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Subdivisions of the Spectra for the Operator $D(r, 0, 0, s)$ over Certain Sequence Spaces

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ABSTRACT: In this paper we have examined the approximate point spectrum, defect spectrum and compression spectrum of the operator $D(r, 0, 0, s)$ on the sequence spaces c_0, c, ℓ_p and $bv_p (1 < p < \infty)$.

Key Words: Fine Spectrum; approximate point spectrum; defect spectrum and compression spectrum.

Contents

1. Preliminaries and Definition

Let X and Y be Banach spaces and $T : X \to Y$ be a bounded linear operator. The set of all bounded linear operators on X into itself is denoted by $B(X)$. The adjoint $T^* : X^* \to X^*$ of T is defined by $(T^*\phi)(x) = \phi(Tx)$ for all $\phi \in X^*$ and $x \in X$. Clearly, T^* is a bounded linear operator on the dual space X^*.

Let $T : D(T) \to X$ a linear operator, defined on $D(T) \subset X$, where $D(T)$ denote the domain of T and X is a complex normed linear space. For $T \in B(X)$ we associate a complex number α with the operator $(T - \alpha I)$ denoted by T_α defined on the same domain $D(T)$, where I is the identity operator. The inverse $(T - \alpha I)^{-1}$, denoted by T_α^{-1} is known as the resolvent operator of T. Many properties of T_α and T_α^{-1} depend on α and spectral theory is concerned with those properties. We are interested in the set of all α in the complex plane such that T_α^{-1} exists. Boundedness of T_α^{-1} is another essential property. We also detemine α's, for which the domain of T_α^{-1} is dense in X.

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A **regular value** is a complex number α of T such that
$(R_1)T_\alpha^{-1}$ exists,
$(R_2)T_\alpha^{-1}$ is bounded and
$(R_3)T_\alpha^{-1}$ is defined on a set which is dense in X.

The **resolvent set** of T is the set of all such regular values α of T, denoted by $\rho(T)$. Its complement is given by $C \setminus \rho(T)$ in the complex plane C is called the **spectrum** of T, denoted by $\sigma(T)$. Thus the spectrum $\sigma(T)$ consists of those values of $\alpha \in C$, for which T_α is not invertible.

The spectra of matrix operators has recently been investigated by Altay and Başar ([1], [2], [3]), Tripathy and Das [9], Tripathy and Paul ([10], [11], [12]), Tripathy and Saikia [13] and others from different aspects.

2. Subdivisions of the spectrum

In this section, we discuss about the point spectrum, continuous spectrum, residual spectrum, approximate point spectrum, defect spectrum and compression spectrum. There are many different ways to subdivide the spectrum of a bounded linear operator.Some of them are motivated by applications to physics, in particular in quantum mechanics.

2.1. The point spectrum, continuous spectrum and residual spectrum

The spectrum $\sigma(T, X)$ is partitioned into three disjoint sets as follows:

(i)The **point(discrete) spectrum** $\sigma_p(T, X)$ is the set of complex numbers α such that T_α^{-1} does not exist. Further $\alpha \in \sigma_p(T, X)$ is called the eigen value of T.

(ii) The **continuous spectrum** $\sigma_c(T, X)$ is the set of complex numbers α such that T_α^{-1} exists and satisfies (R_3) but not (R_2) that is T_α^{-1} is unbounded.

(iii) The **residual spectrum** $\sigma_r(T, X)$ is the set of complex numbers α such that T_α^{-1} exists (and may be bounded or not) but not satisfy (R_3), that is, the domain of T_α^{-1} is not dense in X .

This is to note that in finite dimensional case, continuous spectrum coincides with the residual spectrum and equal to the empty set and the spectrum consists of only the point spectrum.

2.2. The approximate point spectrum, defect spectrum and compression spectrum

Given a bounded linear operator T in a Banach space X, we call a sequence (x_k) in X as a Weyl sequence for T if $||x_k|| = 1$ and $||Tx_k|| \to 0$, as $k \to \infty$.

Appell et al. [4], have given three more classification of spectrum called the approximate point spectrum, defect spectrum and compression spectrum.

(a) The **approximate point spectrum:**
$\sigma_{ap}(T, X) = \{\alpha \in C : \text{there exist a Weyl sequence for } T - \alpha I\}$.

(b) The **defect spectrum:** $\sigma_{\delta}(T, X) = \{\alpha \in C : T - \alpha I \text{ is not surjective}\}$.

(c) The **compression spectrum:** $\sigma_{co}(T, X) = \{\alpha \in C : \overline{R(T - \alpha I)} \neq X\}$.

The two subspectra given by (a) and (b) form a (not necessarily disjoint) subdivisions $\sigma(T, X) = \sigma_{ap}(T, X) \cup \sigma_{co}(T, X)$ of the spectrum.

The compression spectrum gives rise to another (subdivisions not necessarily disjoint) decomposition $\sigma(T, X) = \sigma_{ap}(T, X) \cup \sigma_{co}(T, X)$ of the spectrum.

Clearly $\sigma_p(T, X) \subseteq \sigma_{ap}(T, X)$ and $\sigma_{co}(T, X) \subseteq \sigma_{\delta}(T, X)$. Moreover, comparing these subspectra with $\sigma(T, X) = \sigma_p(T, X) \cup \sigma_c(T, X) \cup \sigma_r(T, X)$ we note that $\sigma_r(T, X) = \sigma_{co}(T, X) \setminus \sigma_p(T, X)$ and$\sigma_c(T, X) = \sigma(T, X) \setminus [\sigma_p(T, X) \cup \sigma_{co}(T, X)]$.

Proposition 2.1. *[4, Proposition 1.3, p.28] Spectra and subspectra of an operator $T \in B(X)$ and its adjoint $T^* \in B(X^*)$ are related by the following relations:*
$(i)\sigma(T^*, X^*) = \sigma(T, X)$.
$(ii)\sigma_c(T^*, X^*) \subseteq \sigma_{ap}(T, X)$.
$(iii)\sigma_{ap}(T^*, X^*) = \sigma_{\delta}(T, X)$.
$(iv)\sigma_{\delta}(T^*, X^*) = \sigma_{ap}(T, X)$.
$(v)\sigma_p(T^*, X^*) = \sigma_{co}(T, X)$.
$(vi)\sigma_{co}(T^*, X^*) \supseteq \sigma_p(T, X)$.
$(vii)\sigma(T, X) = \sigma_{ap}(T, X) \cup \sigma_p(T^*, X^*) = \sigma_p(T, X) \cup \sigma_{ap}(T^*, X^*)$.

2.3. Goldberg's classification of spectrum

If X is a Banach space and $T \in B(X)$, then there are three possibilities for $R(T)$:
$(I)R(T) = X$,
$(II)R(T) \neq \overline{R(T)} = X$,
$(III)\overline{R(T)} \neq X$.
and
$(1)T^{-1}$ exists and is continuous.
$(2)T^{-1}$ exists but is discontinuous.
$(3)T^{-1}$ does not exist.

If these possibilities are combined in all possible ways, nine different states are created. These are labelled by: $I_1, I_2, I_3, II_1, II_2, II_3, III_1, III_2, III_3$. If an operator is in the state III_2 for example, then $\overline{R(T)} \neq X$ and T^{-1} exists but is discontinuous.

By the definitions given above, we can illustrate the subdivisions in the following table:

		1	2	3
		T_α^{-1} exists and is bounded	T_α^{-1} exists and is unbounded	T_α^{-1} does not exists
I	$R(T-\alpha I)=X$	$\alpha\in\rho(T,X)$		$\alpha\in\sigma_p(T,X)$ $\alpha\in\sigma_{ap}(T,X)$
II	$\overline{R(T-\alpha I)}=X$	$\alpha\in\rho(T,X)$	$\alpha\in\sigma_c(T,X)$ $\alpha\in\sigma_{ap}(T,X)$ $\alpha\in\sigma_\delta(T,X)$	$\alpha\in\sigma_p(T,X)$ $\alpha\in\sigma_{ap}(T,X)$ $\alpha\in\sigma_\delta(T,X)$
III	$\overline{R(T-\alpha I)}\neq X$	$\alpha\in\sigma_r(T,X)$ $\alpha\in\sigma_\delta(T,X)$ $\alpha\in\sigma_{co}(T,X)$	$\alpha\in\sigma_r(T,X)$ $\alpha\in\sigma_{ap}(T,X)$ $\alpha\in\sigma_\delta(T,X)$ $\alpha\in\sigma_{co}(T,X)$	$\alpha\in\sigma_p(T,X)$ $\alpha\in\sigma_{ap}(T,X)$ $\alpha\in\sigma_\delta(T,X)$ $\alpha\in\sigma_{co}(T,X)$

Let E and F be two sequence spaces and $A=(a_{nk})$ be an infinite matrix of real or complex numbers a_{nk}, where $n,k\in N_0=\{0,1,2,---\}$. Then, we say that A defines a matrix mapping from E into F, denote by $A:E\to F$, if for every sequence $x=(x_n)\in E$ the sequence $Ax=\{(Ax)_n\}$ is in F where $(Ax)_n=\sum\limits_{k=0}^{\infty}a_{nk}x_k$, provided the right hand side converges for every $n\in N_0$ and $x\in E$.

Throughout the paper $w,\ell_\infty,c,c_0,\ell_p,bv_p$ denote the space of all, bounded, convergent and null, p-absolutly summable and p-bounded variation sequences respectively. The zero sequence is denoted by $\theta=(0,0,0,---)$.

Let $m\in N_0$ be fixed, then Esi and Tripathy [8] have introduced the following type of difference sequence spaces $Z(\Delta_m)=\{x=(x_k):(\Delta_m x_k)\in Z\}$, for $Z=\ell_\infty,c$ and c_0, where $\triangle_m x=(\triangle_m x_k)=(x_k-x_{k+m})$
Taking $m=1$, we have the sequence spaces $\ell_\infty(\triangle),c(\triangle)$ and $c_0(\triangle)$ studied by Kizmaz [7].
Our main focus in this paper is on the operator $D(r,0,0,s)$, where

$$D(r,0,0,s)=\begin{pmatrix} r&0&0&0&0&.&.&.\\ 0&r&0&0&0&.&.&.\\ 0&0&r&0&0&.&.&.\\ s&0&0&r&0&.&.&.\\ 0&s&0&0&r&.&.&.\\ .&.&.&.&.&.&&\\ .&.&.&.&.&&.&\\ .&.&.&.&.&&&. \end{pmatrix}$$

Remark: In particular if we consider $r=-1$ and $s=1$ then $D(r,0,0,s)=\triangle_3$

The spectra of the difference operator has been investigated on different classes of sequences by various authors in the recent past. Altay and Başar ([1], [2], [3]) studied the spectra of difference operator $\triangle$ and generalized difference operator $B(r,s)$ on c_0, c and ℓ_p. Tripathy and Paul [10,11] studied the spectra of the difference operator $D(r,0,0,s)$ over the sequence spaces c_0, c and ℓ_p and bv_p. Recently Tripathy and Paul [12] studied the spectrum of the operator $B(f,g)$ on the vector valued sequence space $c_0(X)$. Başar et.al [5] have studied the subdivisions of the spectra for the generalized difference operator $B(r,s)$ over the sequence spaces c_0, c and ℓ_p and bv_p.

Lemma 2.2. [10] $\sigma(D(r,0,0,s),c_0) = \{\alpha \in C : |\alpha - r| \leq |s|\}$.

Lemma 2.3. [10] $\sigma_p(D(r,0,0,s),c_0) = \phi$.

Lemma 2.4. [10] $\sigma_r(D(r,0,0,s),c_0) = \{\alpha \in C : |\alpha - r| < |s|\}$.

Lemma 2.5. [10] $\sigma(D(r,0,0,s),c) = \{\alpha \in C : |\alpha - r| \leq |s|\}$.

Lemma 2.6. [10] $\sigma_p(D(r,0,0,s),c) = \phi$.

Lemma 2.7. [10] $\sigma_r(D(r,0,0,s),c) = \{\alpha \in C : |\alpha - r| < |s|\} \cup \{r+s\}$.

Lemma 2.8. [11] $\sigma(D(r,0,0,s),\ell_p) = \{\alpha \in C : |\alpha - r| \leq |s|\}$.

Lemma 2.9. [11] $\sigma_p(D(r,0,0,s),\ell_p) = \phi$.

Lemma 2.10. [11] $\sigma_r(D(r,0,0,s),\ell_p) = \{\alpha \in C : |\alpha - r| < |s|\}$.

Lemma 2.11. [11] $\sigma(D(r,0,0,s),bv_p) = \{\alpha \in C : |\alpha - r| \leq |s|\}$.

Lemma 2.12. *The adjoint operator T^* of T is onto if and only if T has a bounded inverse.*

3. Subdivisions of the spectrum of $D(r,0,0,s)$ over c_0

In this section, we give the subdivisions of the spectrum of the difference operator $D(r,0,0,s)$ over the sequence space c_0.

Theorem 3.1. *If $\alpha = r$, then $\alpha \in III_1\sigma(D(r,0,0,s),c_0)$.*

Proof: The operator $D(r,0,0,s) - \alpha I = D(0,0,0,s)$ for $\alpha = r$, and since $\overline{R(D(0,0,0,s)}) \neq c_0$, $D(0,0,0,s)$ is not invertible and hence $D(0,0,0,s) \in III_1$ or III_2. To verify that $D(0,0,0,s)$ has a bounded inverse, it is enough to show that $D(0,0,0,s)$ is bounded below. One can easily prove that for all $x \in c_0$ that $||D(0,0,0,s)|| \geq \frac{|s|}{2}||x||$ which means that $D(0,0,0,s)$ is bounded below. Hence $\alpha \in III_1\sigma(D(r,0,0,s),c_0)$. □

Theorem 3.2. $\sigma_{ap}(D(r,0,0,s),c_0) = \{\alpha \in C : |\alpha - r| \leq |s|\} \setminus \{r\}$.

Proof: Since $\sigma_{ap}(D(r,0,0,s),c_0) = \sigma(D(r,0,0,s),c_0) \setminus III_1\sigma(D(r,0,0,s),c_0)$, $\sigma_{ap}(D(r,0,0,s),c_0) = \{\alpha \in C : |\alpha - r| \le |s|\} \setminus \{r\}$ is obtained by Lemma 2.2 and Theorem 3.1. □

Theorem 3.3. $\sigma_\delta(D(r,0,0,s),c_0) = \{\alpha \in C : |\alpha - r| \le |s|\}$.

Proof: Since $\sigma_\delta(D(r,0,0,s),c_0) = \sigma(D(r,0,0,s),c_0) \setminus I_3\sigma(D(r,0,0,s),c_0)$.
Now, $I_3\sigma(D(r,0,0,s),c_0) = II_3\sigma(D(r,0,0,s),c_0) = III_3\sigma(D(r,0,0,s),c_0) = \sigma_p(D(r,0,0,s),c_0) = \phi$ is obtained by Lemma 2.3. Hence $\sigma_\delta(D(r,0,0,s),c_0) = \sigma(D(r,0,0,s),c_0)$. □

Theorem 3.4. $\sigma_{co}(D(r,0,0,s),c_0) = \{\alpha \in C : |\alpha - r| < |s|\}$.

Proof: $\sigma_{co}(D(r,0,0,s),c_0) = III_1\sigma(D(r,0,0,s),c_0) \cup III_2\sigma(D(r,0,0,s),c_0) \cup III_3\sigma(D(r,0,0,s),c_0)$.
Now,
$III_1\sigma(D(r,0,0,s),c_0) \cup III_2\sigma(D(r,0,0,s),c_0) = \sigma_r(D(r,0,0,s),c_0) = \{\alpha \in C : |\alpha - r| < |s|\}$ is obtained by Lemma 2.4.
Again, $III_3\sigma(D(r,0,0,s),c_0) = \sigma_p(D(r,0,0,s),c_0) = \phi$ is obtained by Lemma 2.3
Hence, $\sigma_{co}(D(r,0,0,s),c_0) = \{\alpha \in C : |\alpha - r| < |s|\}$. □

As a consequence of Proposition 2.1 we have the following result.

Corollary 3.5. *The following results hold:*
$(i)\sigma_{ap}(D(r,0,0,s)^*,\ell_1) = \{\alpha \in C : |\alpha - r| \le |s|\}$.
$(ii)\sigma_\delta(D(r,0,0,s)^*,\ell_1) = \{\alpha \in C : |\alpha - r| \le |s|\}\setminus\{r\}$.

4. Subdivisions of the spectrum of $D(r,0,0,s)$ over c

In this section, we give the subdivisions of the spectrum of the difference operator $D(r,0,0,s)$ over the sequence space c.

Theorem 4.1. *If* $\alpha = r$, *then* $\alpha \in III_1\sigma(D(r,0,0,s),c)$.

Proof: This is obtained in the similar way that is used in the proof of Theorem 3.1. □

Theorem 4.2. $\sigma_{ap}(D(r,0,0,s),c) = \{\alpha \in C : |\alpha - r| \le |s|\}\setminus\{r\}$.

Proof: Since $\sigma_{ap}(D(r,0,0,s),c) = \sigma(D(r,0,0,s),c)\setminus III_1\sigma(D(r,0,0,s),c)$,
$\sigma_{ap}(D(r,0,0,s),c) = \{\alpha \in C : |\alpha - r| \le |s|\}\setminus\{r\}$ is obtained by Lemma 2.5 and Theorem 4.1. □

Theorem 4.3. $\sigma_\delta(D(r,0,0,s),c) = \{\alpha \in C : |\alpha - r| \le |s|\}$.

Proof: Since $\sigma_\delta(D(r,0,0,s),c) = \sigma(D(r,0,0,s),c)\backslash I_3\sigma(D(r,0,0,s),c)$.
Now,
$I_3\sigma(D(r,0,0,s),c) = II_3\sigma(D(r,0,0,s),c) = III_3\sigma(D(r,0,0,s),c)$
$= \sigma_p(D(r,0,0,s),c) = \phi$ is obtained by Lemma 2.6. Hence $\sigma_\delta(D(r,0,0,s),c) = \sigma(D(r,0,0,s),c)$. □

Theorem 4.4. $\sigma_{co}(D(r,0,0,s),c) = \{\alpha \in C : |\alpha - r| < |s|\} \cup \{r+s\}$.

Proof: $\sigma_{co}(D(r,0,0,s),c) = III_1\sigma(D(r,0,0,s),c) \cup III_2\sigma(D(r,0,0,s),c)$
$\cup III_3\sigma(D(r,0,0,s),c)$.
Now,
$III_1\sigma(D(r,0,0,s),c) \cup III_2\sigma(D(r,0,0,s),c) = \sigma_r(D(r,0,0,s),c)$
$= \{\alpha \in C : |\alpha - r| < |s|\} \cup \{r+s\}$ is obtained by Lemma 2.7.
Again, $III_3\sigma(D(r,0,0,s),c) = \sigma_p(D(r,0,0,s),c) = \phi$ is obtained by Lemma 2.6.
Hence, $\sigma_{co}(D(r,0,0,s),c) = \{\alpha \in C : |\alpha - r| < |s|\} \cup \{r+s\}$. □

As a consequence of Proposition 2.1 we have have the following result.

Corollary 4.5. *The following results hold:*
$(i)\sigma_{ap}(D(r,0,0,s)^*,\ell_1) = \{\alpha \in C : |\alpha - r| \leq |s|\}$.
$(ii)\sigma_\delta(D(r,0,0,s)^*,\ell_1) = \{\alpha \in C : |\alpha - r| \leq |s|\}\backslash\{r\}$.

5. Subdivisions of the spectrum of $D(r,0,0,s)$ over $\ell_p(1 < p < \infty)$

In this section, we give the subdivisions of the spectrum of the difference operator $D(r,0,0,s)$ over the sequence space ℓ_p.

Theorem 5.1. *If* $\alpha = r$*, then* $\alpha \in III_1\sigma(D(r,0,0,s),\ell_p)$.

Proof: By Lemma 2.10, $\alpha \in III\sigma(D(r,0,0,s),\ell_p)$ whenever $\alpha = r$. Again by Lemma 2.12, $\alpha = r$ is not in $\sigma_p(D(r,0,0,s),\ell_p)$ and hence $(D(r,0,0,s) - rI)^{-1}$ exists. But $D(r,0,0,s) - rI$ may be continuous or not. We have to show that $(D(r,0,0,s) - rI)^{-1}$ must be continuous, for this it is sufficient to show that $D(r,0,0,s)^* - rI = D(0,0,0,s)^*$ is onto by Lemma 2.12. Given $y = (y_k) \in \ell_q$ we must find $x = (x_k) \in \ell_q$ such that $D(0,0,0,s)^*x = y$. By direct calculation we see that $x_n = \frac{1}{s}y_{n-2}$ which shows that $D(0,0,0,s)^*$ is onto.
This completes the proof. □

Theorem 5.2. $\sigma_{ap}(D(r,0,0,s),\ell_p) = \{\alpha \in C : |\alpha - r| \leq |s|\}\backslash\{r\}$.

Proof: Since $\sigma_{ap}(D(r,0,0,s),\ell_p) = \sigma(D(r,0,0,s),\ell_p)\backslash III_1\sigma(D(r,0,0,s),\ell_p)$,
$\sigma_{ap}(D(r,0,0,s),\ell_p) = \{\alpha \in C : |\alpha - r| \leq |s|\}\backslash\{r\}$ is obtained by Lemma 2.8 and Theorem 5.1. □

Theorem 5.3. $\sigma_\delta(D(r,0,0,s),\ell_p) = \{\alpha \in C : |\alpha - r| \leq |s|\}$.

Proof: Since $\sigma_\delta(D(r,0,0,s),\ell_p) = \sigma(D(r,0,0,s),\ell_p)\backslash I_3\sigma(D(r,0,0,s),\ell_p)$.
Now,
$I_3(D(r,0,0,s),\ell_p) = II_3\sigma(D(r,0,0,s),\ell_p) = III_3\sigma(D(r,0,0,s),\ell_p)$
$= \sigma_p(D(r,0,0,s),\ell_p) = \phi$ is obtained by Lemma 2.9.
Hence $\sigma_\delta(D(r,0,0,s),\ell_p) = \sigma(D(r,0,0,s),\ell_p)$. □

Theorem 5.4. $\sigma_{co}(D(r,0,0,s),\ell_p) = \{\alpha \in C : |\alpha - r| < |s|\}$.

Proof: $\sigma_{co}(D(r,0,0,s),\ell_p) = III_1\sigma(D(r,0,0,s),\ell_p) \cup III_2\sigma(D(r,0,0,s),\ell_p) \cup III_3\sigma(D(r,0,0,s),\ell_p)$.
Now, $III_1\sigma(D(r,0,0,s),\ell_p) \cup III_2 \cup (D(r,0,0,s),\ell_p) = \sigma_r(D(r,0,0,s),\ell_p)$
$= \{\alpha \in C : |\alpha - r| < |s|\}$ is obtained by Lemma 2.10.
Again, $III_3\sigma(D(r,0,0,s),\ell_p) = \sigma_p(D(r,0,0,s),\ell_p) = \phi$ is obtained by Lemma 2.9.
Hence,$\sigma_{co}(D(r,0,0,s),\ell_p) = \{\alpha \in C : |\alpha - r| < |s|\}$. □

As a consequence of Proposition 2.1 we have the following result.

Corollary 5.5. *Let $p^{-1} + q^{-1} = 1$ then, the following are true.*
$(i)\sigma_{ap}(D(r,0,0,s)^*,\ell_q) = \{\alpha \in C : |\alpha - r| \leq |s|\}$.

$(ii)\sigma_\delta(D(r,0,0,s)^*,\ell_q) = \{\alpha \in C : |\alpha - r| \leq |s|\}\backslash\{r\}$.

6. Subdivisions of the spectrum of $D(r,0,0,s)$ over $bv_p(1 < p < \infty)$

In this section, we give the subdivisions of the spectrum of the difference operator $D(r,0,0,s)$ over the sequence space bv_p. Since the subdivisions of the spectrum of the operator $D(r,0,0,s)$ on the sequence space bv_p can be derived by analogy to the space ℓ_p, we omit the detail and give the related results without proof.

Theorem 6.1. *The followings hold:*

$(i)\sigma_{ap}(D(r,0,0,s),bv_p) = \{\alpha \in C : |\alpha - r| \leq |s|\}\backslash\{r\}$.

$(ii)\sigma_\delta(D(r,0,0,s),bv_p) = \{\alpha \in C : |\alpha - r| \leq |s|\}$.

$(iii)\sigma_{co}(D(r,0,0,s),bv_p) = \{\alpha \in C : |\alpha - r| < |s|\}$.

As a consequence of Proposition 2.1 we have the following result.

Corollary 6.2. *The following results hold:*
$(i)\sigma_{ap}(D(r,0,0,s)^*,bv_p^*) = \{\alpha \in C : |\alpha - r| \leq |s|\}$.

$(ii)\sigma_\delta(D(r,0,0,s)^*,bv_p^*) = \{\alpha \in C : |\alpha - r| \leq |s|\}\backslash\{r\}$.

Conclusion: We can generalize our operator

$$(D(r,0,0,..(n-1)times,s) = \begin{pmatrix} r & 0 & 0 & 0 & 0 & 0 & . & . & . \\ 0 & r & 0 & 0 & 0 & 0 & . & . & . \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ s & 0 & . & . & r & 0 & . & . & . \\ 0 & s & 0 & . & . & r & . & . & . \\ . & . & . & . & . & . & . & & \\ . & . & . & . & . & . & & . & \\ . & . & . & . & . & . & & & . \end{pmatrix}$$

If we take $r = -1$ and $s = 1$, then the operator $D(r, 0, 0, ..(n-1)times, s)$ will be the same as the generalized difference operator $\triangle_n$.Further on considering the operator $D(r, 0, 0, ..(n-1)times, s)$ in place of $D(r, 0, 0, s)$, one can get parallel all our results obtained in this paper.

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