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Mathematical Modelling and Estimation of Seasonal Variation of Mosquito Population: A Real Case Study *

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ABSTRACT: A mathematical model is proposed and analyzed for the understanding of growth pattern of mosquito vector looking into its life cycle. The objective of this study is to develop a mathematical model that can fit to the real data provided by DRDE scientist for different month at different stations so that the seasonal variation in population density of mosquitoes can be reported accurately to the estimated data obtained by the proposed mathematical model. The aquatic class (L) and adult stage is divided in two class, indoor population (I) and outdoor population (O). Here we estimated different parameters of our proposed continuous model and numerical simulation is done to compare the estimated data with the actual data.

Key Words: Mosquito Life-cycle; Mathematical Model; Parameter Estimation; Numerical Simulation.

Contents

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1. Introduction

Vector-borne, particularly mosquito-borne diseases, can be extremely fatal to human and animal populations. Mosquitoes are foremost in man's war against insects. As mosquitoes are very climate sensitive, environmental conditions trigger their dynamics and consequently affect disease spread. For the control of mosquito population and prevention and disease, it is very important to understand the vector environment relation. There are about 3200 species of mosquitoes existing in the world. The importance of mosquitoes in human and animal disease has made them an important target of medical, vetinary and conservation research since Patrick Manson and Sir Ronald Ross first implicated mosquitoes in the transmission of filarial nematodes and malaria in the closing decades of the nineteenth century [1,2]. Malaria is one of the most prevalent vector-host diseases, whereby the disease is not transmitted directly from host to host, but through a vector [3,4,5,8,9, 10]. There are many existing models based on climatic conditions considering mosquito control in homogeneous, non-homogeneous stage structured model to study and control the mosquito vector [11,12,16,17,18]. Several models have been developed to predict mosquito population dynamics. The model cited were built for a specific mosquito species in a specific geographic context [19,20,21,22]. The model we proposed represents the mosquito life cycle (aquatic and adult stage)and generic as the model structure is common to all mosquito species and also for all the considered stations simply by changing parameter values and functions. The aim of this paper is to develop a mathematical model to estimate and forecast mosquito population at different stations in accordance of the data provided by DRDE scientist to control vector (mosquito) population.

The life cycle of mosquito involves aquatic(egg, larve and pupae) and terristrial (adult) stages and can be recognized by their special appearance. Mosquitoes lay their eggs on the surface of fresh water. Eggs are laid one at a time and they float on the surface of fresh water. The water may be in any container like cans , barrel, ditches, horse troughs, ornamental ponds, swimming pools, etc. All mosquitoes must have water in which to complete their life cycle. The life cycle of mosquitoes depend on temperature and species characteristics as to how long it takes for development. Larvae commonly called "wigglers" or "wrigglers" live in water and come to the surface to breathe, eats algae and small organism which lives in water. Pupae is a resting non-feeding stage. This is the time the mosquito turns into an adult. It takes about two days before the adult is fully developed. When the development is complete the pupal skin split and the mosquito emerges as an adult and are commonly called "tumblers" and live in water from one to four days. As they are lighter than water therefore floats at the surface. The newly emerged adult rests on the surface of the water for a short time to allow itself to dry and all its parts to harden [23].

The mosquitoes are found in diverse habitats and environmental conditions. Effective bio control agents include predatory fish that feed on mosquito larvae such as mosquitofish (Gambusia affinis) and some cyprinids (carps and minnows) and killifish. Tilapia will also consume mosquito larvae [1,24,25]. In the last few years scientists of DRDE, Gwalior have conducted extensive survey of mosquito populations at 14 Army and Air Force cantonments or stations named: Amritsar, Bathinda, Sri Ganganagar, Bikaner, Jodhpur, Barmer, Gandhinagar, Jamnagar, Bhuj, Pathankot, Karu Leh, Mashimpur, Hathigarh, Nagrota as per suggestions of DGAFMS. In each Army Station and Air Force station, five Army or Air force units and one civilian area adjacent to cantonment had been selected for study on species composition and seasonal variation in population dynamics. Scientits studied the seasonal occurrence, species composition, geographic variation to the mosquito vector ecology. The study area of the selected stations falls under different climatic condition such as western border, north-eastern border northern and cold desert areas. Vector control is one of the effective tools to deduce or suppress their level. The knowledge of vector population in different geographical regions is required to carry out effective vector control strategies and disease management. The distribution and establishment of mosquito species in different areas had been recorded over the year. However, the information of these vectors in the cantonment areas of western border, north-eastern border, northern and cold desert areas is scanty. Some of the areas are remotely located and isolated, due to this become less accessible for exploring the mosquito species prevalent there.

The indoor collections of mosquitoes were carried out with mouth sucking glass aspirators in the morning at $0500-0630hrs$ and in evening at $2000-2330hrs$ during different seasons of the year in each unit of every cantonment, whereas, outdoor collections of mosquitoes were carried out by CDC light traps. Unit of collection of mosquitoes is done in three different ways for larval density in LPL (larvae per ladle), indoor population densities in PMH (per man hour) and outdoor densities in PLT (per light trap).

The genus level identification has been carried out in the field by DRDE scientists. After that, species identification of these mosquitoes was carried out in the Defence Research and Development Establishment, Gwalior. Reference mosquito specimens have been stored in Entomology Division, Defence Research and Development Establishment, Gwalior. Keeping in view of the compartmental modelling approaches [6,7,13,14,15], in this paper, an attempt is made to develop a mathematical model from field data for mosquito population considering all the stage via aquatic and adult stage.

2. Modelling Assumptions and a Generalized Conceptual Model

A mathematical model illustrates the biological perceptive within a computational framework to achieve an improved understanding of how a real world system works. Therefore Modelling is a helpful tool to develop the control mechanism of vectors such as mosquitoes. The integrative nature of it is needed to understand the population dynamics of mosquitoes densities variation in abundance over time and identify the major influential parameters. Mathematical model is being developed that will use a wide range of parameters to forecast the spread and growth of different type of mosquito vector populations. Major parameters for the model are:

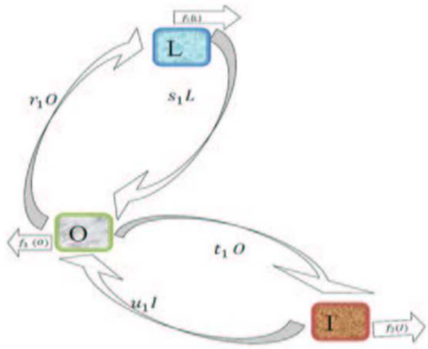


Figure 1: Schematic diagram

a) Larval natural growth rate (r_1),

b) Natural washout rate of larval in presence of zooplankton and fish population (s_1),

c) Conversion rate of larval to adult mosquito (t_1),

d) Migration rate of indoor to outdoor population (u_1).

The schematic flow diagram is shown in figure 1.

A three compartment mathematical model has been proposed and analyzed to study the dynamics of mosquito vector, taking Larva, Indoor and Outdoor mosquitoes as state variable. Here Larva class contains the first three stages of mosquito (i.e., Egg, Larva, Pupa) and adult mosquitoes are divided into Outdoor and Indoor mosquitoes figure 1. The model is being formulated with the help of following system of ordinary differential equations:

$$\frac{dL}{dt} = r_1O - s_1L - f_1(L), \tag{2.1}$$

$$\frac{dI}{dt} = t_1O - u_1I - f_2(I), \tag{2.2}$$

$$\frac{dO}{dt} = s_1L - t_1O + u_1I - f_3(O), \tag{2.3}$$

with non-negative initial conditions $L(0) > 0, I(0) > 0, O(0) > 0$, where $f_1(L) = a_1u_1L$, $f_2(I) = a_2u_1I$,$f_3(O) = a_3u_1O$ and a_1,a_2,a_3 is the natural decay in mosquito populations and u_1 is movement rate (i.e., migration rate of indoor to outdoor population), hence the natural washout rates $f_i, i = 1, 2, 3$; are proportionate to the fraction of the population size of respective compartment and the movement rate of indoor population.

The above system can be non-dimensionalised using τ=u_1t, r=$\frac{r_1}{u_1}$, s=$\frac{s_1}{u_1}$ and t=$\frac{t_1}{u_1}$ we get the following rescaled model-

$$\frac{dL}{d\tau} = rO - sL - a_1L, \quad (2.4)$$

$$\frac{dI}{d\tau} = tO - I - a_2I, \quad (2.5)$$

$$\frac{dO}{d\tau} = sL - tO + I - a_3O. \quad (2.6)$$

Motivated from the parameter estimation approach discussed in [6], we will estimate parameters of our proposed system (2.4)-(2.6) in next two sections.

3. Least Square Curve Fitting of Time Series Data

As a_1, a_2 and a_3 are the washout rate which varies from station to station, depends upon the climatic condition. From the proposed model we calculate the parametric values r, s and t quarter wise. Further, introducing these parametric values in the model we simulate the model and draw the graphs for the different army and air force station. Since different stations have different climatic conditions resulting in different natural washout rate, therefore, there are variations in values of natural washout rate in the model system simulation to fit the available data. To justify the numerical simulation we estimate the value of Larvae, Indoor and Outdoor using cubic least square curve fitting and is given as below:

$$a + bt + ct^2 + dt^3 = \text{Compartment densities}(i.e., L, I, O).$$

The data is estimated using least square curve fit method and simulation is done using well known software Matlab and Mathematica.

4. Parameter Estimation and Mathematical Models

In this section, we will discuss the parameter estimations and analysis of models for two stations, namely, Amritsar and Barmer.

4.1. Amritsar Army Station(ASR)

The model system of Culex quinquefasciatus for Amritsar army station in accordance of (2.4) - (2.6) with washout rate: $a_1 = 2.5$, $a_2 = 1.2$ and $a_3 = 1.7$ is

$$\frac{dL}{d\tau} = rO - sL - 2.5L, \quad (4.1)$$

$$\frac{dI}{d\tau} = tO - uI - 1.2I, \quad (4.2)$$

$$\frac{dO}{d\tau} = sL - tO + uI - 1.7O. \quad (4.3)$$

Seasonal variation at Amritsar army station shown in table AM1.

Table AM1: *Seasonal variation in population density of Culex quinquefasciatus at ASR*

Density	Months			
	April	**June**	**September**	**December**
Larval(LPL)	55.43	29.35	65	61.96
Indoor(PMH)	52.66	38	35.5	77.6
Outdoor(PLT)	13.66	6.8	3.75	7.76

4.1.1. Parameter Estimation.

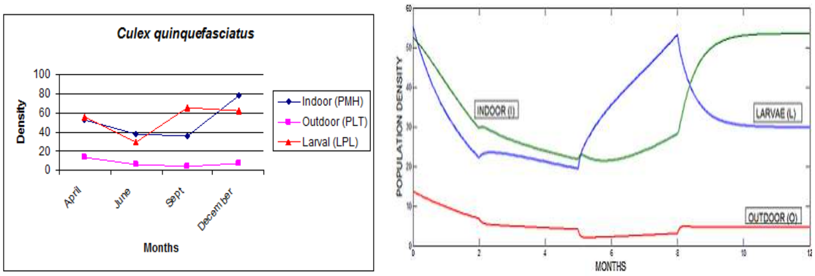
Table AM2: *Rate of change for larval, indoor and outdoor*

Density	Months			
	April	**June**	**September**	**December**
Larval(LPL)	-31.9270	2.2799	13.4609	-23.5134
Indoor(PMH)	-8.4556	-5.6152	5.2742	24.1182
Outdoor(PLT)	-4.5084	-2.3968	0.2618	2.3098

Table AM3: Estimated parametric values for r, s and t

Density	Months			
	April	**June**	**September**	**December**
r	3.1102	7.0257	24.5214	13.9986
s	0.6758	0.8834	0.5409	1.4660
t	6.1273	8.9537	17.9731	20.6080

A graphical comparison of real data and simulated prediction is given in figure 2.



(a) Original Data (b) Estimated Distributions

Figure 2: Seasonal variation at Amritsar Station for Anopheles

4.1.2. Result. The larval, indoor and outdoor densities are calculated for four different months. Estimated diagram of the larval, indoor and outdoor densities of Culex are noticed similar behavior as in the original figure except the smoothness.

The estimated figure showed presence of Culex throughout April to December. This data also suggest that Culex breeding is prevalent throughout the year in Amritsar.

Normally, higher density has been observed from July to October and March and April, whereas lower density has been recorded during May to June and December to January. In Anophelies, the density has been found to be low at all these Army and Air Force stations in comparison to Cx. quinquefasciatus. In general, the adult and larval density has been found to be higher during July to October (rainy and post-rainy season) and March to April with minor fluctuations. In peak summer (May-June) and peak winter (December and January) the Anophelies density has been found to be very low or absent. In some stations, only anophelies density was represented by the presence of very few mosquito larvae.

4.2. Barmer Army Station Culex quinquefasciatus(BM)

The model system of Culex quinquefasciatus for Barmer army station is in accordance of (2.4) - (2.6) with washout rate: $a_1 = 0.66$, $a_2 = 0.7$ and $a_3 = 0.8$.

$$\frac{dL}{d\tau} = rO - sL - 0.66L, \tag{4.4}$$

$$\frac{dI}{d\tau} = tO - I - 0.7I, \tag{4.5}$$

$$\frac{dO}{d\tau} = sL - tO + I - 0.8O. \tag{4.6}$$

Seasonal variation in population density of Culex quinquefasciatus at Barmer army station shown in table BM1.

Table BM1: Seasonal variation in population density of Culex quinquefasciatus in Barmer Army Station (BM)

Density	Months			
	March	**May**	**September**	**December**
Larval(LPL)	8.40	17.80	47.80	35
Indoor(PMH)	13.20	6.40	19.60	16.00
Outdoor(PLT)	2.83	0.2	12.20	1.67

4.2.1. Parameter Estimation.

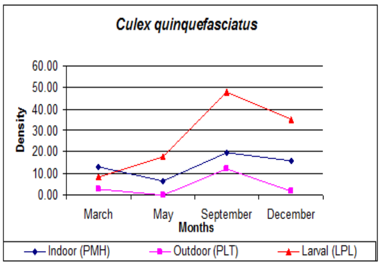
Table BM2: Rate of change for larval, indoor and outdoor

Density	Months			
	March	**May**	**September**	**December**
Larval(LPL)	-1.5134	0.7909	-0.2259	-5.9106
Indoor(PMH)	-2.1806	-0.0324	0.4306	-2.5764
Outdoor(PLT)	-1.1547	0.2068	0.2664	-2.0193

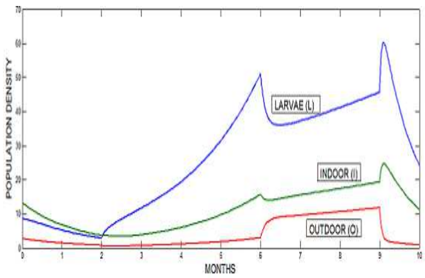
Table BM3: Estimated parametric values for r, s and t

Density	Months			
	March	**May**	**September**	**December**
r	27.6413	129.3261	20.2841	4.1687
s	1.0890	0.7313	1.5882	0.0656
t	10.2194	30.8380	6.1719	18.1181

A graphical comparison of real data and simulated prediction is given in figure 3.



(a) Original Data



(b) Estimated Distributions

Figure 3: Seasonal variation at Barmer Station for Culex

4.2.2. Result. The larval, indoor and outdoor densities are calculated for four different months. The larval and indoor population density of Anopheles shows the same behavior as the original figure except the smoothness. Presence of mosquito was not recorded in light trap. For model validation we replace 0 by 0.2 as the future population depends on the previous population. The overall activity in outdoor density was low.

4.3. Barmer Army Station Anopheles(BM)

The model system of Anopheles for Barmer army station is in accordance of (2.4) - (2.6) with washout rate: $a_1 = 0.66$, $a_2 = 0.7$ and $a_3 = 0.8$.

$$\frac{dL}{d\tau} = rO - sL - 0.66L, \tag{4.7}$$

$$\frac{dI}{d\tau} = tO - I - 0.7I, \tag{4.8}$$

$$\frac{dO}{d\tau} = sL - tO + I - 0.8O. \tag{4.9}$$

Table BM4: Seasonal variation in population density of Culex quinquefasciatus in Barmer Army Station (BM)

Density	Months			
	March	**May**	**September**	**December**
Larval(LPL)	7.13	6.72	10.35	2.20
Indoor(PMH)	4.00	2.00	4.50	2.00
Outdoor(PLT)	1.00	0.20	2.33	0.20

4.3.1. Parameter Estimation.

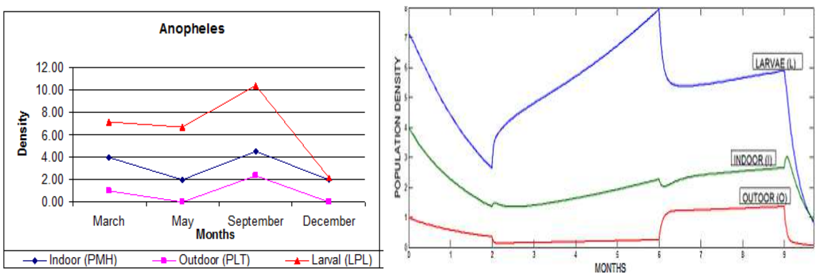
Table BM5: Rate of change for larval, indoor and outdoor

Density	Months			
	March	**May**	**September**	**December**
Larval(LPL)	0.9032	7.5423	3.6397	-14.3206
Indoor(PMH)	-7.9794	0.3974	3.0746	-7.2341
Outdoor(PLT)	-4.1808	1.6750	1.2217	-9.7623

Table BM6: Estimated parametric values for r, s and t

Density	Months			
	March	**May**	**September**	**December**
r	10.7399	63.4947	14.0734	46.8395
s	1.0108	0.6434	1.0158	0.1441
t	7.4419	14.4774	3.7864	16.7460

A graphical comparison of real data and simulated prediction is given in figure 4.



(a) Original Data (b) Estimated Distributions

Figure 4: Seasonal variation at Barmer Station for Culex

4.3.2. Result. The larval, indoor and outdoor densities are calculated during four different months as for Amritsar station and it is found that presence of mosquitoes population decrease in May, increase till September and decreases in December at Barmer station. During the month of May presence of mosquito was not recorded in light trap . For model validation we replace 0 by 0.2 as the future population depends on the previous population. The overall activity in outdoor density was low at Barmer station.

5. Conclusion

In this paper, we developed a mathematical model to study the growth pattern of mosquitoes on the basis of three classes, i.e., aquatic via two adult classes. First includes all the three stages relates to water (egg, larve, pupae) and adult population is shared by indoor and outdoor mosquitoes. Models are developed on seasonal variation corpus of three states (L,I,O) of Culex quinquefasciatus and Anopheles mosquitoes for two stations. A general compartment model is proposed consisting of growth and movement rates between the compartments of mosquito populations. Then model parameters are estimated from DRDE corpus using LSE and finally solving system of equations. We studied the behavior of the two species of mosquitoes(Culex and Anopheles) and it is found that Cx. quinquefasciatus is most dominating species prevalent at all the stations in indoor, outdoor and larval density in comparison to Anopheles. Further it is also observed that the data provided by DRDE is inappropriate for one of the station, because few of the seasonal variation data for larval, indoor and outdoor population are zero and rest of data are nonzero, it is only possible from the migration of adult mosquitoes from nearly region. As the later population of mosquitoes (Culex and Anopheles) depends on previous growth of mosquitoes. Finally models for both the stations (Amritsar and Barmer)are numerically simulated using Matlab software and compared with the original corpus.

The mathematical models are developed and estimated from four data points in each case, which is very less for a continuous model. The model is limited due to following reasons: (i) Data are not uniformly taken with respect to time; (ii) Effect of insecticides is not well defined (periodic or random spaying ?); (iii) Species Migration or diffusion rates are missing. In future one can develop a more realistic model considering the periodic or random effect of insecticides weakly/daily basis data.

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