# New spaces and Continuity via $\hat{\Omega}$-closed sets

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ABSTRACT: In this paper, we introduce new spaces like $_{\hat{\Omega}}T_\delta$ and $_\omega T_{\hat{\Omega}}$. It turns out that the space $_{\hat{\Omega}}T_\delta$ coincide with $semi\text{-}T_1$ and in $_\omega T_{\hat{\Omega}}$-space every closed set is $\hat{\Omega}$-closed set and in $semi\text{-}T_{\frac{1}{2}}$ every $\hat{\Omega}$-closed set is closed in a topological space. Also we introduce some kinds of generalized continuity such as $\hat{\Omega}$-continuity, $\hat{\Omega}$-irresolute and weakly $\hat{\Omega}$-continuity.

Key Words: $\hat{\Omega}$-closed sets, $_{\hat{\Omega}}T_\delta$-spaces, $_\omega T_{\hat{\Omega}}$-spaces, $\hat{\Omega}$-continuity and $\hat{\Omega}$- irresolute.

## Contents

## 1. Introduction

In the recent years, many new separation axioms were introduced by using generalized closed sets such as [3] $T_b$, [12] $T_d$, [2] $semi\text{-}T_1$, $semi\text{-}T_{\frac{1}{2}}$ and [6] $T_{\frac{3}{4}}$. The class of $\hat{\Omega}$-closed sets [8] has been introduced recently by using the $\delta$-closure operator. As the class of $\hat{\Omega}$-closed sets are independent of the class of closed sets, our interest is to find the spaces in which they are interrelated. With these motivation, new separation axioms via $\hat{\Omega}$-closed sets such as $_{\hat{\Omega}}T_\delta$ which coincide with $semi\text{-}T_1$ and $_\omega T_{\hat{\Omega}}$ in which every $\hat{\Omega}$-closed set is closed set are introduced and studied. Moreover, we introduce and study various types of the continuity via $\hat{\Omega}$-closed sets. Also we investigate the role of new spaces and it's impacts on all these continuities.

## 2. Preliminaries

Throughout this paper $(X, \tau)$ (or briefly $X$) represent a topological space on which no separation axioms are assumed unless explicitly stated. For a subset $A$ of $(X, \tau)$, we denote the closure of $A$, the interior of $A$ and the complement of $A$ as $cl(A)$, $int(A)$ and $A^c$ respectively. The family of all open sets in $(X, \tau)$ are denoted by $O(X)$ and $O(X, x) = \{U \in X : x \in U \in O(X)\}$ Let us recall the following definitions, which are useful in the sequel.

**Definition 2.1.** *[7] A subset $A$ of $X$ is called $\delta$-closed in a topological space $(X, \tau)$ if $A = \delta cl(A)$, where $\delta cl(A) = \{x \in X : int(cl(U)) \cap A \neq \emptyset, U \in O(X, x)\}$.*
*The complement of $\delta$-closed set in $(X, \tau)$ is called $\delta$-open set in $(X, \tau)$. From [7], Lemma 3, $\delta cl(A) = \bigcap\{F \in \delta C(X) : A \subseteq F\}$ and from Corollary 4, $\delta cl(A)$ is a $\delta$-closed for a subset $A$ in a topological space $(X, \tau)$.*

The following notations are used in this paper. The family of all $\delta$-open (resp.$\hat{\Omega}$-open) sets on $X$ are denoted by $\delta O(X)$ (resp.$\hat{\Omega}O(X)$).

- $\delta O(X, x) = \{U \in X : x \in U \in \delta O(X)\}$

- $\hat{\Omega}O(X, x) = \{U \in X : x \in U \in \hat{\Omega}O(X)\}$

**Definition 2.2.** *A subset $A$ of a topological space $(X, \tau)$ is called*

(i) *semi open [9] if $A \subseteq cl(int(A))$.*

(ii) *pre open [11] if $A \subseteq int(cl(A))$.*

(iii) *regular open [15] if $A = int(cl(A))$.*

(iv) *a $\delta$ generalised closed (briefly $\delta g$-closed) set [5] if $\delta cl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open in $(X, \tau)$.*

(v) *$g\alpha$-closed set [8] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $\alpha$ open in $(X, \tau)$.*

(vi) *$\hat{g}$ (or) $\omega$-closed set [17] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is semi open in $(X, \tau)$.*

(vii) *$\hat{\Omega}$-closed set [8] if $\delta cl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is semi open in $(X, \tau)$.*

*The complement of semi open (resp.pre open, regular open $\delta g$-closed $\omega$(or $\hat{g}$)-closed) set is called semi closed (resp.pre closed, regular closed, $\delta g$-open, $\omega$(or$\hat{g}$)-open,$\hat{\Omega}$-open).*

**Definition 2.3.** *A function $f \colon (X, \tau) \to (Y, \sigma)$ is called a*

(i) *contra continuous [5] if the inverse image of open set in $Y$ is closed set in $X$.*

(ii) *perfectly continuous [14] if the inverse image of open set in $Y$ is clopen set in $X$.*

(iii) *completely continuous [1] if the inverse image of open set in $Y$ is regular open set in $X$.*

(iv) *super continuous [10] if the inverse image of open set in $Y$ is $\delta$ open set in $X$.*

 (v) *semi continuous [9] if the inverse image of open set in $Y$ is semi open set in $X$.*

 (vi) *pre continuous [11] if the inverse image of open set in $Y$ is pre open set in $X$.*

 (vi) *$\delta g$-continuous [6] if the inverse image of open set in $Y$ is $\delta g$-open set in $X$.*

 (vii) *$\omega$-continuous [17] if the inverse image of open set in $Y$ is $\omega$-open set in $X$.*

(viii) *$g\alpha$-continuous [4] if the inverse image of open set in $Y$ is $g\alpha$-open set in $X$.*

 (ix) *$\delta$-closed mapping [13] if the image of every $\delta$-closed set in $X$ is $\delta$-closed in $Y$.*

**Definition 2.4.** *A space $(X, \tau)$ is said to be $T_{\frac{3}{4}}$ [6] if every $\delta g$-open set is $\delta$-open set in $X$.*

**Definition 2.5.** *A space $(X, \tau)$ is said to be weakly Hausdorff [6] if for every two different point $x$ and $y$, there exists a regular open set $U$ such that $x \in U$, $y \notin U$.*

### 3. $_{\hat{\Omega}}T_\delta$-spaces and $_\omega T_{\hat{\Omega}}$-spaces

In this section, we introduce two new spaces in which we get the relationship between the class of $\hat{\Omega}$-open sets and that of open sets. These spaces provide a new path to class of $\hat{\Omega}$-open sets to travel along with open sets.

**Definition 3.1.** *A space $(X, \tau)$ is called $_{\hat{\Omega}}T_\delta$-space if every $\hat{\Omega}$-closed set in $X$ is $\delta$-closed in $X$.*

**Example 3.2.** $X = \{a, b, c\}, \tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$. *Here,* $\hat{\Omega}O(X) = \tau = \delta O(X)$.
*Therefore, $(Y, \sigma)$ is $_{\hat{\Omega}}T_\delta$-space.*

**Theorem 3.3.** *Every $T_{\frac{3}{4}}$-space is $_{\hat{\Omega}}T_\delta$-space.*

**Proof:** Suppose that $(X, \tau)$ is a $T_{\frac{3}{4}}$-space and $A$ is any $\hat{\Omega}$-closed set in $(X, \tau)$. By [8] Theorem 3.5, $A$ is $\delta g$-closed set in $X$. Since $X$ is $T_{\frac{3}{4}}$-space, $A$ is $\delta$-closed set in $X$.
Thus, $X$ is $_{\hat{\Omega}}T_\delta$-space. $\qquad\qquad\square$

**Remark 3.4.** *Converse is not always possible from the Example 3.2.*

**Example 3.5.** *By [6] the digital line is $T_{\frac{3}{4}}$. By Theorem 3.3, the digital line is $_{\hat{\Omega}}T_\delta$-space.*

Let us give some characterizations of $_{\hat{\Omega}}T_{\delta}$-space.

**Theorem 3.6.** *A topological space $(X, \tau)$ is $_{\hat{\Omega}}T_{\delta}$-space if and only if every singleton set is semi closed in $(X, \tau)$.*

**Proof: Necessity-** Let $x \in X$. By [8] Proposition 4.7, $\{x\}$ is semi closed or $\{x\}^c$ is $\hat{\Omega}$-closed in $X$. Therefore, if $\{x\}$ is not semi closed then $\{x\}^c$ is $\hat{\Omega}$-closed in $X$, By hypothesis, $\{x\}^c$ is $\delta$-closed set in $(X, \tau)$ and hence $\{x\}$ is $\delta$-open in $(X, \tau)$. By [6] Lemma 4.2, $\{x\}$ is regular open and hence $\{x\}$ is semi closed in $X$.
**Sufficiency-** Suppose that $A$ is any $\hat{\Omega}$-closed set in $(X, \tau)$ and $x \in \delta cl(A) \setminus (A)$. Since $A$ is $\hat{\Omega}$-closed set in $(X, \tau)$, [8] by Theorem 4.4, $\delta cl(A) \setminus (A)$ does not contain any non-empty semi closed set in $X$. Therefore, $\{x\}$ is not a semi closed in $(X, \tau)$, a contradiction.
Therefore, $\delta cl(A) = A$ and hence $A$ is $\delta$-closed in $(X, \tau)$.     $\square$

**Theorem 3.7.** *In a topological space $(X, \tau)$, the following statements are equivalent.*

  (i) *$(X, \tau)$ is semi-$T_1$-space.*

 (ii) *$\{x\}$ is semi closed in $X$ for every $x \in X$.*

(iii) *$X$ is $_{\hat{\Omega}}T_{\delta}$.*

**Proof:** By [12], (i)$\Leftrightarrow$ (ii) holds and by Theorem 3.6, (iii) $\Leftrightarrow$ (ii) holds.     $\square$

**Theorem 3.8.** *Every $_{\hat{\Omega}}T_{\delta}$-space is semi-$T_{\frac{1}{2}}$-space.*

**Proof:** Since every semi-$T_1$-space is semi-$T_{\frac{1}{2}}$ and by Theorem 3.6, every $_{\hat{\Omega}}T_{\delta}$-space is semi-$T_{\frac{1}{2}}$-space.     $\square$

**Remark 3.9.** *Converse is not true in general as seen from the following example.*

**Example 3.10.** $X = \{a, b, c\}, \sigma = \{\emptyset, \{a\}, X\}$. *is semi-$T_{\frac{1}{2}}$ but not $_{\hat{\Omega}}T_{\delta}$-space.*

**Theorem 3.11.** *A topological space $(X, \tau)$ is weakly Hausdorff if and only if $(X, \tau)$ is $_{\hat{\Omega}}T_{\delta}$-space and every singleton set is $\hat{\Omega}$-closed in $(X, \tau)$.*

**Proof: Necessity-** By [6] Lemma 4.17, every $\{x\}$ is $\delta$-closed and hence semi closed in $X$. By Theorem 3.7, $X$ is $_{\hat{\Omega}}T_{\delta}$-space. By [8] Theorem 3.2, $\{x\}$ is $\hat{\Omega}$-closed in $(X, \tau)$.
**Sufficiency-** Let $x \in X$. By hypothesis, $\{x\}$ is $\delta$-closed in $X$ and again by [6] Lemma 4.17, $(X, \tau)$ is weakly Hausdorff.     $\square$

**Definition 3.12.** *A space $(X, \tau)$ is called $_{\omega}T_{\hat{\Omega}}$-space if every $\omega$-closed set in $X$ is $\hat{\Omega}$-closed in $X$.*

**Example 3.13.** *Let $X = \{a, b, c, d\}, \tau = \{\emptyset, \{a\}, \{bcd\}, X\}$. Then $(X, \tau)$ is $_\omega T_{\hat{\Omega}}$-space.*

Let us characterize $_\omega T_{\hat{\Omega}}$-space via closed sets.

**Theorem 3.14.** *A space $(X, \tau)$ is $_\omega T_{\hat{\Omega}}$-space if and only if every closed set is $\hat{\Omega}$-closed in $(X, \tau)$.*

**Proof: Necessity-** Suppose that $A$ is any closed set in $(X, \tau)$. Then by [16], $A$ is $\omega$-closed set in $X$. By hypothesis, $A$ is $\hat{\Omega}$-closed set in $X$.
**Sufficiency-** Suppose that $A$ is $\omega$-closed set in $X$. Then by [16], $cl(A) \subseteq sker(A)$. Then $sker(cl(A)) \subseteq sker(sker(A)) = sker(A)$. Since $cl(A)$ is closed set in $X$, by hypothesis, $cl(A)$ is $\hat{\Omega}$-closed set in $X$. By [8] Theorem 4.10, $\delta cl(cl(A)) \subseteq sker(cl(A)) \subseteq sker(A)$. But $\delta cl(A) \subseteq \delta cl(cl(A))$. Therefore, $\delta cl(A) \subseteq sker(A)$. Again by [8] Theorem 4.10, $A$ is $\hat{\Omega}$-closed in $(X, \tau)$. $\square$

**Remark 3.15.** *From the above discussion, a space $(X, \tau)$ is $_\omega T_{\hat{\Omega}}$-space if and only if every open set is $\hat{\Omega}$-open in $(X, \tau)$.*

Let us characterize semi-$T_{\frac{1}{2}}$-space via $\hat{\Omega}$-closed sets.

**Theorem 3.16.** *Every $\hat{\Omega}$-closed set in topological space $(X, \tau)$ is closed in $X$ if and only if every singleton set is either open or semi closed in $(X, \tau)$.*

**Proof: Necessity-** Assume that every $\hat{\Omega}$-closed set in a topological space $(X, \tau)$ is closed set in $X$ and suppose that there exists $x \in X$ such that $\{x\}$ is not semi closed subset of $X$. Then $\{x\}^c$ is not semi open subset of $X$. Here, $X$ is the only semi open set containing $\{x\}^c$ and $\delta cl(\{x\}^c) \subseteq X$. Therefore, $\{x\}^c$ is $\hat{\Omega}$-closed subset of $X$ and by hypothesis, $\{x\}^c$ is closed in $X$. Thus, $\{x\}$ is open in $X$.
**Sufficiency-** Suppose that $A$ is $\hat{\Omega}$-closed set in $X$ and there exists $x \in cl(A) \setminus (A)$.
Case(i)-$\{x\}$ is open in $X$. Then $A \cap \{x\} \neq \emptyset$, a contradiction to $x \in cl(A) \setminus (A)$.
Case(ii)-$\{x\}$ is semi closed in $X$. Since $cl(A) \setminus (A) \subseteq \delta cl(A) \setminus (A)$, $\{x\}$ is a semi closed subset of $\delta cl(A) \setminus (A)$, a contradiction to Theorem 4.4 in [8]. In both cases, we arrive at a contradiction. Therefore, $cl(A) = A$ and hence $A$ is closed in $X$. $\square$

**Theorem 3.17.** *In a topological space $(X, \tau)$, the following statements are equivalent.*

(i) *$(X, \tau)$ is semi-$T_{\frac{1}{2}}$-space.*

(ii) *$\{x\}$ is either open or semi closed in $X$ for every $x \in X$.*

(iii) *Every $\hat{\Omega}$-closed set in $(X, \tau)$ is closed set in $(X, \tau)$.*

(iv) *Every $\omega$-closed set is closed in $X$.*

**Proof:** $(i) \Rightarrow (ii)$ By [16] Theorem 4.8, it follows.

$(ii) \Rightarrow (iii)$ By Theorem 3.16, it follows.

$(iii) \Rightarrow (iv)$ Suppose that $F$ is $\omega$-closed set in $X$. By hypothesis and by Theorem 3.16, every $\{x\}$ is either open or semi closed in $X$. By [17] Theorem 5.01, $F$ is closed in $X$.

$(iv) \Rightarrow (i)$ By [17] Theorem 5.01, it holds. $\qquad\qquad\qquad\qquad$ □

**Remark 3.18.** *From the above discussion, a space $(X,\tau)$ is semi-$T_{\frac{1}{2}}$ if and only if every $\hat{\Omega}$-open set is open in $(X,\tau)$.*

## 4. $\hat{\Omega}$-continuity

**Definition 4.1.** *A function $f\colon (X,\tau) \to (Y,\sigma)$ is called $\hat{\Omega}$-continuous if the inverse image of every open set in $(Y,\sigma)$ is $\hat{\Omega}$-open in $(X,\tau)$.*

**Example 4.2.** $X = Y = \{a,b,c\}, \tau = \{\emptyset, \{a,b\}, X\}, \sigma = \{\emptyset, \{a\}, \{a,b\}, X\}$. *If $f\colon (X,\tau) \to (Y,\sigma)$ is defined by $f(a) = a, f(b) = b, f(c) = b$. Then, $f$ is $\hat{\Omega}$-continuous.*

**Theorem 4.3.** *Let $f\colon (X,\tau) \to (Y,\sigma)$ be any function. Then, the following statements hold.*

(i) *Every super continuous function is $\hat{\Omega}$-continuous function.*

(ii) *Every $\hat{\Omega}$-continuous function is $\delta g$-continuous function.*

(iii) *Every $\hat{\Omega}$-continuous function is $\omega$-continuous function.*

**Proof:** It follows from the Definition 4.1 and by [8] Theorems 3.2 and 3.5. $\qquad$ □
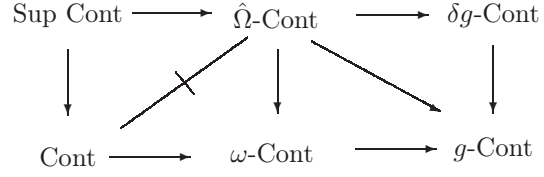
**Remark 4.4.** *The converse is not always true from the following example.*

**Example 4.5.** *Example 4.2 shows that $f$ is $\hat{\Omega}$-continuous but not super continuous. $X = Y = \{a,b,c,d\}, \tau = \{\emptyset, \{a\}, \{a,b\}, \{a,b,c\}, X\}, \sigma = \{\emptyset, \{b,c\}, \{a,b,c\}, \{b,c,d\}, Y\}$.*

*If $f\colon (X,\tau) \to (Y,\sigma)$ is defined by $f(a) = b, f(b) = c, f(c) = f(d) = a$. Then $f$ is $g, \delta g$ and $\omega$-continuous but not $\hat{\Omega}$-continuous.*

**Remark 4.6.** *The notion of $\hat{\Omega}$-continuous and continuous are independent from the examples 4.2 and 4.5.*

**Remark 4.7.** *The pictorial representation of the above discussions and the existing results are given below. The reversible implication is never possible.*

$$\text{Sup Cont} \longrightarrow \hat{\Omega}\text{-Cont} \longrightarrow \delta g\text{-Cont}$$

$$\text{Cont} \longrightarrow \omega\text{-Cont} \longrightarrow g\text{-Cont}$$

The following Theorem gives the conditions under which reversible implication of Theorem 4.3 holds.

**Theorem 4.8.** *In a topological space $(X, \tau)$, the following holds.*

(i) *If $(X, \tau)$ is $_{\hat{\Omega}}T_\delta$-space, then every $\hat{\Omega}$-continuous function is super continuous function.*

(ii) *If $(X, \tau)$ is $T_{\frac{3}{4}}$-space, then every $\delta g$-continuous function is $\hat{\Omega}$-continuous function.*

(iii) *If $(X, \tau)$ is $_{\omega}T_{\hat{\Omega}}$-space, then every $\omega$-continuous function is $\hat{\Omega}$-continuous function .*

**Proof:**

(i) Since in a space $_{\hat{\Omega}}T_\delta$ every $\hat{\Omega}$-open set is $\delta$-open, it follows.

(ii) In a space $T_{\frac{3}{4}}$, every $\delta g$-open set is $\delta$-open. Therefore, by [8] Theorem 3.2, it follows.

(iii) In a space $_{\omega}T_{\hat{\Omega}}$, every $\omega$-open set is $\hat{\Omega}$-open. Therefore, it follows.

$\square$

**Theorem 4.9.** *If a space $(X, \tau)$ is $_{\omega}T_{\hat{\Omega}}$-space, then every continuous function is $\hat{\Omega}$-continuous function.*

**Proof:** It follows from the Definition 4.1 and Theorem 3.14. $\square$

**Theorem 4.10.** *If a space $(X, \tau)$ is semi-$T_{\frac{1}{2}}$-space, then every $\hat{\Omega}$-continuous function is continuous function.*

**Proof:** It follows from the Definition 4.1 and Theorem 3.17. $\square$

Let us decompose perfectly continuous functions via $\hat{\Omega}$-continuous mappings as follows.

**Theorem 4.11.** *If $f\colon (X,\tau) \to (Y,\sigma)$ is a function, then the followings are equivalent.*

(i) *$f$ is perfectly continuous.*

(ii) *$f$ is completely continuous and contra continuous.*

(iii) *$f$ is super continuous and contra continuous.*

(iv) *$f$ is $\hat{\Omega}$-continuous and contra continuous.*

(v) *$f$ is $\omega$-continuous and contra continuous.*

(vi) *$f$ is $g\alpha$-continuous and contra continuous.*

(vii) *$f$ is pre-continuous and contra continuous.*

**Proof:**

(i) Since a clopen set is regular closed and open, it holds.

(ii) Since a regular closed set is $\delta$-closed, it holds.

(iii) Since a $\delta$-closed set is $\hat{\Omega}$-closed, it holds.

(iv) Since a $\hat{\Omega}$-closed set is $\omega$-closed, it holds.

(v) Since a $\omega$-closed set is $g\alpha$-closed, it holds.

(vi) Since a $g\alpha$-closed set is pre-closed, it holds.

(vii) If $A$ is pre-closed and open set in $(X,\tau)$, then $cl(int(A)) \subseteq A$, and $A = intA$. Thus $cl(A) \subseteq A$ and hence $A$ is closed. Thus $A$ is clopen in $X$. Therefore, every pre-continuous and contra continuous function is perfectly continuous.

$\square$

**Remark 4.12.** *The composition of $\hat{\Omega}$-continuous functions is not always $\hat{\Omega}$-continuous function as seen from the following example.*

**Example 4.13.** *Let $X = Y = Z = \{a,b,c\}, \tau = \{\emptyset, \{a\}, \{a,b\}, X\}, \sigma = \{\emptyset, \{a\}, \{b,c\}, Y\}, \eta = \{\emptyset, \{a\}, \{a,b\}, \{a,c\}, Z\}$. If $f\colon (X,\tau) \to (Y,\sigma)$ is defined by $f(a) = b, f(b) = b, f(c) = c$. Then $f$ is $\hat{\Omega}$-continuous function. Define $g\colon (Y,\sigma) \to (Z,\eta)$ as $g(a) = b, g(b) = c, g(c) = a$. Then $g$ is $\hat{\Omega}$-continuous function. But $g \circ f\colon (X,\tau) \to (Z,\eta)$, defined by $(g \circ f)(x) = g(f(x))$ for all $x \in X$ is not a $\hat{\Omega}$-continuous function.*

**Theorem 4.14.** *The following statements are true.*

**(i)** *If $f\colon (X,\tau) \to (Y,\sigma)$ is $\hat{\Omega}$-continuous and $g\colon (Y,\sigma) \to (Z,\eta)$ is continuous, then*
*$g \circ f\colon (X,\tau) \to (Z,\eta)$ is $\hat{\Omega}$-continuous.*

**(ii)** *If $f \colon (X, \tau) \to (Y, \sigma)$ and $g \colon (Y, \sigma) \to (Z, \eta)$ are continuous, then $g \circ f \colon (X, \tau) \to (Z, \eta)$ is $\hat{\Omega}$-continuous provided $(X, \tau)$ is $_\omega T_{\hat{\Omega}}$-space.*

**(iii)** *If $f \colon (X, \tau) \to (Y, \sigma)$ and $g \colon (Y, \sigma) \to (Z, \eta)$ are $\hat{\Omega}$-continuous, then $g \circ f \colon (X, \tau) \to (Z, \eta)$ is $\hat{\Omega}$-continuous whenever $(Y, \sigma)$ is semi-$T_{\frac{1}{2}}$-space.*

**(iv)** *If $f \colon (X, \tau) \to (Y, \sigma)$ is continuous and $g \colon (Y, \sigma) \to (Z, \eta)$ is $\hat{\Omega}$-continuous, then*
*$g \circ f \colon (X, \tau) \to (Z, \eta)$ is $\hat{\Omega}$-continuous provided $(X, \tau)$ is $_\omega T_{\hat{\Omega}}$-space and $(Y, \sigma)$ is semi-$T_{\frac{1}{2}}$-space.*

**(v)** *If $f \colon (X, \tau) \to (Y, \sigma)$ is continuous and $g \colon (Y, \sigma) \to (Z, \eta)$ is $\hat{\Omega}$-continuous then,*

*$g \circ f \colon (X, \tau) \to (Z, \eta)$ is continuous whenever $(Y, \sigma)$ is semi-$T_{\frac{1}{2}}$-space*

**(vi)** *If $f \colon (X, \tau) \to (Y, \sigma)$ is $\hat{\Omega}$-continuous and $g \colon (Y, \sigma) \to (Z, \eta)$ is continuous, then*
*$g \circ f \colon (X, \tau) \to (Z, \eta)$ is continuous whenever $(X, \tau)$ is semi-$T_{\frac{1}{2}}$-space.*

**Proof:** They follow straight forward from their definitions. □

**Theorem 4.15.** *$f \colon (X, \tau) \to (Y, \sigma)$ is a surjective function such that $f(U)$ is $\hat{\Omega}$-open set in $Y$ for any $\hat{\Omega}$-open set in $U$ in $X$ and $g \colon (Y, \sigma) \to (Z, \eta)$ is any function. If $g \circ f \colon (X, \tau) \to (Z, \eta)$ is $\hat{\Omega}$-continuous, then $g$ is $\hat{\Omega}$-continuous function.*

**Proof:** Suppose that $y \in Y$ and $W$ is any open set in $Z$ containing $g(y)$. Since $f$ is surjective, there exists $x \in X$ such that $f(x) = y$. Then $(g \circ f)(x) = g(f(x)) = g(y) \in W$. Since $g \circ f$ is $\hat{\Omega}$-continuous, there exists an $\hat{\Omega}$-open set $U$ in $X$ containing $x$ such that $(g \circ f)(U) \subseteq W$. If we take $V = f(U)$, then by our assumption, $V$ is a $\hat{\Omega}$-open set in $Y$ containing $f(x)$ and $(g \circ f)(U) = g(V) \subseteq W$. Thus, $g$ is $\hat{\Omega}$-continuous function. □

Let us characterize $\hat{\Omega}$-continuous function.

**Theorem 4.16.** *If $f \colon (X, \tau) \to (Y, \sigma)$ is any function, then the following statements are equivalent.*

(i) *$f$ is $\hat{\Omega}$-continuous function.*

(ii) *For each $x \in X$ and each $V \in O(Y, f(x))$, there exists $U \in \hat{\Omega}O(X, x)$ such that $f(U) \subseteq V$.*

(iii) *The inverse image of every closed set in $(Y, \sigma)$ is $\hat{\Omega}$-closed set in $(X, \tau)$.*

(iv) *$f(\hat{\Omega}cl(A)) \subseteq cl(f(A))$ for each subset $A$ of $(X, \tau)$.*

(v) *$\hat{\Omega}cl(f^{-1}(B)) \subseteq f^{-1}(cl(B))$ for each subset $B$ of $(Y, \sigma)$.*

**Proof:** $(i) \Rightarrow (ii)$ Suppose that $x \in X$ and $V$ is any open set in $(Y, \sigma)$ containing $f(x)$. Then $x \in f^{-1}(V)$. Define $U = f^{-1}(V)$. By hypothesis, $U$ is $\hat{\Omega}$-open in $(X, \tau)$ containing $x$. Also $f(U) \subseteq V$.

$(ii) \Rightarrow (i)$ Suppose that $V$ is any open set in $(Y, \sigma)$ and $x \in f^{-1}(V)$ be arbitrary. By hypothesis, there exists $U_x \in \hat{\Omega}O(X, x)$ such that $f(U_x) \subseteq V$. Then $x \in U_x \subseteq f^{-1}(V)$ implies that $f^{-1}(V) = \bigcup\{U_x : x \in f^{-1}(V)\}$. By [8] Theorem 4.16, $f^{-1}(V)$ is $\hat{\Omega}$-open in $(X, \tau)$.

$(i) \Rightarrow (iii)$ Let $F$ be any closed set in $(Y, \sigma)$. Then $Y \setminus F$ is open in $(Y, \sigma)$. By hypothesis, $f^{-1}(Y \setminus F) = X \setminus f^{-1}(F)$ is $\hat{\Omega}$-open in $(X, \tau)$. Therefore, $f^{-1}(F)$ is $\hat{\Omega}$-closed in $(X, \tau)$.

$(iii) \Rightarrow (i)$ Let $V$ be any open set in $(Y, \sigma)$. Then $Y \setminus V$ is closed in $(Y, \sigma)$. By hypothesis, $f^{-1}(Y \setminus V) = X \setminus f^{-1}(V)$ is $\hat{\Omega}$-closed in $(X, \tau)$. Therefore, $f^{-1}(V)$ is $\hat{\Omega}$-open in $(X, \tau)$.

$(iii) \Rightarrow (iv)$ Let $A$ be any subset of $X$. Then $A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(cl(f(A)))$. By hypothesis, $f^{-1}(cl(f(A)))$ is a $\hat{\Omega}$-closed in $(X, \tau)$ containing $A$. By the definition of $\hat{\Omega}$-closure, $\hat{\Omega}cl(A) \subseteq f^{-1}(cl(f(A)))$. Therefore, $f(\hat{\Omega}cl(A)) \subseteq cl(f(A))$.

$(iv) \Rightarrow (iii)$ Suppose that $F$ is any closed set in $(Y, \sigma)$. By hypothesis, $f(\hat{\Omega}cl(f^{-1}(F))) \subseteq cl(f(f^{-1}(F))) \subseteq cl(F) \subseteq F$. Then $\hat{\Omega}cl(f^{-1}(F)) \subseteq f^{-1}(F)$. Thus $f^{-1}(F)$ is $\hat{\Omega}$-closed in $(X, \tau)$.

$(iv) \Rightarrow (v)$ Let $B$ be any subset of $(Y, \sigma)$. By hypothesis, $f(\hat{\Omega}cl(f^{-1}(B))) \subseteq cl(f(f^{-1}(B))) \subseteq cl(B)$. Then $\hat{\Omega}cl(f^{-1}(B)) \subseteq f^{-1}(cl(B))$.

$(v) \Rightarrow (iv)$ Let $A$ be any subset of $(X, \tau)$. If we define $B = f(A)$, then $A \subseteq f^{-1}(f(A)) = f^{-1}(B)$. Then $\hat{\Omega}cl(A) \subseteq \hat{\Omega}cl(f^{-1}(B))$. By hypothesis, $\hat{\Omega}cl(f^{-1}(B)) \subseteq f^{-1}(cl(B))$. Then $\hat{\Omega}cl(A) \subseteq f^{-1}(cl(B)) = f^{-1}(cl(f(A)))$. Therefore, $f(\hat{\Omega}cl(A)) \subseteq cl(f(A))$. □

**Proposition 4.17.** *If $f : (X, \tau) \to (Y, \sigma)$ is a $\hat{\Omega}$-continuous function and $A$ is both open and pre-closed subset of $(X, \tau)$, then the restriction $f|A$ is $\hat{\Omega}$-continuous function.*

**Proof:** Let $F$ be any open subset of $(Y, \sigma)$. By hypothesis, $f^{-1}(F)$ is $\hat{\Omega}$-open subset of $(X, \tau)$. By [8] Theorem 6.10, $f^{-1}(F) \cap A$ is $\hat{\Omega}$-open in $(A, \tau|A)$. Therefore, $(f|A)^{-1}(F)$ is $\hat{\Omega}$-open in $(A, \tau|A)$. Thus, $f|A$ is $\hat{\Omega}$-continuous function. □

**Theorem 4.18.** *Let $\{A_\alpha, \alpha \in \Lambda\}$ be a cover of $X$ by open and preclosed subsets of $(X, \tau)$. If $f|A_\alpha : (A_\alpha, \tau|A_\alpha) \to (Y, \sigma)$ is $\hat{\Omega}$-continuous function for each $\alpha \in \Lambda$, then $f : (X, \tau) \to (Y, \sigma)$ is $\hat{\Omega}$-continuous function.*

**Proof:** Let $V$ be any open subset of $(Y, \sigma)$. Since $f|A_\alpha$ is $\hat{\Omega}$-continuous function, $(f|A_\alpha)^{-1}(V)$ is $\hat{\Omega}$-open subset of $(A_\alpha, \tau|A_\alpha)$. By [8] Theorem 6.9, for each $\alpha \in \Lambda$, $(f|A_\alpha)^{-1}(V)$ is $\hat{\Omega}$-open subset of $(X, \tau)$. Since $f^{-1}(V) = \bigcup\{(f|A_\alpha)^{-1}(V) : \alpha \in \Lambda\}$ and since $\hat{\Omega}O(X)$ is closed under arbitrary union, $f^{-1}(V)$ is $\hat{\Omega}$-open subset of $(X, \tau)$. Thus, $f$ is $\hat{\Omega}$-continuous function. □

Let us prove a significant theorem as follows.

**Theorem 4.19. "The Pasting lemma"** *for $\hat{\Omega}$-continuous function.*
*Let $A$ and $B$ be any two both open and pre-closed subsets of $(X, \tau)$ such that $X = A \cup B$. Let $f : (A, \tau|A) \to (Y, \sigma)$ and $g : (B, \tau|B) \to (Y, \sigma)$ be $\hat{\Omega}$-continuous functions such that $f(x) = g(x)$ for every $x \in A \cap B$. Then the combination $(f \bigtriangledown g)(x) = f(x)$ for every $x \in A$ and $(f \bigtriangledown g)(y) = g(y)$ for every $y \in B$ is $\hat{\Omega}$-continuous function.*

**Proof:** Let $F$ be any closed set in $(Y, \sigma)$. Then $(f \bigtriangledown g)^{-1}(F) = ((f \bigtriangledown g)^{-1}(F) \cap A) \cup ((f \bigtriangledown g)^{-1}(F) \cap B) = f^{-1}(F) \cup g^{-1}(F) = C \cup D$, where $C = f^{-1}(F)$ and $D = g^{-1}(F)$. Since $f$ and $g$ are $\hat{\Omega}$-continuous, $C$ and $D$ are $\hat{\Omega}$-closed sets in $(A, \tau|_A)$ and $(B, \tau|B)$ respectively. By [8], Theorem 6.6, $C$ and $D$ are $\hat{\Omega}$-closed sets in $(X, \tau)$. By [8], Theorem 4.12, $(f \bigtriangledown g)^{-1}(F)$ is $\hat{\Omega}$-closed sets in $(X, \tau)$. Thus, the combination $(f \bigtriangledown g)$ is $\hat{\Omega}$-continuous function. $\qquad\square$

**Theorem 4.20.** *If $f : (X, \tau) \to (Y, \sigma)$ is semi-continuous and $\delta$-closed mapping and if $A$ is $\hat{\Omega}$-closed set in $(X, \tau)$, then $f(A)$ is $\hat{\Omega}$-closed in $(Y, \sigma)$.*

**Proof:** Suppose that $f(A) \subseteq U$, where $U$ is any semi-open subset of $Y$. Then $A \subseteq f^{-1}(U)$. Since $f$ is semi-continuous, $f^{-1}(U)$ is semi open set in $X$. Since $A$ is $\hat{\Omega}$-closed set in $X$, $\delta cl(A) \subseteq f^{-1}(U)$ and hence $f(\delta cl(A)) \subseteq U$. By [7] Corollary 4, $\delta cl(A)$ is $\delta$-closed set and since $f$ is a $\delta$-closed mapping, $\delta cl(f(A)) \subseteq \delta cl(f(\delta cl(A))) \subseteq f(\delta cl(A)) \subseteq U$. Thus, $f(A)$ is $\hat{\Omega}$-closed in $(Y, \sigma)$. $\qquad\square$

## 5. $\hat{\Omega}$-irresolute mappings

**Definition 5.1.** *A function $f : (X, \tau) \to (Y, \sigma)$ is said to be $\hat{\Omega}$-irresolute if $f^{-1}(V)$ is $\hat{\Omega}$-open set in $(X, \tau)$ for every $\hat{\Omega}$-open set $V$ in $(Y, \sigma)$.*

**Example 5.2.** *Let $X = Y = \{a, b, c, d\}, \tau = \{\emptyset, \{b, c\}, \{a, b, c\}, \{b, c, d\}, X\}$ and $\sigma = \{\emptyset, \{a\}, \{a, b\}, Y\}$. Define $f : (X, \tau) \to (Y, \sigma)$ as $f(a) = b, f(b) = a, f(c) = c, f(d) = d$. Here $f$ is $\hat{\Omega}$-irresolute function.*

Let us characterizes $\hat{\Omega}$-irresolute function in the following theorems

**Theorem 5.3.** *A function $f : (X, \tau) \to (Y, \sigma)$ is $\hat{\Omega}$-irresolute if and only $f^{-1}(F)$ is $\hat{\Omega}$-closed set in $(X, \tau)$ for every $\hat{\Omega}$-closed set $F$ in $(Y, \sigma)$.*

**Proof: Necessity-** Suppose that $f$ is $\hat{\Omega}$-irresolute and $F$ is any $\hat{\Omega}$-closed set in $(Y, \sigma)$. Then $(Y \setminus F)$ is $\hat{\Omega}$-open set in $(Y, \sigma)$ and by hypothesis, $f^{-1}(Y \setminus F)$ is $\hat{\Omega}$-open set in $(X, \tau)$. Then $(X \setminus f^{-1}(F))$ is $\hat{\Omega}$-open set in $(X, \tau)$. Thus, $f^{-1}(F)$ is $\hat{\Omega}$-closed set in $(X, \tau)$.
**Sufficiency-** Suppose $V$ is any $\hat{\Omega}$-open set in $(Y, \sigma)$. Then $(Y \setminus V)$ is $\hat{\Omega}$-closed set in $(Y, \sigma)$. By hypothesis, $f^{-1}(Y \setminus V)$ is $\hat{\Omega}$-closed set in $(X, \tau)$ and hence $(X \setminus f^{-1}(V))$ is $\hat{\Omega}$-closed set in $(X, \tau)$. Thus, $f^{-1}(V)$ is $\hat{\Omega}$-open set in $(X, \tau)$. $\qquad\square$

**Theorem 5.4.** *A function $f : (X, \tau) \to (Y, \sigma)$ is $\hat{\Omega}$-irresolute if and only if $f(\hat{\Omega}cl(A)) \subseteq \hat{\Omega}cl(f(A))$ for every subset $A$ of $X$.*

**Proof: Necessity-** Suppose that $f$ is $\hat{\Omega}$-irresolute and $A$ is any subset of $X$. Let $x \in \hat{\Omega}cl(A)$ and $f(x) \notin \hat{\Omega}cl(f(A))$. By [8] Theorem 5.11, there exists a $\hat{\Omega}$-open set $V$ in $Y$ containing $f(x)$ such that $V \cap f(A) = \emptyset$. Then $f^{-1}(V) \cap A = \emptyset$. Since $f$ is $\hat{\Omega}$-continuous, $f^{-1}(V)$ is $\hat{\Omega}$-open set in $X$ containing $x$. Since $x \in \hat{\Omega}cl(A)$ and by [8] Theorem 5.11, $f^{-1}(V) \cap A \neq \emptyset$, a contradiction. Therefore, $f(\hat{\Omega}cl(A)) \subseteq \hat{\Omega}cl(f(A))$.
**Sufficiency-** Let $F$ be any $\hat{\Omega}$-closed set in $Y$. If $x \in \hat{\Omega}cl(f^{-1}(F))$, then $f(x) \in f(\hat{\Omega}cl(f^{-1}(F)))$. By hypothesis, $f(x) \in \hat{\Omega}cl(f(f^{-1}(F))) \subseteq \hat{\Omega}cl(F) = F$. Then $x \in f^{-1}(F)$. By [8] Theorem 5.3 (v), $f^{-1}(F)$ is $\hat{\Omega}$-closed set in $X$ and hence $\hat{\Omega}$-continuous. $\qquad\square$

**Theorem 5.5.** *A function $f\colon (X,\tau) \to (Y,\sigma)$ is $\hat{\Omega}$-irresolute iff $\hat{\Omega}cl(f^{-1}(B)) \subseteq f^{-1}(\hat{\Omega}cl(B))$ for each subset $B$ of $(Y,\sigma)$.*

**Proof: Necessity-** For any set $B$ in $Y$ by Theorem 5.4, $f(\hat{\Omega}cl(f^{-1}(B))) \subseteq \hat{\Omega}cl(f(f^{-1}(B))) \subseteq \hat{\Omega}cl(B)$. Therefore, $\hat{\Omega}cl(f^{-1}(B)) \subseteq f^{-1}(\hat{\Omega}cl(B))$.
**Sufficiency-** It suffices to show that inverse image of a closed subset of $Y$ is $\hat{\Omega}$-closed subset of $X$. Suppose that $F$ is any closed set in $X$. Then by [8] Theorem 5.3, $F = \hat{\Omega}cl(F)$. By hypothesis, $\hat{\Omega}cl(f^{-1}(F)) \subseteq f^{-1}(\hat{\Omega}cl(F)) = f^{-1}(F)$. Again by [8] Theorem 5.3, $f^{-1}(F)$ is $\hat{\Omega}$-closed in $X$. $\qquad\square$

**Theorem 5.6.** *$f\colon (X,\tau) \to (Y,\sigma)$ is $\hat{\Omega}$-continuous function if and only if for every $x \in X$ and for every $V \in \hat{\Omega}O(Y,f(x))$, there exists $U \in \hat{\Omega}O(X,x)$ such that $f(U) \subseteq V$.*

**Proof: Necessity-** Suppose that $x \in X$ and $V$ is any $\hat{\Omega}$-open set in $(Y,\sigma)$ containing $f(x)$. Then $x \in f^{-1}(V)$. Define $U = f^{-1}(V)$. By hypothesis, $U$ is $\hat{\Omega}$-open in $(X,\tau)$ containing $x$. Also $f(U) \subseteq V$.
**Sufficiency-** Suppose that $V$ is any $\hat{\Omega}$-open set in $(Y,\sigma)$ and $x \in f^{-1}(V)$. By hypothesis, there exists $U_x \in \hat{\Omega}O(X,x)$ such that $f(U_x) \subseteq V$. Then $x \in U_x \subseteq f^{-1}(V)$ implies that $f^{-1}(V) = \bigcup_{x \in f^{-1}(V)} U_x$. By [8] Theorem 4.16, $f^{-1}(V)$ is $\hat{\Omega}$-open in $(X,\tau)$.

$\qquad\square$

**Remark 5.7.** *From the following examples, it is shown that the notion of $\hat{\Omega}$-irresolute and $\hat{\Omega}$-continuous functions are independent. Moreover, the concept of $\hat{\Omega}$-irresolute and continuous functions are independent.*

**Example 5.8.** *$X = Y = \{a,b,c,d\}, \tau = \{\emptyset, \{b,c\}, \{a,b,c\}, \{b,c,d\}, X\}, \sigma = \{\emptyset, \{a\}, \{a,b\}.Y\}$. If $f\colon (X,\tau) \to (Y,\sigma)$ is defined by $f(a) = b, f(b) = a, f(c) = c, f(d) = d$. Then $f$ is $\hat{\Omega}$-irresolute but not continuous.*

**Example 5.9.** *$X = Y = \{a,b,c\}, \tau = \{\emptyset, \{a\}, X\}, \sigma = \{\emptyset, \{a\}, \{b,c\}, Y\}$. If $f\colon (X,\tau) \to (Y,\sigma)$ is defined by $f(a) = b, f(b) = c, f(c) = b$. Then $f$ is $\hat{\Omega}$-continuous and continuous but not $\hat{\Omega}$-irresolute.*

**Example 5.10.** $X = Y = \{a, b, c, d\}, \tau = \{\emptyset, \{c\}, \{c, d\}, X\}, \sigma = \{\emptyset, \{a\}, \{a, b\}.Y\}$. If $f: (X, \tau) \to (Y, \sigma)$ is defined by $f(a) = c, f(b) = d, f(c) = a, f(d) = b$. Then $f$ is $\hat{\Omega}$-irresolute but not $\hat{\Omega}$-continuous.

**Theorem 5.11.** Let $(Y, \sigma)$ be a $_\omega T_{\hat{\Omega}}$-space. If $f: (X, \tau) \to (Y, \sigma)$ is $\hat{\Omega}$-irresolute then it is $\hat{\Omega}$-continuous. Moreover, if $(X, \tau)$ is semi-$T_{\frac{1}{2}}$-space, then $f$ is continuous.

**Proof:** Suppose that $V$ is open in $Y$. Since $Y$ is $_\omega T_{\hat{\Omega}}$, $V$ is $\hat{\Omega}$-open in $Y$. Since $f$ is $\hat{\Omega}$-irresolute, $f^{-1}(V)$ is $\hat{\Omega}$-open in $X$. Therefore, $f$ is $\hat{\Omega}$-continuous. If $X$ is $semiT_{\frac{1}{2}}$, then $f^{-1}(V)$ is open in $X$ and hence $f$ is continuous. $\square$

**Theorem 5.12.** Let $(Y, \sigma)$ be a semi-$T_{\frac{1}{2}}$-space. If $f: (X, \tau) \to (Y, \sigma)$ is $\hat{\Omega}$-continuous then it is $\hat{\Omega}$-irresolute.

**Proof:** It follows from their definitions. $\square$

**Theorem 5.13.** Suppose $(Y, \sigma)$ is a semi-$T_{\frac{1}{2}}$-space and $(X, \tau)$ is $_\omega T_{\hat{\Omega}}$-space. If $f: (X, \tau) \to (Y, \sigma)$ is continuous then it is $\hat{\Omega}$-irresolute.

**Proof:** It follows from their definitions. $\square$

Composition theorems.

**Theorem 5.14.** *The following statements are true.*

(i) *If $f: (X, \tau) \to (Y, \sigma)$ is $\hat{\Omega}$-irresolute and $g: (Y, \sigma) \to (Z, \eta)$ is $\hat{\Omega}$-irresolute, then*
    $g \circ f: (X, \tau) \to (Z, \eta)$ *is $\hat{\Omega}$-irresolute.*

(ii) *If $f: (X, \tau) \to (Y, \sigma)$ is continuous and $g: (Y, \sigma) \to (Z, \eta)$ is $\hat{\Omega}$-irresolute function, then $g \circ f: (X, \tau) \to (Z, \eta)$ is $\hat{\Omega}$-irresolute whenever $(Y, \sigma)$ is semi-$T_{\frac{1}{2}}$-space and $(X, \tau)$ is $_\omega T_{\hat{\Omega}}$-space.*

(iii) *If $f: (X, \tau) \to (Y, \sigma)$ is $\hat{\Omega}$-continuous and $g: (Y, \sigma) \to (Z, \eta)$ is $\hat{\Omega}$-irresolute, then*
    $g \circ f: (X, \tau) \to (Z, \eta)$ *is $\hat{\Omega}$-irresolute whenever $(Y, \sigma)$ is semi-$T_{\frac{1}{2}}$-space.*

(iv) *If $f: (X, \tau) \to (Y, \sigma)$ is $\hat{\Omega}$-irresolute and $g: (Y, \sigma) \to (Z, \eta)$ is continuous function, then $g \circ f: (X, \tau) \to (Z, \eta)$ is $\hat{\Omega}$-continuous whenever $(Y, \sigma)$ is $_\omega T_{\hat{\Omega}}$-space.*

(v) *If $f: (X, \tau) \to (Y, \sigma)$ is $\hat{\Omega}$-irresolute and $g: (Y, \sigma) \to (Z, \eta)$ is $\hat{\Omega}$-continuous, then*
    $g \circ f: (X, \tau) \to (Z, \eta)$ *is $\hat{\Omega}$-continuous.*

(vi) If $f\colon (X,\tau) \to (Y,\sigma)$ is $\hat{\Omega}$-irresolute and $g\colon (Y,\sigma) \to (Z,\eta)$ is $\hat{\Omega}$-continuous, then
$g \circ f\colon (X,\tau) \to (Z,\eta)$ is continuous whenever $(X,\tau)$ is semi-$T_{\frac{1}{2}}$-space..

**Proof:** They follow straight forward from their definitions. $\qquad\Box$

Let us prove an application of $\hat{\Omega}$-irresolute functions as follows.

**Theorem 5.15. "The Pasting lemma"** *for $\hat{\Omega}$-irresolute function.*
*Let $A$ and $B$ be any two both open and pre-closed sets in $(X,\tau)$ such that $X = A \cup B$. Let $f\colon (A,\tau \mid_A) \to (Y,\sigma)$ and $g\colon (B,\tau \mid_B) \to (Y,\sigma)$ be $\hat{\Omega}$-irresolute functions such that $f(x) = g(x)$ for every $x \in A \cap B$. Then the combination $(f \triangledown g)(x) = f(x)$ for every $x \in A$ and $(f \triangledown g)(y) = g(y)$ for every $y \in B$ is $\hat{\Omega}$-irresolute function.*

**Proof:** It is similar to that of 4.18. $\qquad\Box$

**Theorem 5.16.** *Let $(X,\tau)$ be a Hausdorff space and if $f\colon (X,\tau) \to (Y,\sigma)$ is $\hat{\Omega}$-continuous, then the set $A = \{x \in X : f(x) = x\}$ is $\hat{\Omega}$-closed set in $(X,\tau)$.*

**Proof:** Suppose that $x \in \hat{\Omega}cl(A) \setminus A$. Then $x \neq f(x)$. Since $X$ is Hausdorff, there exists two disjoint open sets $U$ and $V$ in $X$ such that $x \in U$ and $f(x) \in V$. Since $f$ is $\hat{\Omega}$-continuous and by [8] Theorem 4.12, $f^{-1}(V) \cap U$ is $\hat{\Omega}$-open set containing $x$. Since $x \in \hat{\Omega}cl(A)$, by [8] Theorem 5.11, $f^{-1}(V) \cap U \cap A \neq \emptyset$. Therefore, $U \cap V \neq \emptyset$ a contradiction. Thus $A$ is $\hat{\Omega}$-closed set in $(X,\tau)$. $\qquad\Box$

**Corollary 5.17.** *Let $(X,\tau)$ be a Hausdorff and $_\omega T_{\hat{\Omega}}$-space and if $f\colon (X,\tau) \to (Y,\sigma)$ is $\hat{\Omega}$-irresolute, then the set $A = \{x \in X : f(x) = x\}$ is $\hat{\Omega}$-closed set in $(X,\tau)$.*

**Theorem 5.18.** *If $f\colon (X,\tau) \to (Y,\sigma)$ and $g\colon (X,\tau) \to (Y,\sigma)$ are $\hat{\Omega}$-irresolute from $X$ into a Hausdorff space $Y$, then the set $A = \{x \in X : f(x) = g(x)\}$ is $\hat{\Omega}$-closed set in $(X,\tau)$.*

**Proof:** Let $x \in (X \setminus A)$. Then $f(x) \neq g(x)$. Since $Y$ is Hausdorff space, there exists two disjoint open sets $U$ and $V$ in $Y$ such that $f(x) \in U$ and $g(x) \in V$. Since $f$ is $\hat{\Omega}$-continuous and by [8] Theorem 4.12, $f^{-1}(U) \cap g^{-1}(V)$ is $\hat{\Omega}$-open set containing $x$. Moreover, $f^{-1}(U) \cap g^{-1}(V) \cap A = \emptyset$. Therefore, $x \in f^{-1}(U) \cap g^{-1}(V) \subseteq (X \setminus A)$. This implies that $(X \setminus A)$ is $\hat{\Omega}$-open in $(X,\tau)$ and hence $A$ is $\hat{\Omega}$-closed set in $(X,\tau)$. $\qquad\Box$

**Corollary 5.19.** *Let $(Y,\sigma)$ be a $\hat{\Omega}$-$T_2$-space and if $f\colon (X,\tau) \to (Y,\sigma)$ and $g\colon (X,\tau) \to (Y,\sigma)$ are $\hat{\Omega}$-irresolute, then the set $A = \{x \in X : f(x) = g(x)\}$ is $\hat{\Omega}$-closed set in $(X,\tau)$.*

**Theorem 5.20.** *Let $(X,\tau)$ be a semi-$T_{\frac{1}{2}}$-space. If $f\colon (X,\tau) \to (Y,\sigma)$ is $\hat{\Omega}$-continuous injective and $Y$ is Hausdorff, then $X$ is Hausdorff.*

**Proof:** Suppose that $x, y \in X$ such that $x \neq y$. Since $f$ is injective, $f(x) \neq f(y)$ and since $Y$ is Hausdorff space, there exists two disjoint open sets $U$ and $V$ in $Y$ such that $f(x) \in U$ and $f(y) \in V$. Since $f$ is $\hat{\Omega}$-continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint $\hat{\Omega}$-open sets in $X$ containing $x$ and $y$ respectively. Since $X$ is *semi-$T_{\frac{1}{2}}$-space*, $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint open sets in $X$ containing $x$ and $y$ respectively. Thus, $(X, \tau)$ is Hausdorff. $\qquad\square$

**Definition 5.21.** *A space is said to be $\hat{\Omega}$-$T_2$, if for every distinct pair of points $x, y \in X$ there exists disjoint $\hat{\Omega}$-open sets in $X$ such that $x \in U, y \in V$.*

**Corollary 5.22.** *Let $(Y, \sigma)$ be Hausdorff space. If $f: (X, \tau) \to (Y, \sigma)$ is $\hat{\Omega}$-continuous injective, then $X$ is $\hat{\Omega}$-$T_2$.*

**Corollary 5.23.** *Let $(X, \tau)$ be semi-$T_{\frac{1}{2}}$-space and $(Y, \sigma)$ be $\hat{\Omega}$-$T_2$-space. If $f: (X, \tau) \to (Y, \sigma)$ is $\hat{\Omega}$-irresolute injective, then $X$ is Hausdorff.*

**Corollary 5.24.** *If $f: (X, \tau) \to (Y, \sigma)$ is $\hat{\Omega}$-irresolute injective and $Y$ is $\hat{\Omega}$-$T_2$, then $X$ is $\hat{\Omega}$-$T_2$.*

## 6. weakly $\hat{\Omega}$-continuous

**Definition 6.1.** *A function $f: (X, \tau) \to (Y, \sigma)$ is weakly $\hat{\Omega}$-continuous at $x \in X$ if for each $G \in O(Y, f(x))$, there exists $U \in \hat{\Omega}O(X, x)$ such that $f(U) \subseteq cl(G)$. If $f$ is weakly $\hat{\Omega}$ continuous at each $x \in X$, then $f$ is weakly $\hat{\Omega}$-continuous on $X$.*

**Example 6.2.** $X = Y = \{a, b, c\}, \tau = \{\emptyset, \{a\}, \{a, b\}, X\}, \sigma = \{\emptyset, \{a\}, \{a, b\}, \{a, c\}, Y\}$. *Define $f: (X, \tau) \to (Y, \sigma)$ as an identity function. Then $f$ is weakly $\hat{\Omega}$-continuous.*

**Theorem 6.3.** *Every $\hat{\Omega}$-continuous function is weakly $\hat{\Omega}$-continuous function.*

**Proof:** Let $x \in X$ and $V \in O(Y, f(x))$. By hypothesis, their exists $U \in \hat{\Omega}O(X, x)$ such that $f(U) \subseteq V \subseteq cl(V)$. Thus, $f$ is weakly $\hat{\Omega}$-continuous function. $\qquad\square$

**Remark 6.4.** *The converse is not possible in general, as seen from the example 6.2.*

The following theorem gives the conditions under which the reversible implication of Theorem 6.3 is true.

**Theorem 6.5.** *If $(Y, \sigma)$ is regular space, then every weakly $\hat{\Omega}$-continuous function $f: (X, \tau) \to (Y, \sigma)$ is $\hat{\Omega}$-continuous function.*

**Proof:** Let $x \in X$ and $V$ be any open set in $Y$ containing $f(x)$. Since $Y$ is regular, there exists an open set $W$ in $Y$ such that $f(x) \in W \subseteq cl(W) \subseteq V$. Since $f$ is weakly $\hat{\Omega}$-continuous, there exists $\hat{\Omega}$-open set in $U$ in $X$ containing $x$ such that $f(U) \subseteq cl(W) \subseteq V$. Therefore, $f$ is $\hat{\Omega}$-continuous function. $\qquad\square$

The following theorem states the characterization of weakly $\hat{\Omega}$-continuous function.

**Theorem 6.6.** *The following statements are equivalent.*

(i) $f\colon (X,\tau) \to (Y,\sigma)$ *is weakly* $\hat{\Omega}$-*continuous function.*

(ii) $\hat{\Omega}cl(f^{-1}(int(cl(V)))) \subseteq f^{-1}(cl(V))$ *for every subset* $V$ *of* $Y$.

(iii) $\hat{\Omega}cl(f^{-1}(int(F))) \subseteq f^{-1}(F)$ *for every regular closed subset* $F$ *of* $Y$.

(iv) $\hat{\Omega}cl(f^{-1}(U) \subseteq f^{-1}(cl(U))$ *for every open subset* $U$ *of* $Y$.

(v) $f^{-1}(U) \subseteq \hat{\Omega}int(f^{-1}(cl(U)))$ *for every open subset* $U$ *of* $Y$.

**Proof:**
    $(i) \Rightarrow (ii)$ Suppose that $V$ is any subset of $Y$ and $x \notin f^{-1}(cl(V))$. Then $f(x) \notin cl(V)$ implies that there exists an open set $U$ in $Y$ containing $f(x)$ such that $U \cap V = \emptyset$. Therefore $U = int(U) \subseteq int(X \setminus V) = (X \setminus cl(V))$. Then $cl(U) \cap int(cl(V)) = \emptyset$ and hence $f^{-1}(int(cl(V))) \subseteq (X \setminus f^{-1}(cl(U)))$. Since $f$ is weakly $\hat{\Omega}$-continuous, there exists $\hat{\Omega}$-open set $W$ in $X$ containing $x$ such that $f(W) \subseteq cl(U)$ and then $W \subseteq f^{-1}(cl(U))$. Therefore, $f^{-1}(int(cl(V))) \subseteq (X \setminus W)$ where $(X \setminus W)$ is a $\hat{\Omega}$-closed set in $X$. By [8] the definition of $\hat{\Omega}$ closure, $\hat{\Omega}cl(f^{-1}(int(cl(V)))) \subseteq (X \setminus W)$. Then $W \cap \hat{\Omega}cl(f^{-1}(int(cl(V)))) = \emptyset$. Since $x \in W, x \notin \hat{\Omega}cl(f^{-1}(int(cl(V))))$.
    $(ii) \Rightarrow (iii)$ Suppose that $F$ is any regular closed set in $Y$. By hypothesis, $\hat{\Omega}cl(f^{-1}(int(cl(int(F))))) \subseteq f^{-1}(cl(int(F)))$. Then, $\hat{\Omega}cl(f^{-1}(int(F))) \subseteq f^{-1}(F)$.
    $(iii) \Rightarrow (iv)$ Suppose that $U$ is any open set in $Y$ and then $cl(U)$ is regular closed set in $Y$. By hypothesis, $\hat{\Omega}cl(f^{-1}(int(cl(U)))) \subseteq f^{-1}(cl(U))$. Since $U$ is open, $\hat{\Omega}cl(f^{-1}(U) \subseteq \hat{\Omega}cl(f^{-1}(int(cl(U))) \subseteq f^{-1}(cl(U))$. Thus, $\hat{\Omega}cl(f^{-1}(U)) \subseteq f^{-1}(cl(U))$.
    $(iv) \Rightarrow (v)$ Suppose that $U$ is open in $Y$ and then $(Y \setminus cl(U))$ is open in $Y$. By hypothesis, $\hat{\Omega}cl(f^{-1}(Y \setminus cl(U)) \subseteq f^{-1}(cl(Y \setminus cl(U)))$. That is, $\hat{\Omega}cl(X \setminus f^{-1}(cl(U))) \subseteq f^{-1}(cl(Y \setminus cl(U)))$. By [8] Theorem 5.15 (i), $(X \setminus \hat{\Omega}int(f^{-1}(cl(U))) \subseteq f^{-1}(cl(Y \setminus cl(U))) \subseteq (X \setminus f^{-1}(U))$. Therefore, $f^{-1}(U)) \subseteq \hat{\Omega}int(f^{-1}(cl(U)))$.
    $(v) \Rightarrow (i)$ Suppose that $x \in X$ and $U$ be any open set in $Y$ containing $f(x)$. By hypothesis, $f^{-1}(U) \subseteq \hat{\Omega}int(f^{-1}(cl(U)))$. By [8] Remark 5.13, $\hat{\Omega}int(f^{-1}(cl(U)))$ is a $\hat{\Omega}$ open set in $X$ such that $\hat{\Omega}int(f^{-1}(cl(U))) \subseteq f^{-1}(cl(U))$. Therefore, $f(\hat{\Omega}int(f^{-1}(cl(U)))) \subseteq cl(U)$. If we take $W = \hat{\Omega}int(f^{-1}(cl(U)))$, then $W$ is $\hat{\Omega}$-open set in $X$ containing $x$ such that $f(W) \subseteq cl(U)$.                                                  $\square$

**Theorem 6.7.** $f\colon (X,\tau) \to (Y,\sigma)$ *is weakly* $\hat{\Omega}$-*continuous function if and only if* $\hat{\Omega}cl(f^{-1}(int(F))) \subseteq f^{-1}(F)$ *for every closed subset* $F$ *of* $Y$.

**Proof: Necessity-** Suppose that $F$ is any closed set in $Y$ and then $(Y \setminus F)$ is open set in $Y$. By Theorem 6.6, (v), $f^{-1}(Y \setminus F) \subseteq \hat{\Omega}int(f^{-1}(cl(Y \setminus F)))$. Then, $(X \setminus f^{-1}(F)) \subseteq \hat{\Omega}int(f^{-1}(Y \setminus intF))) = \hat{\Omega}int[(f^{-1}(Y)) \setminus f^{-1}(intF)] = \hat{\Omega}int[(X \setminus f^{-1}(intF)] = (X \setminus \hat{\Omega}cl(f^{-1}(intF)))$. Therefore, $\hat{\Omega}cl(f^{-1}(intF)) \subseteq f^{-1}(F)$.
**Sufficiency-** Suppose that $x \in X$ and $U$ be any open set in $Y$ containing $f(x)$.

Then, $(Y \setminus U)$ is closed in $Y$ not containing $f(x)$. By hypothesis, $\hat{\Omega}cl(f^{-1}(int(Y \setminus U))) \subseteq f^{-1}(Y \setminus U)$. Since $x \notin f^{-1}(int(Y \setminus U))$, $x \notin \hat{\Omega}cl(f^{-1}(int(Y \setminus U)))$. By [8] Theorem 5.11, there exists $\hat{\Omega}$-open set $V$ in $X$ such that $V \cap f^{-1}(int(Y \setminus U)) = \emptyset$. Also $f(V) \cap int(Y \setminus U) = \emptyset$ implies that $f(V) \subset cl(U)$. Thus $f$ is weakly $\hat{\Omega}$-continuous function. $\square$

**Remark 6.8.** *If $f$ and $g$ are weakly $\hat{\Omega}$-continuous functions, then their composition is not always weakly $\hat{\Omega}$-continuous function from the following example. However, composition is possible if we consider $g$ as a continuous function.*

**Example 6.9.** $X = Y = Z = \{a, b, c, d\}$ and $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}, \sigma = \{\emptyset, \{a\}, \{bcd\}, Y\}, \eta = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, Z\}$. *Define* $f \colon (X, \tau) \to (Y, \sigma)$ *and* $g \colon (Y, \sigma) \to (Z, \eta)$ *by* $f(a) = b, f(b) = c, f(c) = d, f(d) = b, g(a) = a, g(b) = a, g(c) = d, g(d) = b$. *Then $f$ and $g$ are weakly $\hat{\Omega}$-continuous functions where as $g \circ f \colon (X, \tau) \to (Z, \eta)$, defined by $(g \circ f)(x) = g(f(x))$ for all $x \in X$ is not a weakly $\hat{\Omega}$-continuous function.*

**Theorem 6.10.** *If $f \colon (X, \tau) \to (Y, \sigma)$ is weakly $\hat{\Omega}$-continuous function and $g$ is continuous, then $g \circ f \colon (X, \tau) \to (Z, \eta)$ is weakly $\hat{\Omega}$-continuous function.*

**Proof:** Suppose that $x \in X$ and $W$ is any open set in $Z$ containing $(g \circ f)(x)$. Since $g$ is continuous, $g^{-1}(W)$ is open set in $Y$ containing $f(x)$. Since $f$ is weakly $\hat{\Omega}$-continuous, there exists $\hat{\Omega}$-open set $U$ in $X$ containing $x$ such that $f(U) \subseteq cl(g^{-1}(W))$. Since $g$ is continuous, $(g \circ f)(U) \subseteq g(f(U)) \subseteq g(cl(g^{-1}(W))) \subseteq cl(gg^{-1}(W)) \subseteq cl(W)$. Therefore, $g \circ f \colon (X, \tau) \to (Z, \eta)$ is weakly $\hat{\Omega}$-continuous function. $\square$

**Theorem 6.11.** $f \colon (X, \tau) \to (Y, \sigma)$ *is a surjective function such that $f(U)$ is $\hat{\Omega}$-open set in $Y$ for any $\hat{\Omega}$-open set in $U$ in $X$ and $g \colon (Y, \sigma) \to (Z, \eta)$ is any function. If $g \circ f \colon (X, \tau) \to (Z, \eta)$ is weakly $\hat{\Omega}$-continuous, then $g$ is weakly $\hat{\Omega}$-continuous function.*

**Proof:** Suppose that $y \in Y$ and $W$ is any open set in $Z$ containing $g(y)$. Since $f$ is surjective, there exists $x \in X$ such that $f(x) = y$. Then $(g \circ f)(x) = g(f(x)) = g(y) \in W$. Since $g \circ f$ is weakly $\hat{\Omega}$-continuous, there exists an $\hat{\Omega}$-open set $U$ in $X$ containing $x$ such that $(g \circ f)(U) \subseteq cl(W)$. If we take $V = f(U)$, then by our assumption, $V$ is a $\hat{\Omega}$-open set in $Y$ containing $f(x)$ and $(g \circ f)(U) = g(V) \subseteq cl(W)$. Thus, $g$ is weakly $\hat{\Omega}$-continuous function. $\square$

**Theorem 6.12.** *If $f \colon (X, \tau) \to (Y, \sigma)$ is $\hat{\Omega}$-irresolute function and $g$ is weakly $\hat{\Omega}$-continuous, then $g \circ f \colon (X, \tau) \to (Z, \eta)$ is weakly $\hat{\Omega}$-continuous function.*

**Proof:** Suppose that $x \in X$ and $W$ is any open set in $Z$ containing $(g \circ f)(x)$. Since $g$ is weakly $\hat{\Omega}$-continuous, there exists $\hat{\Omega}$-open set $V$ in $Y$ containing $f(x)$ such that $g(V) \subseteq cl(W)$. Since $f$ is $\hat{\Omega}$-irresolute, there exists $\hat{\Omega}$-open set $U$ in $X$ containing $x$ such that $f(U) \subseteq V$. Then $(g \circ f)(U) \subseteq g(f(U)) \subseteq g(V) \subseteq cl(W)$. Therefore, $g \circ f \colon (X, \tau) \to (Z, \eta)$ is weakly $\hat{\Omega}$-continuous function. $\square$

**Theorem 6.13.** *If $f\colon (X,\tau) \to (Y,\sigma)$ is weakly $\hat{\Omega}$-continuous and $Y$ is Hausdorff, then $f$ has $\hat{\Omega}$-closed points inverses.*

**Proof:** Suppose that $y \in Y$ and $x \in \{x \in X : f(x) \neq y\}$. Since $f(x) \neq y$ in a Hausdorff space $Y$, there exists two disjoint open sets $U$ and $V$ in $Y$ such that $f(x) \in U$ and $y \in V$. Moreover, $cl(U) \cap V = \emptyset$ implies that $y \notin cl(U)$. Since $f$ is weakly $\hat{\Omega}$-continuous, there exists $\hat{\Omega}$-open set $W$ in $X$ containing $x$ such that $f(W) \subseteq cl(U)$. If $W \nsubseteq \{x \in X : f(x) \neq y\}$, then choose a point $z \in W$ such that $f(z) = y$. Then $y = f(z) \in f(W) \subseteq cl(U)$, a contradiction. Hence $x \in W \subseteq \{x \in X : f(x) \neq y\}$ and hence $\{x \in X : f(x) \neq y\}$ is $\hat{\Omega}$-open in $X$. Therefore, $f^{-1}(y) = \{x \in X : f(x) = y\}$ is $\hat{\Omega}$-closed in $X$. $\qquad\square$

**Definition 6.14.** *If $A \subseteq X$, weakly $\hat{\Omega}$-continuous retraction of $X$ onto $A$ is a weakly $\hat{\Omega}$-continuous function $f\colon X \to A$ such that $f(a) = a$ for each $a \in A$.*

**Theorem 6.15.** *If $A \subseteq X$, and $f\colon X \to A$ is a weakly $\hat{\Omega}$-continuous retraction of a Hausdorff space $X$ onto $A$, then $A$ is $\hat{\Omega}$-closed in $X$.*

**Proof:** Suppose that there exists $x \in \hat{\Omega}cl(A) \setminus A$. Then $x \neq f(x)$. Since $X$ is Hausdorff, there exists two disjoint open sets $U$ and $V$ in $X$ such that $x \in U$ and $f(x) \in V$. Also $U \cap cl(V) = \emptyset$. Let $W$ be an arbitrary $\hat{\Omega}$-open set in $X$ containing $x$. By [8] Theorem 4.12, $U \cap W$ is a such that $\hat{\Omega}$-open set in $X$ containing $x$. Since $x \in \hat{\Omega}cl(A)$, by [8] Theorem 5.11, $U \cap W \cap A \neq \emptyset$. If we choose $y \in U \cap W \cap A$, then $y \in A$ implies that $y = f(y) \in U$ and hence $f(y) \notin cl(V)$. That is, there exists $y \in W$ such that $f(y) \notin cl(V)$. Therefore, $f(W) \nsubseteq cl(V)$ for any $\hat{\Omega}$-open set in $X$ containing $x$, a contradiction. Thus, $A$ is $\hat{\Omega}$-closed in $X$. $\qquad\square$

**Definition 6.16.** *A space $(X,\tau)$ is called Urysohn space if for every pair of distinct points $x$ and $y$ in $X$, there exists open sets $U$ and $V$ in $X$ such that $x \in U$, $y \in V$ and $cl(U) \cap cl(V) = \emptyset$.*

**Theorem 6.17.** *If $f\colon (X,\tau) \to (Y,\sigma)$ is injective weakly $\hat{\Omega}$-continuous from $X$ into a Urysohn space $Y$, then $X$ is $\hat{\Omega}$-$T_2$-space.*

**Proof:** Suppose that $x, y \in X$ such that $x \neq y$. Since $f$ is injective, $f(x) \neq f(y)$ and since $Y$ is Urysohn space, there exists two open sets $U$ and $V$ in $Y$ such that $f(x) \in U, f(y) \in V$ and $cl(U) \cap cl(V) = \emptyset$. Since $f$ is weakly $\hat{\Omega}$-continuous, there exists $\hat{\Omega}$-open set $U_1$ and $V_1$ in $X$ containing $x$ and $y$ respectively such that $f(U_1) \subseteq cl(U), f(V_1) \subseteq cl(V)$. Moreover, $U_1 \cap V_1 = \emptyset$. Thus, $X$ is $\hat{\Omega}$-$T_2$-space. $\quad\square$

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