteq \mathcal{F}\text{-}Int[\mathcal{F}\text{-}Cl(\psi_\Omega(A))]$ that is $A \in \tau^{\psi}_{\mathcal{F}\mathcal{G}}$. This complete the proof of theorem. □

Theorem 4.6. *Let $(X, \mathcal{F}, \mathcal{G})$ be a grill-filter space, where $\mathcal{F} \subseteq \mathcal{G}$. Then $SO(X, \tau_{\mathcal{F}\mathcal{G}}) = \{A \subseteq X : A \subseteq \mathcal{F}\text{-}Cl(\psi_\Omega(A))\} = \psi_\Omega(X, \mathcal{F})$.*

Proof: Let $A \in SO(X, \tau_{\mathcal{F}\mathcal{G}})$. Then $A \subseteq Cl_{\mathcal{F}\mathcal{G}}[Int_{\mathcal{F}\mathcal{G}}(A)] = Cl_{\mathcal{F}\mathcal{G}}(A \cap \psi_\Omega(A))$ [14]$\subseteq Cl_{\mathcal{F}\mathcal{G}}(\psi_\Omega(A)) = [\psi_\Omega(A) \cup \Omega(\psi_\Omega(A))] = \Omega(\psi_\Omega(A))$, since $\psi_\Omega(A) \in \mathcal{F}$. This implies that $A \subseteq \mathcal{F}\text{-}Cl(\psi_\Omega(A))$. Hence $A \in \psi_\Omega(X, \mathcal{F})$. So

$$SO(X, \tau_{\mathcal{F}\mathcal{G}}) \subseteq \psi_\Omega(X, \mathcal{F}) \text{——————(i).}$$

Suppose that $A \in \psi_\Omega(X, \mathcal{F})$ and we show that $A \in SO(X, \tau_{\mathcal{F}\mathcal{G}})$. Let $x \notin Cl_{\mathcal{F}\mathcal{G}}(Int_{\mathcal{F}\mathcal{G}}(A))$. Then there exists $U \in \tau_{\mathcal{F}\mathcal{G}}$ containing x such that $U \cap Int_{\mathcal{F}\mathcal{G}}(A) = \phi$. And also there exists $F \in \mathcal{F}$ and $G \notin \mathcal{G}$ such that $x \in F \setminus G \subset U$; hence $(F \setminus G) \cap Int_{\mathcal{F}\mathcal{G}}(A) = \phi$. By Theorem 2.7, $\phi = \psi_\Omega(\phi) = \psi_\Omega((F \cap Int_{\mathcal{F}\mathcal{G}}(A)) \setminus G)$. Since $G \notin \mathcal{G}$, $\phi = \psi_\Omega(F \cap Int_{\mathcal{F}\mathcal{G}}(A)) = \psi_\Omega(F) \cap \psi_\Omega(Int_{\mathcal{F}\mathcal{G}}(A))$. Since $F \in \mathcal{F} \subseteq \tau_{\mathcal{F}\mathcal{G}}$, $F \subseteq \psi_\Omega(F)$. And also $\psi_\Omega(Int_{\mathcal{F}\mathcal{G}}(A)) = \psi_\Omega(A \cap \psi_\Omega(A)) = \psi_\Omega(A) \cap \psi_\Omega(\psi_\Omega(A)) = \psi_\Omega(A)$. Therefore, we obtain $F \cap \psi_\Omega(A) = \phi$. Since $x \in F \in \mathcal{F}$, $x \notin \mathcal{F}\text{-}Cl(\psi_\Omega(A))$ and by hypothesis $x \notin A$. Consequently, we obtain $A \subset Cl_{\mathcal{F}\mathcal{G}}(Int_{\mathcal{F}\mathcal{G}}(A))$ and hence $A \in SO(X, \tau_{\mathcal{F}\mathcal{G}})$.

$$\psi_\Omega(X, \mathcal{F}) \subseteq SO(X, \tau_{\mathcal{F}\mathcal{G}}) \text{——————(ii).}$$

From (i) and (ii), $\psi_\Omega(X, \mathcal{F}) = SO(X, \tau_{\mathcal{F}\mathcal{G}})$. □

Remark 4.1. *Let $x \in X$, then $\{x\} \in SO(X, \tau_{\mathcal{F}\mathcal{G}})$ if and only if $\{x\} \in \tau^{\psi}_{\mathcal{F}\mathcal{G}}$, where $\mathcal{F} \subseteq \mathcal{G}$.*

Proof: Proof is obvious from Theorem 4.5 and the Theorem 4.6. □

Theorem 4.7. *$\tau^{\psi}_{\mathcal{F}\mathcal{G}}$ is exactly the collection such that A belongs to $\tau^{\psi}_{\mathcal{F}\mathcal{G}}$ and B belongs $SO(X, \tau_{\mathcal{F}\mathcal{G}})$ implies $A \cap B$ belongs to $SO(X, \tau_{\mathcal{F}\mathcal{G}})$, where $\mathcal{F} \subseteq \mathcal{G}$.*

Proof: Proof is obvious from Corollary 4.1 and the Theorem 4.6. □

Now we shall discuss the relation between $(\tau_{\mathcal{F}\mathcal{G}})^{\alpha}$ and $\tau^{\psi}_{\mathcal{F}\mathcal{G}}$. For this we mention a remarkable theorem owing to O. Njastad.

Theorem 4.8. *[18]. Let (X,τ) be a topological space. τ^{α} consists of exactly those sets A for which $A \cap B \in SO(X,\tau)$ for all $B \in SO(X,\tau)$.*

From Theorem 4.7 and Theorem 4.8 follows;

Corollary 4.2. *Let $(X,\mathcal{F},\mathcal{G})$ be a grill-filter space, where $\mathcal{F} \subseteq \mathcal{G}$. Then $\tau^{\psi}_{\mathcal{F}\mathcal{G}} = (\tau_{\mathcal{F}\mathcal{G}})^{\alpha}$.*

5. Continuity on grill-filter topological spaces

In this last section, we shall define a generalized continuity on grill-filter space and interrelate it with usual continuity. We also characterize this generalized continuity.
Definition of generalized continuity is:

Definition 5.1. *Let $(X,\tau_{\mathcal{F}\mathcal{G}},\mathcal{F},\mathcal{G})$ and $(Y,\tau_{\mathcal{F}_1\mathcal{G}_1},\mathcal{F}_1,\mathcal{G}_1)$ be two grill-filter topological spaces. A map $f:(X,\tau_{\mathcal{F}\mathcal{G}},\mathcal{F},\mathcal{G}) \longrightarrow (Y,\tau_{\mathcal{F}_1\mathcal{G}_1},\mathcal{F}_1,\mathcal{G}_1)$ is called $\mathcal{F}$-continuous if $f^{-1}(V)$ is $\mathcal{F}$-open in $(X,\tau_{\mathcal{F}\mathcal{G}},\mathcal{F},\mathcal{G})$, for every $V \in \tau_{\mathcal{F}_1\mathcal{G}_1}$.*

Properties of $\mathcal{F}$-continuity:
We know that $\mathcal{F} \subseteq \tau_{\mathcal{F}\mathcal{G}}$, then it is obvious that every $\mathcal{F}$-continuous map is always a continuous map. Again from Observation 3.2, each continuous map is not necessarily a $\mathcal{F}$-continuous map.

Theorem 5.1. *Let $(X,\tau_{\mathcal{F}\mathcal{G}},\mathcal{F},\mathcal{G})$, $(Y,\tau_{\mathcal{F}_1\mathcal{G}_1},\mathcal{F}_1,\mathcal{G}_1)$ and $(Z,\tau_{\mathcal{F}_2\mathcal{G}_2},\mathcal{F}_2,\mathcal{G}_2)$ be three grill-filter topological spaces. If $f:(X,\tau_{\mathcal{F}\mathcal{G}},\mathcal{F},\mathcal{G}) \longrightarrow (Y,\tau_{\mathcal{F}_1\mathcal{G}_1},\mathcal{F}_1,\mathcal{G}_1)$ and $g:(Y,\tau_{\mathcal{F}_1\mathcal{G}_1},\mathcal{F}_1,\mathcal{G}_1) \longrightarrow (Z,\tau_{\mathcal{F}_2\mathcal{G}_2},\mathcal{F}_2,\mathcal{G}_2)$ are two $\mathcal{F}$-continuous maps, then gof is a $\mathcal{F}$-continuous map.*

Proof: Consider $(gof)^{-1}(V)$, where $V \in \tau_{\mathcal{F}_2\mathcal{G}_2}$. Now $g^{-1}(V)$ is $\mathcal{F}$-open in $(Y,\tau_{\mathcal{F}_1\mathcal{G}_1},\mathcal{F}_1,\mathcal{G}_1)$, again it is obvious that $g^{-1}(V)$ is open in $(Y,\tau_{\mathcal{F}_1\mathcal{G}_1},\mathcal{F}_1,\mathcal{G}_1)$. So $f^{-1}(g^{-1}(V))$ is $\mathcal{F}$-open in $(X,\tau_{\mathcal{F}\mathcal{G}},\mathcal{F},\mathcal{G})$. Hence the result. □

Corollary 5.1. *Let $(X,\tau_{\mathcal{F}\mathcal{G}},\mathcal{F},\mathcal{G})$, $(Y,\tau_{\mathcal{F}_1\mathcal{G}_1},\mathcal{F}_1,\mathcal{G}_1)$ and $(Z,\tau_{\mathcal{F}_2\mathcal{G}_2},\mathcal{F}_2,\mathcal{G}_2)$ be three grill-filter topological spaces. If $f:(X,\tau_{\mathcal{F}\mathcal{G}},\mathcal{F},\mathcal{G}) \longrightarrow (Y,\tau_{\mathcal{F}_1\mathcal{G}_1},\mathcal{F}_1,\mathcal{G}_1)$ is continuous map and $g:(Y,\tau_{\mathcal{F}_1\mathcal{G}_1},\mathcal{F}_1,\mathcal{G}_1) \longrightarrow (Z,\tau_{\mathcal{F}_2\mathcal{G}_2},\mathcal{F}_2,\mathcal{G}_2)$ is $\mathcal{F}$-continuous map, then gof is a continuous map.*

Proof: Proof is obvious from the fact that $g^{-1}(V)$ is open in $(Y,\tau_{\mathcal{F}_1\mathcal{G}_1},\mathcal{F}_1,\mathcal{G}_1)$, for every open set V in $(Z,\tau_{\mathcal{F}_2\mathcal{G}_2},\mathcal{F}_2,\mathcal{G}_2)$. □

Corollary 5.2. *Let* $(X, \tau_{\mathcal{F}\mathcal{G}}, \mathcal{F}, \mathcal{G})$, $(Y, \tau_{\mathcal{F}_1\mathcal{G}_1}, \mathcal{F}_1, \mathcal{G}_1)$ *and* $(Z, \tau_{\mathcal{F}_2\mathcal{G}_2}, \mathcal{F}_2, \mathcal{G}_2)$ *be three grill-filter topological spaces. If* $f : (X, \tau_{\mathcal{F}\mathcal{G}}, \mathcal{F}, \mathcal{G}) \longrightarrow (Y, \tau_{\mathcal{F}_1\mathcal{G}_1}, \mathcal{F}_1, \mathcal{G}_1)$ *is* $\mathcal{F}$*-continuous map and* $g : (Y, \tau_{\mathcal{F}_1\mathcal{G}_1}, \mathcal{F}_1, \mathcal{G}_1) \longrightarrow (Z, \tau_{\mathcal{F}_2\mathcal{G}_2}, \mathcal{F}_2, \mathcal{G}_2)$ *is continuous map, then* gof *is a* $\mathcal{F}$*-continuous map.*

Proof: Proof is obvious from the fact that $g^{-1}(V)$ is open in $(Y, \tau_{\mathcal{F}_1\mathcal{G}_1}, \mathcal{F}_1, \mathcal{G}_1)$, for every open set V in $(Z, \tau_{\mathcal{F}_2\mathcal{G}_2}, \mathcal{F}_2, \mathcal{G}_2)$. □

Theorem 5.2. *Let* $f : (X, \tau_{\mathcal{F}\mathcal{G}}, \mathcal{F}, \mathcal{G}) \longrightarrow (Y, \tau_{\mathcal{F}_1\mathcal{G}_1}, \mathcal{F}_1, \mathcal{G}_1)$ *be a map then following conditions are equivalent:*

1. f *is* $\mathcal{F}$*-continuous;*
2. $f[\mathcal{F}\text{-}Cl(A)] \subset Cl[f(A)]$*;*
3. *For every closed set* B *of* $(Y, \tau_{\mathcal{F}_1\mathcal{G}_1}, \mathcal{F}_1, \mathcal{G}_1)$, $f^{-1}(B)$ *is* $\mathcal{F}$*- closed in* $(X, \tau_{\mathcal{F}\mathcal{G}}, \mathcal{F}, \mathcal{G})$.

Proof: $1 \Longrightarrow 2$. Let $x \in \mathcal{F}\text{-}Cl(A)$. Let V be an open set containing $f(x)$ in $(Y, \tau_{\mathcal{F}_1\mathcal{G}_1}, \mathcal{F}_1, \mathcal{G}_1)$. Then $f^{-1}(V)$ is a $\mathcal{F}$-open set containing x in $(X, \tau_{\mathcal{F}\mathcal{G}}, \mathcal{F}, \mathcal{G})$(from definition of $\mathcal{F}$-continuity). Therefore $f^{-1}(V) \cap A \neq \phi$(by Theorem 2.1), and hence $V \cap f(A) \neq \phi$. So $f(x) \in Cl[f(A)]$, implies that $f[\mathcal{F}\text{-}Cl(A)] \subset Cl[f(A)]$.
$2 \Longrightarrow 3$. Let B be a closed set of $(Y, \tau_{\mathcal{F}_1\mathcal{G}_1}, \mathcal{F}_1, \mathcal{G}_1)$ and let $A = f^{-1}(B)$. Now we shall show that A is $\mathcal{F}$-closed set of $(X, \tau_{\mathcal{F}\mathcal{G}}, \mathcal{F}, \mathcal{G})$. Now $f(A) = f(f^{-1}(B)) \subseteq B$. Therefore for $x \in \mathcal{F}\text{-}Cl(A), f(x) \in f[\mathcal{F}\text{-}Cl(A)] \subseteq Cl[f(A)] \subseteq Cl(B) = B$. This implies that $x \in f^{-1}(B) = A$. Hence $\mathcal{F}\text{-}Cl(A) = A$.
$3 \Longrightarrow 1$. Let V be an open set in $(Y, \tau_{\mathcal{F}_1\mathcal{G}_1}, \mathcal{F}_1, \mathcal{G}_1)$. Then $f^{-1}(Y \setminus V) = X \setminus f^{-1}(V)$ is a $\mathcal{F}$-closed set in $(X, \tau_{\mathcal{F}\mathcal{G}}, \mathcal{F}, \mathcal{G})$, and hence $f^{-1}(V)$ is an $\mathcal{F}$-open set in $(X, \tau_{\mathcal{F}\mathcal{G}}, \mathcal{F}, \mathcal{G})$. So f is $\mathcal{F}$-continuous map. □

Finally we shall give a sufficient condition:

Theorem 5.3. *Let* $f : (X, \tau_{\mathcal{F}\mathcal{G}}, \mathcal{F}, \mathcal{G}) \longrightarrow (Y, \tau_{\mathcal{F}_1\mathcal{G}_1}, \mathcal{F}_1, \mathcal{G}_1)$ *be a* $\mathcal{F}$*-continuous map. Then* f *is continuous if* $\mathcal{G} = \wp(X) \setminus \{\phi\}$.

Proof: Let V be an open set in $(Y, \tau_{\mathcal{F}_1\mathcal{G}_1}, \mathcal{F}_1, \mathcal{G}_1)$. Then $f^{-1}(V)$ is $\mathcal{F}$-open in $(X, \tau_{\mathcal{F}\mathcal{G}}, \mathcal{F}, \mathcal{G})$. Again $f^{-1}(V)$ is open in $(X, \tau_{\mathcal{F}\mathcal{G}}, \mathcal{F}, \mathcal{G})$(using Theorem 2.5 and the condition $\mathcal{G} = \wp(X) \setminus \{\phi\}$). □

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