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Foliation by G-orbits*

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abstract: We study the properties of the normal bundle defined by the bundle of the G-orbits of the action of a semisimple Lie group G on a pseudo-Riemannian manifold M, as a consequence we obtain that the foliation induced by the normal bundle is integrable and totally geodesic.

Key Words: semisimple Lie groups, Riemannian foliation, local freeness.

Contents

1. Introduction

It is known that locally free G-actions preserving pseudo-Riemannian metrics induce bi-invariant metrics on the orbits which define bi-invariant forms when lifted to the Lie group G, ([9]). Therefore to study the geometry of the orbits for actions on pseudo-Riemannian manifolds it is important to understand the properties of bi-invariant metrics on Lie groups.

In a previous paper ([8]) we inquired about the relationship of the pseudo-riemannian invariants of G and M, where G is a semisimple Lie group and M is a pseudo-Riemananian manifold. We restricted our attention to the signature, which we denoted with (m_1, m_2) and (n_1, n_2) for M and G, respectively. We obtained an estimate between the signatures of M and G, in the case $G = G_1 \cdots G_l$ and each G_i a connected simple Lie group. If we denote $n_0^i = \min(n_1^i, n_2^i)$ and $m_0 = \min(m_1, m_2)$, then $n_0^1 + \cdots + n_0^l \leq m_0$.

If we assume that $n_0^1 + \cdots + n_0^l = m_0$ and the G-action is topologically transitive on M, then the G-orbits are nondegenerate with respect to the metric on M. This is the key to consider a normal foliation on M.

We consider the transverse or normal bundle $T\Delta^\perp$ to the orbits and we obtain certain properties of this bundle by using Gromov's machinery on G-actions([2], [7]).

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2. G-orbits

Let G be a Lie group and let F be a symmetric bilinear form on $\mathfrak{g}$. For $g \in G$ and $X, Y \in \mathfrak{g}$, we define

$$(\mathrm{Ad}(g)F)(X,Y) = F(\mathrm{Ad}(g^{-1})X, \mathrm{Ad}(g^{-1})Y).$$

We say that F is $\mathrm{Ad}(g)$-**invariant**, if $\mathrm{Ad}(g)F = F$. Let $\mathrm{Symm}^2(\mathfrak{g})$ be the space of symmetric bilinear forms on $\mathfrak{g}$. Now we define a map $\phi : M \to \mathrm{Symm}^2(\mathfrak{g})$ by $\phi(m)(X,Y) = \langle X^*_m, Y^*_m \rangle_m$ for $X, Y \in \mathfrak{g}$, where $\langle \cdot \mid \cdot \rangle$ is the pseudo Riemannian metric on M. For $X \in \mathfrak{g}$, we denote with X^* the vector field on the manifold M whose one-parameter group of diffeomorphisms is given by $(\exp(tX))_t$ through the action on M. The map ϕ is smooth, and equivariant with respect to the adjoint action of G on $\mathrm{Symm}^2(\mathfrak{g})$.

Lemma 2.1. *Let G be a connected Lie group with Lie algebra $\mathfrak{g}$ and M a connected pseudo-Riemannian manifold acted upon smoothly and isometrically by G. Then $\phi(g \cdot m) = g \cdot \phi(m)$, for each $m \in M$.*

Proof: Let $\langle \cdot \mid \cdot \rangle$ be the pseudo Riemannian metric on M. For $X, Y \in \mathfrak{g}$, and $g \in G$ we have:

$$\begin{aligned}
\phi(g \cdot m)(X,Y) =& \langle X^*_g m, Y^*_g m \rangle_{gm} \\
=& \langle dg^{-1}_{gm}(X^*_{gm}), dg^{-1}_{gm}(Y^*_{gm}) \rangle_m \\
=& \langle \mathrm{Ad}(g^{-1})(X)^*_m, \mathrm{Ad}(g^{-1})(Y)^*_m \rangle_m \\
=& \phi(m)(\mathrm{Ad}(g^{-1})(X), \mathrm{Ad}(g^{-1})(Y)) \\
=& (\mathrm{Ad}(g)\phi(m))(X,Y) \\
=& (g \cdot \phi(m))(X,Y),
\end{aligned}$$

at every $m \in M$. □

The following lemma, taken from [5], will be usefull in the next theorem.

Lemma 2.2. *Let $F \colon \mathfrak{g} \times \mathfrak{g} \to \mathbb{R}$ be a symmetric bilinear form that is* $\mathrm{Ad}(G)$*-invariant, then $F([W,X],X) = 0$ for $W, X \in \mathfrak{g}$. The converse holds if G is connected.*

Let $\mathrm{Symm}^2(\mathfrak{g})^{\mathrm{Ad}(G)}$ be the space of $\mathrm{Ad}(G)$-invariant symmetric bilinear forms on $\mathfrak{g}$. We want to impose conditions on G and M under which the image of the map ϕ lies in $\mathrm{Symm}^2(\mathfrak{g})^{\mathrm{Ad}(G)}$.

The following result is due to Zimmer (see [12]) but we present a extended version. Also this result appears in [9] but our proof is totally diferent.

Theorem 2.1. *Let M be a pseudo-Riemannian manifold with a finite volume, and let G be a connected semisimple Lie group without compact factors acting smoothly by isometries on M. If any normal subgroup acts nontrivially on M and G has finite center, then the map ϕ is* $\mathrm{Ad}(G)$*-invariant.*

Proof: By Corollary 3.2.4 from [7] there is a dense subset S of M such that for every $m \in S$, there is an open neighborhood U_m of m, and a Lie subalgebra $\mathcal{L}_m$, of smooth vector fields on U_m, such that $\mathcal{L}_m$ is isomorphic, as Lie algebra, with $\mathfrak{g}$. This isomorphism, ρ_m, for each $m \in S$, satisfies the following condition:

$$(\mathrm{ad}(\rho_m(X))Y)^* = [X, Y^*],$$

for each $Y \in \mathfrak{g}$, and $X \in \mathcal{L}_m$.

For all $X, W \in \mathfrak{g}$, we have:

$$\begin{aligned} \phi(m)(\mathrm{ad}(W)X, X) &= \langle (\mathrm{ad}(W)X)^*_m, X^*_m \rangle_m \\ &= \langle (\mathrm{ad}(W^*)X^*)_m, X^*_m \rangle_m \\ &= -\langle X^*_m, (\mathrm{ad}(W)X)^*_m \rangle_m \\ &= -\phi(m)(\mathrm{ad}(W)X, X). \end{aligned}$$

The result follows from lemma 2.2 for each $m \in S$. By the density of S in M, we conclude that $\phi(m) \in \mathrm{Symm}^2(\mathfrak{g})^{\mathrm{Ad}(G)}$ for each $m \in M$. □

Note that $\phi(m)$ may be degenerate. If we impose the extra condition that the G-action on M has a dense orbit, then $\phi(m)$ is always non degenerate. This gives us a bi-invariant pseudo-Riemannian metric on G that does not depend on the G-orbit. Our method employs results contained in the beautiful book of Zimmer (see [10]).

Corollary 2.1. *Let M be a pseudo-Riemannian manifold with a finite volume, and let G be a connected semisimple Lie group without compact factors acting smoothly by isometries on M. If any normal subgroup acts nontrivially on M, G has finite center, and there is a dense orbit, then $\phi(m) = \phi(n)$, for all $n, m \in M$.*

Proof: Suposse that ϕ is constant on G-orbits, i.e, $\phi(g \cdot m) = \phi(m)$ for all $g \in G$ and $m \in M$. Note that ϕ is a continuous function and M is connected, then ϕ is constant on M.

We know that ϕ is G-equivariant and $\phi(M)$ lies in G-fixed points, then ϕ is G-invariant, i.e, $\phi(g \cdot m) = \phi(m)$ for all $m \in M$. □

Definition 2.1. *A tangent vector $v \in T_m(M)$ is spacelike if $\langle v|v \rangle > 0$, or $v = 0$. If $\langle v|v \rangle < 0$ we say that v is timelike. The signature of the pseudo-Riemannian metric on M depends on the index of the pseudo-Riemannian metric. This index is the largest integer that is the dimension of a subspace on which the pseudo - Riemannian metric is negative definite. We denote with (m_1, m_2) the signature for M. It is convenient for us to denote with $m_0 = \min\{m_1, m_2\}$. This number is the dimension of maximal lightlike tangent subspace for M.*

On G we have that the signature (n_1, n_2) depends on the choice of the metric. If we take the Killing-Cartan form, then any other pseudo-Riemannian metric on G has a signature (n_1, n_2) or (n_2, n_1). This was remarked in [2].

From now on $G = G_1 \cdots G_l$ will be a connected noncompact semisimple Lie group with Lie algebra $\mathfrak{g} = \mathfrak{g}_1 \oplus \mathfrak{g}_2 \oplus \cdots \oplus \mathfrak{g}_l$. We know that G admits bi-invariant pseudoRiemannian metrics and all of them can be described in terms of the Killing-Cartan form on each $\mathfrak{g}_i$.

If M is a connected compact smooth manifold, and G acts smoothly, faithfully and preserving a finite measure on M, then we are interested in comparing the numbers m_0 and $n_0^1 + \cdots n_0^l$, where for each $i = 1, \ldots, l$ the numbers $n_0^i = \min(n_1^i, n_2^i)$.

Theorem 2.2. *If $n_0^1 + \cdots + n_0^l = m_0$, no factor of G acts trivially, and the G-action is topologically transitive on M, then the G-orbits are nondegenerate with repect to the metric on M.*

Proof: By theorem 2.1, for every $m \in M$ we obtain an $\mathrm{Ad}(G)$-invariant form in $\mathfrak{g}$ from the metric restricted to $T_m(Gm)$. Let A be the kernel of such a form. It is clear that A is an ideal of $\mathfrak{g}$. In particular, the metric $\langle \cdot, \cdot \rangle$ restricted to $T_m(Gm)$ is either zero or nondegenerate. Since G is semisimple only one of these possibilities is true. By theorem 5 from [8], for every $m \in M$, the metric restricted to $T_m(Gm)$ is nondegenerate. We can conclude the G-orbits are nondegenerate with repect to the metric on M. □

3. The Normal Bundle

We recall the definition of a foliation as found in ([4]).

Definition 3.1. *Let M be a smooth manifold. A codimension-k foliation $\mathfrak{F}$ of M is a decomposition of M into a union of disjoint connected codimension-k submanifolds $M = \cup_{L \in \mathfrak{F}} L$, called the leaves of the foliation, such that for each $m \in M$, there is a neighborhood U of M and a smooth submersion $f_U : U \to \mathbb{R}^k$ with $f_U^{-1}(x)$ a leaf of $\mathfrak{F}_U$ for each $x \in \mathbb{R}^k$.*

We obtain a foliation of M by orbits from the action of G on M which we denote with $\mathfrak{F}$.

If we restrict the given metric on M to each orbit of M we obtain a nondegenerate metric by using theorem 2. We denote with $T\Delta$ the tangent bundle to the orbits of the G-action on M, i.e

$$T\Delta = \bigcup_{m \in M} T_m(G \cdot m).$$

The following is the first property about the normal bundle $T\Delta^{\perp}$, i.e,

$$T\Delta^{\perp} = \bigcup_{m \in M} T_m(G \cdot m)^{\perp}.$$

Proposition 3.1. *If $n_0^1 + \cdots + n_0^l = m_0$, no factor of G acts trivially, and the G-action is topologically transitive on M, then:*

1. *for each $m \in M$, $T_m(M) = T_m(Gm) \oplus T_m(Gm)^{\perp}$, and*

2. the normal bundle $T\Delta^{\perp}$ is Riemannian or antiRiemannian.

Proof: The first part is obvious because the G-orbits are nondegenerate. For the second part we use the first one. If $T_m(Gm)^{\perp}$ has a nullvector, then it follows easily that $m_0 \geq n_0 + 1$. This contradicts our hypothesis and the following lemma. □

Lemma 3.1. *Let (V, g) be a scalar product space, i.e, V is a finite dimensional vector space and g a nondegenerate symmetric bilinear form. Suppose that $V = V_1 \oplus \cdots \oplus V_l$, where each V_i is a subspace of V and $g = g_1 \oplus \cdots \oplus g_l$, where each g_i is a scalar product in V_i, for $i = 1, \ldots, l$. Let n_0 be the dimension of the maximal subspace of null vectors with respect to g in V, and n_0^i, for $i = 1, \ldots, l$, is defined in a similar way for each V_i. Then the following inequality holds: $n_0 \geq n_0^1 + \cdots + n_0^l$*

Proof: The idea for this is to realize that for each i we have $n_0^i = \min\{n_-^i, n_+^i\}$, where n_-^i is the number of -1 and n_+^i the number of $+1$ when g is diagonalized. Without loss of generality we can suppose that for $i = 1, \ldots, k$ we have that $n_0^i = n_-^i$, and for $j = k+1, \ldots, l$ also $n_0^j = n_+^j$. It follows that

$$\begin{aligned} n_- &= n_-^1 + \cdots + n_-^l \\ &\geq n_0^1 + \cdots + n_0^l, \end{aligned}$$

and in a similar way we have

$$\begin{aligned} n_+ &= n_+^1 + \cdots + n_+^l \\ &\geq n_0^1 + \cdots + n_0^l. \end{aligned}$$

From this it follows that $n_0 = \min\{n_-, n_+\} \geq n_0^1 + \cdots + n_0^l$. □

The following theorem is dued to Gromov, and its proof is based in a version of the Gromov's Centralizer Theorem, see [7].

Theorem 3.1. *Suppose G is a semisimple Lie group acting topologically transitive on M preserving its pseudo-Riemannian metric and satisfying $n_0^1 + \cdots n_0^l = m_0$. Then $T\Delta^{\perp}$ is integrable.*

Proof: Let $\omega : TM \to \mathfrak{g}_1 \oplus \mathfrak{g}_2 \oplus \cdots \oplus \mathfrak{g}_l$ be the $\mathfrak{g}$-valued 1-form on M given by

$$\begin{aligned} TM &= T\Delta \oplus T\Delta^{\perp} \\ &\to T\Delta \\ &\cong M \times \mathfrak{g}_1 \oplus \mathfrak{g}_2 \oplus \cdots \oplus \mathfrak{g}_l \\ &\to \mathfrak{g}_1 \oplus \mathfrak{g}_2 \oplus \cdots \oplus \mathfrak{g}_l, \end{aligned}$$

where the two arrows are the natural projections.

Define the curvature of ω by the 2-form $\Omega = d\omega \mid_{T\Delta^{\perp} \times T\Delta^{\perp}}$.

It is very easy to show that $T\Delta^{\perp}$ is integrable if and only if $\Omega = 0$, where Ω is the curvature form. From [3, Ch.I] we obtain

$$2\Omega(X,Y) \,=\, X\omega(Y) - Y\omega(X) - \omega([X,Y]),$$

for $X, Y \in T_x(G\cdot x)^{\perp}$, then $\omega([X,Y]) = 0$ if and only if $\Omega(X,Y) = 0$. We conclude that $[X,Y] \in T_x(G\cdot x)^{\perp}$ if and only if $\Omega(X,Y) = 0$. Using the Frobenius theorem [3, Ch.I]) it is clear that $T\Delta^{\perp}$ is integrable if and only if $\Omega = 0$

We are going to use the local version of the Gromov's Centralizer theorem (see [7, Ch.III]) to obtain a dense subset S of M so that for every $x \in S$ there is an open neighborhood U_x and a Lie subalgebra $\mathcal{L}_x$ of Killing vector fields on U_x such that:

- if $Z \in \mathcal{L}_x$, then $Z_x = 0$.
- $\mathcal{L}_x \simeq \mathfrak{g}$, as Lie algebras.
- the local one-parameter subgroups of $\mathcal{L}_x$ preserve the G-orbits of x.
- $T_x(G\cdot x) \simeq \mathfrak{g}$, as $\mathfrak{g}$-modules.

Let X be an element of $\mathcal{L}_x$. If ϕ_t is the flow of X, then it is easy to see that $\phi_t(x) = x$, and $d\phi_t$ conmutes with ω_x. For every pair of sections X_1, X_2 of $T\Delta^{\perp}$ we have:

$$\begin{aligned}
2\Omega(d\phi_t X_1, d\phi_t X_2) &= d\phi_t X_1(\omega(d\phi_t X_2)) - d\phi_t X_2(\omega(d\phi_t X_1)) \\
&\quad - \omega([d\phi_t X_1, d\phi_t X_2]) \\
&= -\omega([d\phi_t X_1, d\phi_t X_2]) \\
&= -d\phi_t(\omega([X_1, X_2])) \\
&= d\phi_t(X_1(\omega(X_2))) - d\phi_t(X_2(\omega(X_1))) \\
&\quad - d\phi_t(\omega([X_1, X_2])) \\
&= d\phi_t 2\Omega(X_1, X_2),
\end{aligned}$$

thus showing that $\Omega_x(d\phi_t X_1, d\phi_t X_2) = d\phi_t \Omega_x(X_1, X_2)$. Differentiating with respect to t and using the Lie derivation it follows that Ω_x is a $\mathfrak{g}$-module homomorphism, for each $x \in S$.

On the other hand, we have:

$$\Omega_x(\rho_1(X)(X_1, X_2)) = \rho_1(X)\Omega_x(X_1, X_2).$$

It follows that every projection $\Omega_x^i : T\Delta^{\perp} \times T\Delta^{\perp} \to \mathfrak{g}_i$ is zero for every x in S, and then Ω_x is zero, for every x in the same set. Hence, $\Omega = 0$ on all M and $T\Delta^{\perp}$ is integrable. $\square$

4. Main result

In [6] are introduced the fundamental tensors T and A associated to a Riemannian submersion $\pi : M \to B$. The properties stated for these tensor also hold for pseudo-Riemannian manifolds. We denote by ∇ the Levi-Civita connection of M and ∇^* the Levi-Civita connection of B. For $E, F \in \mathfrak{X}(M)$

1. $T_E F := \mathcal{H}\nabla_{\mathcal{V}E}(\mathcal{V}F) + \mathcal{V}\nabla_{\mathcal{V}E}(\mathcal{H}F)$.
2. $A_E F := \mathcal{V}\nabla_{\mathcal{H}E}(\mathcal{H}F) + \mathcal{H}\nabla_{\mathcal{H}E}(\mathcal{V}F)$.

The main property of the fundamental tensors that we are going to use is the following:

Lemma 4.1. *Let X, Y horizontal vector fields on a pseudo Riemannian manifold M. Then*

$$A_X Y = \frac{1}{2}\mathcal{V}[X, Y].$$

Proof: Since $[X, Y] = \nabla_X Y - \nabla_Y X$, we have

$$A_X Y - A_Y X = \mathcal{V}\nabla_X Y - \mathcal{V}\nabla_Y X = \mathcal{V}(\nabla_X Y - \nabla_Y X) = \mathcal{V}[X, Y].$$

Using polarization is sufficient to show that $A_X X = 0$ for X horizontal vector field.

Suppose X is the horizontal lift of $Z \in \mathfrak{X}(B)$. Hence

$$X < X, V >=< V, [X, X] >=< X, [X, V] >= 0,$$

for any vertical V, and then by Koszul's formulae

$$2 < \nabla_V X, X >= V < X, X >,$$

but $V < X, X >= V(< Z, Z, > \circ \pi) = 0 < Z, Z, > \circ \pi = 0$, hence $< \nabla_V X, X >= 0$.

The field $[V, X] = \nabla_V X - \nabla_X V$ is vertical because of $d\phi_m[V, X]_m = 0$, and then

$$\begin{aligned} < \nabla_V X, X >= & < [V, X] + \nabla_X V, X > \\ = & < \nabla_X V, X > \\ = & X < V, X > - < V, \nabla_X X > \\ = & - < V, \nabla_X X > \\ = & - < V, \mathcal{V}\nabla_X X > \\ = & - < V, A_X X > \\ = & 0. \end{aligned}$$

Since $A_X X$ is vertical, then $A_X X = 0$, and the result follows. □

The following definiton is taken from [4].

Definition 4.1. *The metric $\langle \cdot | \cdot \rangle$ is said to be bundle-like for the foliation $\mathcal{F}$ it is has the following property: for any open set U of M and for all vectors fields Y, Z on U that are foliated and perpendicular to the leaves, the function $\langle Y|Z\rangle$ is basic on U.*

We note that the results in [4] are stated for Riemannnian metric only, but those we use here extended to pseudo-Riemannian metrics without change.

Theorem 4.1. *Suppose G is a semisimple Lie group acting topologically transitive on preserving its pseudo-Riemannian metric and satisfying $n_0^1 + \cdots + n_0^l = m_0$. Then the foliation induced by $T\Delta^{\perp}$ is totally geodesic.*

Proof:

By theorem 3.1 the normal bundle $T\Delta^{\perp}$ is integrable. By Frobenius's theorem there exist a induced foliation $\mathfrak{F}^{\perp}$.

We will prove that its leaves are totally geodesic submanifolds of M.

With h we will denote the metric on M preserved by G. If $X \in \mathfrak{g}$, we define X^* the infinitesimal generator as the vector field on M induced by X,(see [1]). We consider Y, Z horizontal vector fields or local sections of $T\Delta^{\perp}$ that preserve the foliation, then $[X^*, Y]$ and $[X^*, Z]$ are vertical vector fields or local sections of $T\Delta$. Hence, $h([X^*, Y], Z) = h(Y, [X^*, Z]) = 0$.

Now note that the function $h(Y, Z)$ is constant along the G-orbits because $X^*(h(Y, Z)) = 0$. We conclude that the metric h is bundle-like metric for the foliation $\mathfrak{F}$.

By results in [4] we obtain a transverse metric to the foliation $\mathfrak{F}^{\perp}$ from h. From [[4], Prop.2.1] we can get at every point of M a pseudo-riemannian submersion $\pi : U \to B$, where U is a open set in M, such that the fibers of π define the foliation $\mathfrak{F}^{\perp}$.

We will use Lemma 4.1 in order to conclude the proof. Let A the associated fundamental tensor as define above. The second fundamental tensor for the leaves of the foliation $\mathfrak{F}^{\perp}$ is given by $A_X Y$, for X, Y tangent vector fields to $\mathfrak{F}$. Using Lemma 4.1 we obtain that $A_X Y$ take values in $T\Delta$.

By Theorem 3.1 we know that $T\Delta^{\perp}$ is integrable and hence A vanishes on vertical vector fields to $\mathfrak{F}^{\perp}$, and therefore the leaves of the foliation $\mathfrak{F}^{\perp}$ are totally geodesic. □

We obtain the following corollary.

Corollary 4.1. *If M is complete, then the leaves of the foliation $\mathfrak{F}^{\perp}$ are complete.*

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