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New approach of ideal topological generalized closed sets

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ABSTRACT: In this paper, we introduce and investigate the notions of $I_{\hat{\omega}}$-closed sets and $I_{\hat{\omega}}$-continuous functions,maximal $I_{\hat{\omega}}$-closed sets and maximal $I_{\hat{\omega}}$-continuous functions in ideal topological spaces.We also introduce a new class of spaces called $MT_{\hat{\omega}}$-spaces.

Key Words: $I_{\hat{\omega}}$-closed sets,$I_{\hat{\omega}}$-continuous functions,maximal $I_{\hat{\omega}}$-closed sets and maximal $I_{\hat{\omega}}$-continuous functions.

Contents

1. Introduction

Ideals in topological spaces have been considered since 1930.In 1990,Jankovic and Hamlett [5] once again investigated applications of topological ideals.The notion of I_g-closed sets was first introduced by Dontchev et al [3]in 1999.Navaneethakrishnan and Joseph [9] further investigated and characterized I_g-closed sets and I_g-open sets by the use of local functions.Recently the notion of I_{rg}-closed sets was introduced and investigated by Navaneethakrishnan and Sivaraj [10].In this paper we define and characterize $I_{\hat{\omega}}$-closed sets,$I_{\hat{\omega}}$-continuous functions,maximal$I_{\hat{\omega}}$-closed sets and maximal $I_{\hat{\omega}}$-continuous functions.We also introduce a new class of spaces called $MT_{\hat{\omega}}$-spaces.Finally we obtain a decomposition of $*$-continuity.

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2. Preliminaries

An ideal on a set X is a nonempty collection of subsets of X which satisfies (i)$A \in I$ and $B \subseteq A$ implies $B \in I$ and (ii)$A \in I$ and $B \in I$ implies $A \cup B \in I$.If I is an ideal on X, then (X, τ, I) is called an ideal topological space(an ideal space).For an ideal space (X, τ, I) and $A \subseteq X, A^*(I, \tau) = \{x \in X : U \cap A \notin I$for every open set U containing x } is called the local function [6] of A with respect to I and τ.We simply write A* instead of $A^*(I, \tau)$ in case there is no confusion.For every ideal topological space (X, τ, I),there exists a topology $\tau^*(I)$,finer than τ generated by $\beta(I, \tau) = \{U - J : U \in \tau$ and J$\in I\}$.A subset A of an ideal topological space (X, τ, I) is said to be τ^*-closed [5] or simply $*$-closed (resp.$*$-perfect in itself [4]) if $A^* \subseteq$ A(resp.A=A*).A Kuratowski closure operator cl*() for a topology $\tau^*(I, \tau)$called the $*$-topology is defined by cl*(A)=$A \cup A^*(X, \tau)$.

Definition 2.1. *A subset A of a space (X, τ) is called*

(i)a-open set [2] if $A \subseteq int(cl(int_\delta(A))$.

(ii)regular open if A=int(cl(A)) and A is said to be regular closed if A=cl(int(A)).

(iii)strongly-I-locally closed [12] (briefly strongly-I-LC) if $A = U \cap V$ where U is regular open and V is $*$-closed.

The complement of a-open set is called a-closed set.The a-closure of a subset A of (X, τ) is the intersection of all a-closed sets containing A and is denoted by acl(A).

Definition 2.2. *A subset A of a space (X, τ) is called δ-closed [15] if $A = cl_\delta(A)$,where $cl_\delta(A) = \{x\in X : int(cl(U)) \cap A \neq \phi, U \in \tau$ and $x \in U\}$.The complement of δ-closed set is called δ-open set.*

Definition 2.3. *A subset A of a space (X, τ) is called a*

(i)α-generalized closed (briefly αg-closed) set[7] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).

(ii)$\hat{g}$-closed set[14] if cl(A)$\subseteq U$ whenever $A \subseteq U$ and U is semi-open in (X, τ).

(iii)$\alpha\hat{g}$-closed set [1] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\hat{g}$-open in (X, τ).

(iv)$\hat{\omega}$-closed set [8] if $acl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\alpha\hat{g}$-open in (X, τ).

The complement of αg-closed($\hat{g}$-closed,$\alpha\hat{g}$-closed and $\hat{\omega}$-closed)set is called αg-open ($\hat{g}$-open,$\alpha\hat{g}$-open and $\hat{\omega}$-open).

Definition 2.4. *A subset A of an ideal space (X, τ, I) is said to be*

(i)I_g-closed [3]if $A^* \subseteq U$ whenever $A \subseteq U$ and U is open in X.

(ii)I_{rg}-closed [10]if $A^* \subseteq U$ whenever $A \subseteq U$ and U is regular open in X.

(iii)$I_{\alpha gg}$-closed [13]if $A^* \subseteq U$ whenever $A \subseteq U$andUisαg-open in X.

Definition 2.5. *A function $f : (X, \tau, I) \to (Y, \sigma)$ is said to be*

(i)$*$-continuous [12] if $f^{-1}(A)$ is $*$-closed in X for every closed set A of Y.

(ii)strongly-I-LC-continuous [12] if $f^{-1}(A)$ is strongly-I-LC in X for every closed set A of Y.

Lemma 2.6. *[5] If A and B are subsets of an ideal space ,then $(A \cup B)^* = A^* \cup B^*$*

3. $I_{\hat{\omega}}$-Closed sets

In this section we introduce $I_{\hat{\omega}}$-closed sets and investigate some properties of such sets.

Definition 3.1. *A subset A of an ideal space (X, τ, I) is said to be $I_{\hat{\omega}}$-closed if $A^* \subseteq U$ whenever $A \subseteq U$ and U is $\hat{\omega}$-open in X.The complement of an $I_{\hat{\omega}}$-closed set is said to be an $I_{\hat{\omega}}$-open set.*

Example 3.2. *Let X={a,b,c,d},$\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}, X\}$and $I = \{\phi\}$.Then $I_{\hat{\omega}}$-closed sets are $\phi, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}$ and X.*

Remark 3.3. *(i)Since $A^* = \phi$ for every $A \in I$,every $A \in I$ is $I_{\hat{\omega}}$-closed.*

(ii)Every $*$-closed set is $I_{\hat{\omega}}$-closed,but not conversely.

(iii)A^* is an $I_{\hat{\omega}}$-closed set for every subset A of (X, τ, I)

Remark 3.4. *Converse of Remark(ii) need not be true as shown below.*

Example 3.5. *Let X={a,b,c},$\tau = \{\phi, \{a\}, \{a, c\}, X\}$and $I = \{\phi, \{a\}\}$.Then $\{c\}$ is $I_{\hat{\omega}}$-closed,but not $*$-closed.*

Theorem 3.6. *Let (X, τ, I) be an ideal space.Then every $I_{\hat{\omega}}$-closed,$\hat{\omega}$-open set is $*$-closed set.*

Proof: Assume that A is $I_{\hat{\omega}}$-closed and $\hat{\omega}$-open set.Then $A^* \subseteq A$ whenever $A \subseteq A$ and A is $\hat{\omega}$-open.Thus A is $*$-closed. □

Theorem 3.7. *Let (X, τ, I) be an ideal space.Then either $\{x\}$ is $\hat{\omega}$-closed or $\{x\}^c$ is $I_{\hat{\omega}}$-closed for every $x \in X$.*

Proof: Suppose $\{x\}$ is not $\hat{\omega}$-closed, then $\{x\}^c$ is not $\hat{\omega}$-open and the only $\hat{\omega}$-open set containing $\{x\}^c$is X and hence $(\{x\}^c)^* \subseteq X$.Thus $\{x\}^c$ is $I_{\hat{\omega}}$-closed. □

Theorem 3.8. *Let (X, τ, I) be an ideal space.Then every $I_{\hat{\omega}}$-closed set is I_{rg}-closed.*

Proof: Let A be an $I_{\hat{\omega}}$-closed set.Let A⊆U where U is regular open.By theorem [8],every regular open set is $\hat{\omega}$-open.Since A is $I_{\hat{\omega}}$-closed,we have $A^* \subseteq U$ and so A is I_{rg}-closed. □

The converse of the above theorem need not be true as shown in the following example.

Example 3.9. *Let X={a,b,c},* $\tau = \{\phi, \{a\}, \{b, c\}, X\}$ *and* $I = \{\phi, \{c\}\}$*.Then {b} is* I_{rg}*-closed but not* $I_{\hat{\omega}}$*-closed.*

Theorem 3.10. *Let* (X, τ, I) *be an ideal space.Then every* $I_{\alpha gg}$*-closed set is* $I_{\hat{\omega}}$*-closed.*

Proof: Let A be an $I_{\alpha gg}$-closed set.Let A⊆U where U is $\hat{\omega}$-open.Since $\hat{\omega}$-open set is αg-open [8], A⊆U where U is αg-open.Since A is $I_{\alpha gg}$-closed,we have $A^* \subseteq U$ and so A is $I_{\hat{\omega}}$-closed. □

The converse of the above theorem need not be true as shown in the following example.

Example 3.11. *Let X={a,b,c},* $\tau = \{\phi, \{c\}, X\}$ *and* $I = \{\phi\}$ *Then {b} is* $I_{\hat{\omega}}$*-closed,but not* $I_{\alpha gg}$*-closed.*

The following examples show that $I_{\hat{\omega}}$-closed sets and I_g-closed sets are independent.

Example 3.12. *Let X={a,b,c,d},* $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ *and* $I = \{\phi, \{b\}, \{c\}, \{b, c\}\}$*. Then {a,b,c} is* I_g*-closed,but not* $I_{\hat{\omega}}$*-closed.*

Example 3.13. *Let X={a,b,c,d},* $\tau = \{\phi, \{a\}, \{a, b\}, X\}$ *and* $I = \{\phi\}$*.Then {b} is* $I_{\hat{\omega}}$*-closed,but not* I_g*-closed.*

Remark 3.14. *The following table shows the relationships of* $I_{\hat{\omega}}$*-closed sets with other known existing sets.The symbol"1" in a cell means that a set implies the other maps and the symbol"0" means that a set does not imply the other sets.*

sets	$I_{\hat{\omega}}$	I_{rg}	$I_{\alpha gg}$	I_g
$I_{\hat{\omega}}$	1	1	0	0
I_{rg}	0	1	0	0
$I_{\alpha gg}$	1	1	1	1
I_g	0	1	0	1

Theorem 3.15. *Let* (X, τ, I) *be an ideal space.Then every subset of* (X, τ, I) *is* $I_{\hat{\omega}}$*-closed if and only if every* $\hat{\omega}$*-open set is* $*$*-closed.*

Proof: Necessity. Assume that every subset of (X, τ, I) is $I_{\hat{\omega}}$-closed.If A is $\hat{\omega}$-open,then by hypothesis A is $I_{\hat{\omega}}$-closed and so $A^* \subseteq A$which implies A is $*$-closed. Sufficiency.Suppose every $\hat{\omega}$-open set is $*$-closed.Let A be a subset of X and $A \subseteq U$ where U is $\hat{\omega}$-open.Then $A^* \subseteq U^* \subseteq U$.Thus A is $I_{\hat{\omega}}$-closed. □

Theorem 3.16. *If A and B are* $I_{\hat{\omega}}$*-closed sets in an ideal space* (X, τ, I)*,then* $A \cup B$ *is also an* $I_{\hat{\omega}}$*-closed set.*

Proof: Let U be a $\hat{\omega}$-open subset of (X, τ, I) containing $A \cup B$.Then $A \subseteq U$ and $B \subseteq U$.Since A and B are $I_{\hat{\omega}}$-closed,$A^* \subseteq U$ and $B^* \subseteq U$.By lemma 2.6,$(A \cup B)^* = A^* \cup B^*$.Thus $(A \cup B)^* \subseteq U$ which implies $A \cup B$ is $I_{\hat{\omega}}$-closed. □

Remark 3.17. *The intersection of two $I_{\hat{\omega}}$-closed sets need not be $I_{\hat{\omega}}$-closed.*

Example 3.18. *Let X={a,b,c},$\tau = \{\phi, \{c\}, X\}$and $I = \{\phi\}$.Then $A = \{a, c\}$and $B = \{b, c\}$are $I_{\hat{\omega}}$-closed ,but $A \cap B = \{c\}$ is not $I_{\hat{\omega}}$-closed.*

Theorem 3.19. *Let A be an $I_{\hat{\omega}}$-closed set in an ideal space (X, τ, I).Then $cl^*(A) - A$ contains no nonempty $\hat{\omega}$-closed set.*

Proof: Suppose A is an $I_{\hat{\omega}}$-closed set in X and F be a $\hat{\omega}$-closed subset of $cl^*(A) - A$.Then $A \subseteq X - F$ and X-F is $\hat{\omega}$-open.Since A is $I_{\hat{\omega}}$-closed,$cl^*(A) \subseteq X - F$ and so $F \subseteq X - cl^*(A)$.Thus $F \subseteq cl^*(A) \cap (X - cl^*(A)) = \phi$.Hence $cl^*(A) - A$ contains no nonempty $\hat{\omega}$-closed set. □

Theorem 3.20. *Let A be an $I_{\hat{\omega}}$-closed set in an ideal space (X, τ, I).Then the following are equivalent.*

(a)A is a $*$-closed set.

(b)$cl^*(A) - A$ is a $\hat{\omega}$-closed set.

(c)$A^* - A$ is a $\hat{\omega}$-closed set.

Proof: (a)⇒(b)If A is $*$-closed,then $cl^*(A) - A = \phi$ and so $cl^*(A) - A$ is $\hat{\omega}$-closed.
(b)⇒(a)Suppose $cl^*(A) - A$ is a $\hat{\omega}$-closed.Since A is $I_{\hat{\omega}}$-closed,by theorem 3.19, $cl^*(A) - A = \phi$ and so A is $*$-closed.
(b)⇔(c) Follows from $cl^*(A) - A = (A \cup A^*) - A = (A \cup A^*) \cap (X - A) = A^* \cap (X - A) = A^* - A$. □

Theorem 3.21. *Let A be an $I_{\hat{\omega}}$-closed set in an ideal space (X, τ, I)such that $A \subseteq B \subseteq A^*$.Then B is also an $I_{\hat{\omega}}$-closed set.*

Proof: Let U be an $\hat{\omega}$-open of X such that $B \subseteq U$.Then $A \subseteq U$.Since A is $I_{\hat{\omega}}$-closed,$A^* \subseteq U$.Now $B^* \subseteq (A^*)^* \subseteq A^* \subseteq U$.Consequently B is $I_{\hat{\omega}}$-closed. □

Theorem 3.22. *Let A be an $I_{\hat{\omega}}$-closed set in an ideal space (X, τ, I).Then $A \cup (X - A^*)$ is also an $I_{\hat{\omega}}$-closed set.*

Proof: Let U be a $\hat{\omega}$-open set such that $A \cup (X - A^*) \subseteq U$.Then $X - U \subseteq X - (A \cup (X - A^*)) = X \cap (A \cup (X - A^*))^c = X \cap (A \cup (A^*)^c)^c = X \cap (A^c \cap A^*) = A^* \cap A^c = A^* - A$.Since X-U is $\hat{\omega}$-closed and A is $I_{\hat{\omega}}$-closed,by theorem 3.19,$X - U = \phi$ and so X =U.Thus X is the only $\hat{\omega}$-open set containing $A \cup (X - A^*)$.Hence $A \cup (X - A^*)$ is $I_{\hat{\omega}}$-closed. □

Theorem 3.23. *Let (X, τ, I)be an ideal space (X, τ, I) and $A \subseteq X$.Then $A \cup (X - A^*)$ is $I_{\hat{\omega}}$-closed if and only if $A^* - A$ is $I_{\hat{\omega}}$-open.*

Proof: Follows from $A \cup (X - A^*) = X - (A^* - A)$. □

The following theorem gives a characterization of $I_{\hat{\omega}}$-open sets.

Theorem 3.24. *A subset A of an ideal space (X,τ,I) is $I_{\hat{\omega}}$-open if and only if $F \subseteq int^*(A)$ whenever F is $\hat{\omega}$-closed and $F \subseteq A$.*

Proof: Necessity.Suppose A is $I_{\hat{\omega}}$-open and F be a $\hat{\omega}$-closed set contained in A.Then $X-A \subseteq X-F$ and hence $cl^*(X-A) \subset X-F$.Thus $F \subseteq X-cl^*(X-A) = int^*(A)$.
Sufficiency.Suppose $X-A \subseteq U$ where U is $\hat{\omega}$-open.Then $X-U \subseteq A$ and X-U is $\hat{\omega}$-closed.Then $X-U \subseteq int^*(A)$ which implies $cl^*(X-A) \subseteq U$.Consequently X-A is $I_{\hat{\omega}}$-closed and so A is $I_{\hat{\omega}}$-open. □

Theorem 3.25. *If A is an $I_{\hat{\omega}}$-open set of an ideal space (X,τ,I) and $int^*(A) \subseteq B \subseteq A$,Then B is also an $I_{\hat{\omega}}$-open set of (X,τ,I).*

Proof: Suppose $F \subseteq B$ where F is $\hat{\omega}$-closed set of (X,τ,I).Then $F \subseteq A$.Since A is $I_{\hat{\omega}}$-open,$F \subseteq int^*(A)$.Since $int^*(A) \subseteq int^*(B)$,we have $F \subseteq int^*(B)$.By theorem 3.24, B is $I_{\hat{\omega}}$-open. □

4. $I_{\hat{\omega}}$-continuity and $I_{\hat{\omega}}$-irresoluteness

Definition 4.1. *A function $f:(X,\tau,I) \to (Y,\sigma)$ is said to be $I_{\hat{\omega}}$-continuous if $f^{-1}(V)$ is $I_{\hat{\omega}}$-closed in X for every closed set V of Y.*

Example 4.2. *Let X=Y={a,b,c},$\tau = \{\phi,\{a,b\},X\}, I = \{\phi,\{a\}\}$ and $\sigma = \{\phi,\{a\},\{a,b\},\{a,c\},Y\}$.Then the function $f:(X,\tau,I) \to (Y,\sigma)$ defined by $f(a)=c, f(b)=a$ and $f(c)=b$ is $I_{\hat{\omega}}$-continuous.*

Definition 4.3. *A function $f:(X,\tau,I_1) \to (Y,\sigma,I_2)$ is said to be $I_{\hat{\omega}}$-irresolute if $f^{-1}(V)$ is $I_{\hat{\omega}}$-closed in X for every $I_{\hat{\omega}}$-closed set V of Y.*

Example 4.4. *Let $X = Y = \{a,b,c\}$,$\tau = \{\phi,\{a\},\{a,c\},X\}, I = \{\phi,\{a\}\}$ and $\sigma = \{\phi,\{a\},\{b,c\},Y\}$,$J = \{\phi,\{c\}\}$.Then the function $f:(X,\tau,I) \to (Y,\sigma,J)$ defined by $f(a)=c, f(b)=a$ and $f(c)=b$ is $I_{\hat{\omega}}$-irresolute.*

Theorem 4.5. *A subset A of an ideal space (X,τ,I) is $I_{\hat{\omega}}$-continuous if and only if $f^{-1}(V)$ is $I_{\hat{\omega}}$-open in X for every open set V of Y.*

Proof: Let $f:(X,\tau) \to (Y,\sigma)$ be a $I_{\hat{\omega}}$-continuous function and V be any open set in (Y,σ).Then $f^{-1}(V^c)$ is $I_{\hat{\omega}}$-closed in (X,τ).Since $f^{-1}(V^c) = [f^{-1}(V)]^c, f^{-1}(V)$ is $I_{\hat{\omega}}$-open in (X,τ).
Converse is similar. □

Remark 4.6. *The composition of two $I_{\hat{\omega}}$-continuous functions need not be $\hat{\omega}$-continuous and this is shown by the following example.*

Example 4.7. *Let X ={a,b,c}=Y=Z, $\tau = \{\phi,\{a,b\},X\}, I = \{\phi,\{c\}\}, \sigma = \{\phi,\{a\},Y\}, J = \{\phi,\{b\}\}$ and $\eta = \{\phi,\{a\},\{a,b\},\{a,c\},Z\}$.Let $f:(X,\tau,I) \to (Y,\sigma)$ be the function defined by $f(a)=a, f(b)=c$ and $f(c)=b$ and $g:(Y,\sigma,J) \to (Z,\eta)$*

be the function defined by $g(a) = a, g(b) = c$ and $g(c) = b$.Then both f and g are $I_{\hat{\omega}}$-continuous.But $g \circ f : (X, \tau, I) \to (Z, \eta)$ is not a $I_{\hat{\omega}}$-continuous function since $(g \circ f)^{-1}(\{b\}) = f^{-1}(g^{-1}(\{b\})) = f^{-1}\{c\} = \{b\}$ is not $I_{\hat{\omega}}$-closed in (X, τ, I) where $\{b\}$ is a closed set in(Z, η).

Theorem 4.8. *Let $f : (X, \tau, I_1) \to (Y, \sigma, I_2)$and $g : (Y, \sigma, I_2) \to (Z, \eta, I_3)$be any two functions.Then the following hold.*

(i)$g \circ f$ is $I_{\hat{\omega}}$-continuous if f is $I_{\hat{\omega}}$-continuous and g is continuous.

(ii)$g \circ f$ is $I_{\hat{\omega}}$-continuous if f is $I_{\hat{\omega}}$-irresolute and g is $I_{\hat{\omega}}$-continuous.

(iii)$g \circ f$ is $I_{\hat{\omega}}$-irresolute if f is $I_{\hat{\omega}}$-irresolute and g is $I_{\hat{\omega}}$-irresolute.

Proof:

(i)Let V be closed in Z. Since g is continuous,$g^{-1}(V)$ is closed in Y.$I_{\hat{\omega}}$-continuity of f implies $f^{-1}(g^{-1}(V))$ is $I_{\hat{\omega}}$-closed in X and hence $g \circ f$ is $I_{\hat{\omega}}$-continuous.

(ii)Let V be closed in Z. Since g is $I_{\hat{\omega}}$-continuous,$g^{-1}(V)$ is $I_{\hat{\omega}}$-closed in Y.Since f is $I_{\hat{\omega}}$-irresolute,$f^{-1}(g^{-1}(V))$ is $I_{\hat{\omega}}$-closed in X and hence $g \circ f$ is $I_{\hat{\omega}}$-continuous.

(iii)Let V be $I_{\hat{\omega}}$-closed in Z. Since g is $I_{\hat{\omega}}$-irresolute,$g^{-1}(V)$ is $I_{\hat{\omega}}$-closed in Y.Since f is $I_{\hat{\omega}}$-irresolute,$f^{-1}(g^{-1}(V))$ is $I_{\hat{\omega}}$-closed in X and hence $g \circ f$ is $I_{\hat{\omega}}$-irresolute.

□

Recall that $f : (X, \tau) \to (Y, \sigma)$ is said to be $\hat{\omega}^*$-closed if f(V) is $\hat{\omega}$-closed in Y for every $\hat{\omega}$-closed set V in X.

Theorem 4.9. *If $f : (X, \tau, I) \to (Y, \sigma)$ is $I_{\hat{\omega}}$-continuous and $\hat{\omega}^*$-closed then f is $I_{\hat{\omega}}$-irresolute.*

Proof: Suppose V is $I_{\hat{\omega}}$-closed in Y and $f^{-1}(V) \subseteq U$ where U is $\hat{\omega}$-open in X.Then $X - U \subseteq X - f^{-1}(V) = f^{-1}(Y - V)$ and hence $f(X - U) \subseteq Y - V$.Since f is $\hat{\omega}^*$-closed,f(X-U) is $\hat{\omega}$-closed.By theorem 3.24,$f(X - U) \subseteq int^*(Y - V) = Y - cl^*(V)$.Now $X - U \subseteq f^{-1}(f(X - U)) \subseteq f^{-1}(Y - cl^*(V)) = X - f^{-1}(cl^*(V))$ which implies $f^{-1}(cl^*(V)) \subseteq U$.Since f is $I_{\hat{\omega}}$-continuous,$f^{-1}(cl^*(V))$ is $I_{\hat{\omega}}$-closed and so $cl^*(f^{-1}(cl^*(V))) \subseteq U$.Hence $cl^*(f^{-1}(V)) \subseteq cl^*(f^{-1}(cl^*(V))) \subseteq U$.Thus $f^{-1}(V)$ is $I_{\hat{\omega}}$-closed and so f is $I_{\hat{\omega}}$-irresolute. □

Definition 4.10. *A collection $\{A_\alpha : \alpha \in \wedge\}$ of $I_{\hat{\omega}}$-open sets in an ideal topological space (X, τ, I) is said to be an $I_{\hat{\omega}}$-open cover of a subset V of X if $V \subset \cup\{A_\alpha : \alpha \in \wedge\}$ holds.*

Definition 4.11. *An ideal space (X, τ, I) is said to be $I_{\hat{\omega}}$-compact if for every $I_{\hat{\omega}}$-open cover $\{V_\alpha : \alpha \in \Lambda\}$,there exists a finite subset Λ_0 of Λ of X such that $X - \cup\{V_\alpha : \alpha \in \Lambda_0\} \in I$.*

Lemma 4.12. *[11]For any function $f : (X, \tau, I) \to (Y, \sigma)$, f(I) is an ideal on Y.*

Theorem 4.13. *The image of a $I_{\hat{\omega}}$-compact space under a $I_{\hat{\omega}}$-continuous surjective function is f(I)-compact.*

Proof: Let $f : (X, \tau, I) \to (Y, \sigma)$ be a $I_{\hat{\omega}}$-continuous surjective function from a $\hat{I}_{\hat{\omega}}$-compact space.Let$\{V_\alpha : \alpha \in \Lambda\}$ be an open cover of Y. Then $\{f^{-1}(V_\alpha) : \alpha \in \Lambda\}$is an $I_{\hat{\omega}}$-open cover of X.Since X is $I_{\hat{\omega}}$-compact,there exists a finite subset Λ_0 of Λ of X such that $X - \cup\{f^{-1}(V_\alpha) : \alpha \in \Lambda_0\} \in I$.Hence $Y - \cup\{V_\alpha : \alpha \in \Lambda_0\} \in$ f(I) and so (Y, σ) is f(I) compact. □

Definition 4.14. *A topological space (X, τ, I) is said to be $I_{\hat{\omega}}$-connected if X cannot be written as a disjoint union of two non-empty $I_{\hat{\omega}}$-open subsets.A subset of X is $I_{\hat{\omega}}$-connected if it is $I_{\hat{\omega}}$-connected as a subspace.*

Theorem 4.15. *If $f : (X, \tau, I) \to (Y, \sigma)$ is a $I_{\hat{\omega}}$-continuous surjection and X is $I_{\hat{\omega}}$-connected,then Y is connected.*

Proof: Suppose $Y = A \cup B$ where A and B are disjoint,open sets in Y.Since f is $I_{\hat{\omega}}$-continuous and onto,$X = f^{-1}(A) \cup f^{-1}(B)$ where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint,non-empty $I_{\hat{\omega}}$-open sets in X,a contradiction since X is $I_{\hat{\omega}}$-connected.Hence Y is connected. □

Definition 4.16. *An ideal space (X, τ, I) is said to be $I_{\hat{\omega}}$-normal if for each pair of non-empty disjoint closed sets A and B of X, there exists disjoint $I_{\hat{\omega}}$-open subsets U and V of X such that $A \subseteq U$ and $B \subseteq V$.*

Theorem 4.17. *If $f : (X, \tau, I) \to (Y, \sigma)$ is a $I_{\hat{\omega}}$-continuous,closed injection and Y is normal,then X is $I_{\hat{\omega}}$-normal.*

Proof: Let A and B be disjoint closed subsets of X.Since f is closed and injective, f(A) and f(B) are disjoint,closed subsets of Y.Since Y is normal, there exists disjoint open subsets U and V of X such that $f(A) \subseteq U$ and $f(B) \subseteq V$.Hence $A \subseteq f^{-1}(U)$ and $B \subseteq f^{-1}(V)$ and $f^{-1}(U) \cap f^{-1}(V) = \phi$.Since f is $I_{\hat{\omega}}$-continuous,$f^{-1}(U)$ and $f^{-1}(V)$ are $I_{\hat{\omega}}$-open in X which implies X is $I_{\hat{\omega}}$-normal. □

5. Maximal $I_{\hat{\omega}}$-closed sets

Definition 5.1. *A proper nonempty $I_{\hat{\omega}}$-closed subset U of an ideal space (X, τ, I) is said to be maximal $I_{\hat{\omega}}$-closed if any $I_{\hat{\omega}}$-closed set containing U is either X or U.*

Example 5.2. *Let X={a,b,c,d },$\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}, I = \{\phi, \{b\}, \{c\}, \{b, c\}\}$. Then $I_{\hat{\omega}}$-closed sets are $\phi, \{b\}, \{c\}, \{b, c\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}, X$.Here $\{a, c, d\}$ and $\{b, c, d\}$ are maximal $I_{\hat{\omega}}$-closed sets.*

Remark 5.3. *Every maximal $I_{\hat{\omega}}$-closed set is a $I_{\hat{\omega}}$-closed set but not conversely.*

Example 5.4. *In example 5.2,$\{b\}$ is a $I_{\hat{\omega}}$-closed set but not a maximal $I_{\hat{\omega}}$-closed set.*

Theorem 5.5. *The following statements are true for any ideal space (X, τ, I).*

(i)Let F be a maximal $I_{\hat{\omega}}$-closed set and G be a $I_{\hat{\omega}}$-closed set.Then $F \cup G = X$ or $G \subset F$.

(ii)Let F and G be maximal $I_{\hat{\omega}}$-closed sets.Then $F \cup G = X$ or $F = G$.

Proof:

(i)Let F be a maximal $I_{\hat{\omega}}$-closed set and G be a $I_{\hat{\omega}}$-closed set.If $F \cup G = X$,then there is nothing to prove.Assume that $F \cup G \neq X$.Now $F \subset F \cup G$.By theorem 3.16,$F \cup G$ is a $I_{\hat{\omega}}$-closed set.Since F is a maximal $I_{\hat{\omega}}$-closed set,we have $F \cup G = X$ or $F \cup G = F$.Hence $F \cup G = F$ and so $G \subset F$.

(ii)Let F and G be maximal $I_{\hat{\omega}}$-closed sets.If $F \cup G = X$,then there is nothing to prove.Assume that $F \cup G \neq X$.Then by(i) $F \subset G$ and $G \subset F$ which implies F=G.

□

Definition 5.6. *An ideal space (X, τ, I) is said to be a $MT_{\hat{\omega}}$-space if every nonempty proper $I_{\hat{\omega}}$-closed subset of X is a maximal $I_{\hat{\omega}}$-closed set.*

Example 5.7. *Let X={a,b,c,d },$\tau = \{\phi, \{a\}, \{b, c, d\}, X\}, I = \{\phi\}$.Then $I_{\hat{\omega}}$-closed sets are $\phi, \{a\}, \{b, c, d\}, X$.Here $\{a\}$ and $\{b, c, d\}$are maximal $I_{\hat{\omega}}$-closed sets. Hence (X, τ, I) is a $MT_{\hat{\omega}}$-space.*

Definition 5.8. *A function $f : (X, \tau, I) \to (Y, \sigma)$ is said to be maximal $I_{\hat{\omega}}$-continuous if $f^{-1}(V)$is a maximal $I_{\hat{\omega}}$-closed set in X for every nonempty proper closed set V of Y.*

Theorem 5.9. *Every surjective maximal $I_{\hat{\omega}}$-continuous function is $I_{\hat{\omega}}$-continuous.*

Proof: Let $f : (X, \tau, I) \to (Y, \sigma)$ be a surjective maximal $I_{\hat{\omega}}$-continuous function.The inverse image of ϕ and Y are always $I_{\hat{\omega}}$-closed sets in X. Let V be a proper closed set in Y.Now f is a maximal $I_{\hat{\omega}}$-continuous function implies $f^{-1}(V)$ is a maximal $I_{\hat{\omega}}$-closed set in X.Since every maximal $I_{\hat{\omega}}$-closed set is a $I_{\hat{\omega}}$-closed set,the proof follows. □

Remark 5.10. *The converse of the above theorem need not be true as seen from the following example.*

Example 5.11. *Let X=Y={a,b,c},$\tau = \{\phi, \{a, b\}, X\}, I = \{\phi, \{a\}\}$and $\sigma = \{\phi, \{a\}, \{a, b\}, \{a, c\}, Y\}$.Then the function $f : (X, \tau, I) \to (Y, \sigma)$ defined by $f(a) = c, f(b) = a$ and $f(c) = b$ is $I_{\hat{\omega}}$-continuous, but not maximal $I_{\hat{\omega}}$-continuous.*

Theorem 5.12. *If $f : (X, \tau, I) \to (Y, \sigma)$ is a surjective,$I_{\hat{\omega}}$-continuous function where X is a $MT_{\hat{\omega}}$-space,then f is a maximal $I_{\hat{\omega}}$-continuous function.*

Proof: Let V be a nonempty proper closed subset of Y.Since f is surjective and $I_{\hat{\omega}}$-continuous,$f^{-1}(V)$ is a nonempty proper $I_{\hat{\omega}}$-closed subset of X.Now X is a $MT_{\hat{\omega}}$-space implies $f^{-1}(V)$ is a maximal $I_{\hat{\omega}}$-closed subset of X.Thus f is a maximal $I_{\hat{\omega}}$-continuous function. □

Remark 5.13. *The composition of maximal $I_{\hat{\omega}}$-continuous functions need not be a maximal $I_{\hat{\omega}}$-continuous function.*

Example 5.14. *Let X={a,b,c}=Y=Z,$\tau = \{\phi, \{a\}, \{b, c\}, X\}, I = \{\phi\}, \sigma = \{\phi, \{c\}, Y\}, J = \{\phi\}$ and $\eta = \{\phi, \{a\}, Z\}$.Let $f : (X, \tau, I) \to (Y, \sigma)$ be the function defined by $f(a) = c, f(b) = b$ and $f(c) = a$and $g : (Y, \sigma, J) \to (Z, \eta)$ be the function by $g(a) = b, g(b) = a$ and $g(c) = c$.Then both f and g are maximal $I_{\hat{\omega}}$-continuous.But $g \circ f : (X, \tau, I) \to (Z, \eta)$ is not a maximal $I_{\hat{\omega}}$-continuous function since $(g \circ f)^{-1}(\{b, c\}) = f^{-1}(g^{-1}(\{b, c\})) = f^{-1}\{a, c\} = \{a, c\}$ is not maximal $I_{\hat{\omega}}$-closed in (X, τ, I) where $\{b, c\}$ is a closed set in(Z, η).*

Theorem 5.15. *Let $f : (X, \tau, I) \to (Y, \sigma)$ be a maximal $I_{\hat{\omega}}$-continuous function and $g : (Y, \sigma, J) \to (Z, \eta)$ be a surjective, continuous function.Then $g \circ f : (X, \tau, I) \to (Z, \eta)$ is a maximal $I_{\hat{\omega}}$-continuous function.*

Proof: Let V be a nonempty proper closed set in Z.Since g is continuous,$g^{-1}(V)$ is a nonempty proper closed set in Y.Now f is maximal $I_{\hat{\omega}}$-continuous implies $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is a maximal $I_{\hat{\omega}}$-closed set in X.Hence $g \circ f$ is a maximal $I_{\hat{\omega}}$-continuous function. □

Theorem 5.16. *Let $f : (X, \tau, I) \to (Y, \sigma)$ be a surjective,$I_{\hat{\omega}}$-continuous function and X be a $MT_{\hat{\omega}}$-space and $g : (Y, \sigma, J) \to (Z, \eta)$ be a surjective,continuous function.Then $g \circ f : (X, \tau, I) \to (Z, \eta)$ is a maximal $I_{\hat{\omega}}$-continuous function.*

Proof: Let V be a nonempty proper closed set in Z.Since g is continuous,$g^{-1}(V)$ is a nonempty proper closed set in Y.Now f is $I_{\hat{\omega}}$-continuous implies $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is a proper $I_{\hat{\omega}}$-closed set in X.Since X is a $MT_{\hat{\omega}}$-space,$(g \circ f)^{-1}(V)$is a maximal $I_{\hat{\omega}}$-closed set in X.Hence $g \circ f$ is a maximal $I_{\hat{\omega}}$-continuous function. □

6. Applications

In this section, we introduce $\hat{C}_I$-sets and $\hat{D}_I$-sets and investigate some
of their properties.

Definition 6.1. *A subset A of an ideal space (X, τ, I) is said to be a*

(i) $\hat{C}_I$-set if $A = U \cap V$ where U is an $\hat{\omega}$-open set and V is a $*$-closed set.

(ii)$\hat{D}_I$-set if $A = U \cap V$ where U is an $\hat{\omega}$-open set and V is a $*$-perfect set.

Remark 6.2. *(i)Every $\hat{\omega}$-open set is a $\hat{C}_I$-set.*

(ii)Every $*$*-closed set is a* $\hat{C}_I$*-set.*

(iii)Every $*$*-perfect set is a* $\hat{D}_I$*-set.*

(iv)Every $\hat{D}_I$*-set set is a* $\hat{C}_I$*-set.*

Remark 6.3. *The converses of the above remark 6.2 need not be true as shown in the following example.*

Example 6.4. *Let X={a,b,c,d},*$\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ *and* $I = \{\phi, \{b\}, \{c\}, \{b, c\}\}$*. Then*

(i)$\{b, c, d\}$ *is a* $\hat{C}_I$*-set,but not a* $\hat{\omega}$*-open set.*

(ii)$\{a, b, d\}$ *is a* $\hat{C}_I$*-set,but not a* $*$*-closed set.*

(iii)$\{a\}$ *is a* $\hat{D}_I$*-set,but not a* $*$*-perfect set.*

Theorem 6.5. *Every strongly-I-LC is a* $\hat{C}_I$*-set in an ideal space* (X, τ, I)*.*

Proof: Since every regular open set is $\hat{\omega}$-open [8],the proof follows. □

The converse of the above theorem 6.5 need not be true as shown in the following example.

Example 6.6. *Let X={a,b,c,d},*$\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ *and* $I = \{\phi, \{b\}, \{c\}, \{b, c\}\}$*. Then* $\{a, b, c\}$ *is a* $\hat{C}_I$*-set,but not strongly-I-LC.*

Theorem 6.7. *Let A be a subset of an ideal space* (X, τ, I)*.If A is a* $\hat{D}_I$*-set and an* $I_{\hat{\omega}}$*-closed set, then A is a* $*$*-closed set.*

Proof: Since A is a $\hat{D}_I$-set,$A = U \cap V$ where U is an $\hat{\omega}$-open set and V is a $*$-perfect set.Now $A \subseteq U$ where U is an $\hat{\omega}$-open set and A is an $I_{\hat{\omega}}$-closed set implies that $A^* \subseteq U$.Also $A \subseteq V$ where V is a $*$-perfect set implies that $A^* \subseteq V$.Thus $A^* \subseteq U \cap V = A$ and so A is a $*$-closed set. □

Remark 6.8. *The converse of the above theorem need not be true as seen from the following example.*

Example 6.9. *Let* (X, τ) *and I be same as in example 6.4.Then {b,c}is a* $*$*-closed set,but not a* $\hat{D}_I$*-set.*

Theorem 6.10. *Let A be a subset of an ideal space* (X, τ, I)*.Then A is a* $\hat{*}$*-closed set if and only if A is a* $\hat{C}_I$*-set and an* $I_{\hat{\omega}}$*-closed set.*

Proof: Necessity.Suppose A is a $*$-closed set.Then $A = X \cap A$ where X is $\hat{\omega}$-open set and A is a $*$-closed set.Hence A is a $\hat{C}_I$-set.By remark 3.3,A is also $I_{\hat{\omega}}$-closed. Sufficiency.Let A be a $\hat{C}_I$-set and an $I_{\hat{\omega}}$-closed set.Since A is a $\hat{C}_I$-set, $A = U \cap V$ where U is an $\hat{\omega}$-open set and V is a $*$-closed set.Now $A \subseteq U$ where U is an $\hat{\omega}$-open set and A is an $I_{\hat{\omega}}$-closed set implies that $A^* \subseteq U$.Also $A \subseteq V$ where V is a $*$-closed set implies that $A^* \subseteq V^* \subseteq V$.Thus $A^* \subseteq U \cap V = A$ and so A is a $*$-closed set. □

Theorem 6.11. *Let (X, τ, I) be an ideal space and A be a $\hat{C}_I$-set in X.Then the following hold.*

(i)If B is $*$-closed,then $A \cap B$ is a $\hat{C}_I$-set.

(ii)If B is $\hat{\omega}$-open,then $A \cap B$ is a $\hat{C}_I$-set.

(iii)If B is $\hat{C}_I$-set,then $A \cap B$ is a $\hat{C}_I$-set.

Proof:

(i)Since A is a $\hat{C}_I$-set, $A = U \cap V$ where U is an $\hat{\omega}$-open set and V is a $*$-closed set.Hence $A \cap B = (U \cap V) \cap B = U \cap (V \cap B)$ where $V \cap B$is $*$-closed.Thus $A \cap B$ is a $\hat{C}_I$-set.

(ii)Since A is a $\hat{C}_I$-set, $A = U \cap V$ where U is an $\hat{\omega}$-open set and V is a $*$-closed set.Hence $A \cap B = (U \cap V) \cap B = (U \cap B) \cap V$ where $U \cap B$ is $\hat{\omega}$-open.Thus $A \cap B$ is a $\hat{C}_I$-set.

(iii)Since B is a $\hat{C}_I$-set, $A = U_1 \cap V_1$ where U_1 is an $\hat{\omega}$-open set and V_1 is a $*$-closed set.Hence $A \cap B = (U \cap V) \cap (U_1 \cap V_1) = (U \cap U_1) \cap (V \cap V_1)$ where $U \cap U_1$is $\hat{\omega}$-open and $V \cap V_1$ is$*$-closed .Thus $A \cap B$ is a $\hat{C}_I$-set.

□

Definition 6.12. *A function $f : (X, \tau, I) \to (Y, \sigma)$ is said to be $\hat{C}_I$-continuous if $f^{-1}(V)$is $\hat{C}_I$-set in (X, τ, I) for every closed set A of Y.*

Theorem 6.13. *Every strongly-I-LC-continuous is $\hat{C}_I$-continuous.*

Proof: Follows from theorem 6.5 □

Theorem 6.14. *Let $f : (X, \tau, I) \to (Y, \sigma)$ be a function.Then the following are equivalent.*

(i)f is $*$-continuous.

(ii)f is $\hat{C}_I$-continuous and $I_{\hat{\omega}}$-continuous.

Proof: Follows from theorem 6.10 □

Theorem 6.15. *Let $f : (X, \tau, I) \to (Y, \sigma)$ be a function.Then the following are equivalent.*

(i) f is $*$-continuous.

(ii) f is strongly-I-LC-continuous and $I_{\hat{\omega}}$-continuous.

Proof: (i)⇒(ii)Follows from the fact that every $*$-closed set is strongly-I-LC [12] and from Remark 3.3
(ii)⇒(i)Let V be a closed set in Y.Since f is strongly-I-LC-continuous and $I_{\hat{\omega}}$-continuous, $f^{-1}(V)$ is a strongly-I-LC and $I_{\hat{\omega}}$-closed set.By theorem 6.5,$f^{-1}(V)$ is a $\hat{C}_I$-set and $I_{\hat{\omega}}$-closed set and hence f is $\hat{C}_I$-continuous and $I_{\hat{\omega}}$-continuous.By theorem 6.14,f is $*$-continuous. □

Definition 6.16. *A function $f : (X, \tau, I) \to (Y, \sigma)$ is said to be $\hat{D}_I$-continuous if $f^{-1}(V)$is $\hat{D}_I$-set in (X, τ, I) for every closed set V of Y.*

Theorem 6.17. *If function $f : (X, \tau, I) \to (Y, \sigma)$ is $\hat{D}_I$-continuous and $I_{\hat{\omega}}$-continuous,then f is $*$-continuous.*

Proof: Follows from Remark 6.2(iv) and theorem 6.14 □

Remark 6.18. *The converse of the above theorem need not be true as seen from the following example.*

Example 6.19. *Let X={a,b,c}=Y,$\tau = \{\phi, \{a\}, \{a, c\}, X\}$,and $I = \{\phi, \{a\}\}, \sigma = \{\phi, \{a\}, \{a, b\}, \{a, c\}, Y\}$,Then the function $f : (X, \tau, I) \to (Y, \sigma)$ defined by $f(a) = b, f(b) = c$,and $f(c) = a$ is $*$-continuous,but not $\hat{D}_I$-continuous,since $f^{-1}\{b, c\} = \{a, b\}$is not a $\hat{D}_I$-set where {b,c} is a closed set in Y.*

Theorem 6.20. *Let $f : (X, \tau, I) \to (Y, \sigma)$ be a surjective,$\hat{C}_I$-continuous and maximal $I_{\hat{\omega}}$-continuous function.Then f is $*$-continuous.*

Proof: Follows from theorem 5.9 and theorem 6.14 □

Remark 6.21. *The converse of the above theorem need not be true as seen from the following example.*

Example 6.22. *Let X=Y={a,b,c},$\tau = \{\phi, \{a\}, \{a, c\}, X\}$and $I = \{\phi, \{a\}\}, \sigma = \{\phi, \{b\}, \{c\}, \{b, c\}, Y\}$. Then the function $f : (X, \tau, I) \to (Y, \sigma)$ defined by $f(a) = b, f(b) = a$ and $f(c) = c$ is surjective and $*$-continuous,but not maximal $I_{\hat{\omega}}$-continuous,since $f^{-1}(\{a\}) = \{b\}$is not a maximal $I_{\hat{\omega}}$-closed set in X where $\{a\}$ is a closed set in Y.*

7. Conclusion

The study of ideal topological generalized closed sets via $\hat{\omega}$-open sets is applicable in most areas of pure and applied mathematics.This study would open up the academic flood gates and new vistas in the field of bitopology and fuzzy topology for further research studies.

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