Bol. Soc. Paran. Mat. (3s.) **v. 35** 1 (2017): 33–41.
©SPM –ISSN-2175-1188 on line ISSN-00378712 in press
SPM: www.spm.uem.br/bspm doi:10.5269/bspm.v35i1.25317

$^*\wedge_\mu$-sets and $^*\vee_\mu$-sets in Generalized Topological Spaces

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ABSTRACT: In this paper, we introduce and study some properties of the new sets namely $^*\wedge_\mu$- sets, $^*\vee_\mu$-sets, $^*\lambda_\mu$-closed sets, $^*\lambda_\mu$-open sets in a generalized topological space.

Key Words: Generalized topology, $^*\wedge_\mu$-set, $^*\vee_\mu$-set, $^*\lambda_\mu$-closed set, $^*\lambda_\mu$-open set.

Contents

1. Introduction

A study on generalized topology (briefly, GT) was introduced by *Á.Császár* [7]. He defined some basic operators on a generalized topological spaces. It is observed there are a large number of papers devoted to the study of generalized open-like sets of a topological space, containing the class of open sets and possessing properties more or less similar to those of open sets. The research on generalized open sets is still being continued by different mathematicians. For example, the concepts of $\wedge$ -sets, $\wedge_\delta$ -sets,$\wedge_s$ -sets, pre-$\wedge$ -sets, $\wedge_b$ -sets,$\wedge_\alpha$ -sets,$\wedge_{sp}$-sets,$\wedge_m$ -sets have been studied in [1,2,3,4,5,9,10]. A generalized topology μ on a nonempty set X is a collection of subsets of X such that $\phi \in \mu$ and μ is closed under arbitrary union. Elements of μ are called μ-open sets. A set X with a GT μ is called a generalized topological space (briefly GTS) and is denoted by (X, μ). If A is a subset of a space (X, μ), then $c_\mu(A)$ is the smallest μ-closed set containing A and $i_\mu(A)$ is the largest μ-open set contained in A. Clearly, A is μ-open if and only if $A = i_\mu(A)$ and A is μ-closed if and only if $A = c_\mu(A)$ (see [7,8]).

Definition 1.1. [11]. *Let (X, μ) be a GTS. For $A \subset X$, define $\wedge_\mu(A) = \cap\{U \subset X | A \subset U$ and $U \in \mu\}$ if there exists $U \in \mu$ such that $A \subset U$, otherwise X and $\vee_\mu(A) = \cup\{U \subset X | U \subset A$ and U is μ-closed$\}$ if there exists a μ-closed set U such that $U \subset A$, otherwise $\emptyset$.*

2000 *Mathematics Subject Classification:* Primary 54A05

Definition 1.2. [11]. *Let (X, μ) be a GTS. A subset A of X is called a $\vee_\mu$-set if $A = \vee_\mu(A)$ and A is called a $\wedge_\mu$-set if $A = \wedge_\mu(A)$.*

Definition 1.3. [11]. *A subset A of a GTS (X, μ) is said to be λ_μ-closed if $A = T \cap C$, where T is a $\wedge_\mu$-set and C is a μ-closed set. The complement of a λ_μ-closed set is called a λ_μ-open set.[For convenience, we use the notation "$\lambda_\mu-$closed" instead of "($\wedge$,μ)-closed"]. We denote the collection of all λ_μ-open (resp.λ_μ-closed) sets of X by $\lambda_\mu O(X, \mu)$ (resp.$\lambda_\mu C(X, \mu)$).*

Remark 1.4. *It is obvious that $\emptyset \in \lambda_\mu O(X, \mu)$. It follows from Observation 2.7 of [11] that $\lambda_\mu O(X, \mu)$ is closed under arbitrary union. Therefore, $\lambda_\mu O(X, \mu)$ is a GTS.*

2. $^*\wedge_\mu$-sets and $^*\vee_\mu$-sets

In this section we define the new sets $^*\wedge_\mu$-sets and $^*\vee_\mu$-sets in GTS and study some of their properties.

Definition 2.1. *Let (X, μ) be a GTS. For $A \subset X$ we define $^* \wedge_\mu (A) = \cap\{G|A \subset G, G \in \lambda_\mu O(X, \mu)\}$ and $^* \vee_\mu (A) = \cup\{G|G \subset A, G \in \lambda_\mu C(X, \mu)\}$.*

Theorem 2.2 gives the properties of the operator $^*\wedge_\mu$.

Theorem 2.2. *Let A, B and $\{C_i|i \in I\}$ be subsets of a GTS (X, μ). Then the following hold:*
a). $A \subset^* \wedge_\mu (A) \subset \wedge_\mu(A)$.
b). If $A \subset B$ then $^* \wedge_\mu (A) \subset^* \wedge_\mu (B)$.
c). $^* \wedge_\mu (^*\wedge_\mu(A)) =^* \wedge_\mu (A)$.
d). $^* \wedge_\mu (\cup\{C_i|i \in I\}) = \cup\{^*\wedge_\mu(C_i|i \in I)\}$.
e). $^* \wedge_\mu (\cap\{C_i|i \in I\}) \subset \cap\{^*\wedge_\mu(C_i|i \in I)\}$.
f). If A is μ-open then $^* \wedge_\mu (A) = \wedge_\mu(A) = A$.
g). If $A \in \lambda_\mu O(X, \mu)$ then $A =^* \wedge_\mu(A)$.
h). If A is μ-closed, then $A =^* \wedge_\mu(A)$.

Proof: a). By the definition of $^*\wedge_\mu$, $A \subset^* \wedge_\mu(A)$. Suppose $x \notin \wedge_\mu(A)$. Then there exists $G \in \mu$ such that $A \subset G$ and $x \notin G$. Since every μ-open set is a λ_μ-open set, $G \in \lambda_\mu O(X, \mu)$ such that $A \subset G$ and $x \notin G$ and therefore $x \notin^* \wedge_\mu(A)$ which proves (a).
b). Suppose $x \notin^* \wedge_\mu(B)$. Then there exists $G \in \lambda_\mu O(X, \mu)$ such that $B \subset G$ and $x \notin G$. Since $A \subset B$ and $G \in \lambda_\mu O(X, \mu)$,$A \subset G$ and $x \notin G$ and therefore $x \notin^* \wedge_\mu(A)$ which proves (b).
c). By (a), $A \subset^* \wedge_\mu(A)$ and by (b) $^* \wedge_\mu (A) \subset^* \wedge_\mu(^*\wedge_\mu(A))$. Let $x \notin^* \wedge_\mu(A)$. Then there exists $G \in \lambda_\mu O(X, \mu)$ such that $A \subset G$ and $x \notin G$ which implies that $^* \wedge_\mu (A) \subset G$ and $x \notin G$. Therefore $x \notin^* \wedge_\mu(^*\wedge_\mu(A))$, which implies that $^* \wedge_\mu (^*\wedge_\mu(A)) \subset^* \wedge_\mu(A)$. This completes the proof.
d). Clearly by (a),$\cup\{^*\wedge_\mu(C_i)|i \in I\} \subset^* \wedge_\mu(\cup\{C_i|i \in I)\}$. Conversely, suppose $x \notin \cup\{^*\wedge_\mu(C_i)|i \in I\}$. Then $x \notin^* \wedge_\mu(C_i)$ for every $i \in I$. Therefore for every $i \in I$, there exists $G_i \in \lambda_\mu O(X, \mu)$ such that $C_i \subset G_i$ and $x \notin G_i$. Let $G = \cup\{G_i|i \in I\}$.

Then $x \notin G$ and $\cup\{C_i|i \in I\} \subset G$ which implies that $x \notin^* \wedge_\mu(\cup\{C_i|i \in I\})$. This completes the proof.
e). The proof follows from (b).
f). Let A be μ-open. Then $A = \wedge_\mu(A)$. Hence the proof follows from (a).
g). The proof follows from the definition of $^*\wedge_\mu$.
h). By Obeservation 2.4 (ii) and 2.7 (i) of [9], every μ-colsed set is λ_μ-open and hence the proof is immediate by (g). □

The proof of Theorem 2.3 is similar to that of Theorem 2.2 and hence the proof is omitted.

Theorem 2.3. *Let A, B and $\{C_i| \in I\}$ be subsets of a GTS (X, μ). Then the following hold:*
a). $\vee_\mu(A) \subset^* \vee_\mu(A) \subset A$.
b). If $A \subset B$ then $^* \vee_\mu (A) \subset^* \vee_\mu(B)$.
c). $^* \vee_\mu (^*\vee_\mu(A)) =^* \vee_\mu(A)$.
d). $^* \vee_\mu (\cup\{C_i|i \in I\}) \supset \cup\{^*\vee_\mu(C_i)|i \in I\}$.
e). $^* \vee_\mu (\cap\{C_i|i \in I\}) = \cap\{^*\vee_\mu(C_i)|i \in I\}$.
f). If A is μ-closed then $A = \vee_\mu(A) =^* \vee_\mu(A)$.
g). If $A \in \lambda_\mu C(X, \mu)$ then $A =^* \vee_\mu(A)$.
h). If A is μ-open then $A =^* \vee_\mu(A)$.

Example 2.4 shows that the two sets in (e) of Theorem 2.2 and in (d) of Theorem 2.3 are not equal.

Example 2.4. *Let $X = \{a, b, c\}$ and $\mu = \{\phi, \{a\}\}$. If $A = \{c\}$, $B = \{a, b\}$ then $^* \wedge_\mu (A) = \{b, c\},^* \wedge_\mu(B) = X$ and $^* \wedge_\mu (A \cap B) =^* \wedge_\mu(\phi) = \phi$.*
Since $^* \wedge_\mu (A) \cap^* \wedge_\mu(B) = \{b, c\}$, $^* \wedge_\mu (A) \cap^* \wedge_\mu(B) \neq^* \wedge_\mu(A \cap B)$.
If $A = \{b\}$, $B = \{c\}$ then $^* \vee_\mu (A) =^* \vee_\mu(B) = \phi$ and $^* \vee_\mu (A \cup B) =^* \vee_\mu(\{b, c\}) = \{b, c\}$. Hence $^* \vee_\mu (A \cup B) \neq^* \vee_\mu(A) \cup^* \vee_\mu(B)$.

Example 2.5 shows that the converses of f), g), h) in Theorem 2.2 and g), h) in Theorem 2.3 are not true.

Example 2.5. *Let $X = \{a, b, c, d\}$ and $\mu = \{\phi, \{a, b, \}\{b, c\}, \{a, b, c\}\}$. If $A = \{b\}$ then $A =^* \wedge_\mu(A) = \wedge_\mu(A)$ but A is not μ-open. If $A = \{a\}$ then $A =^* \wedge_\mu(A)$ but A is not λ_μ-open. If $A = \{a, c\}$ then $A =^* \wedge_\mu(A)$ but A is not μ-closed. If $A = \{a, b, d\}$ then $A =^* V_\mu(A)$ but A is neither λ_μ-closed nor μ-open.*

Example 2.6 shows that the converse of Theorem 2.3(f) is not true.

Example 2.6. *Let $X = \{a, b, c, d\}$ and $\mu = \{\phi, \{a, b\}, \{b, c\}, \{a, b, c\}, \{b, c, d\}, X\}$. If $A = \{a, c, d\}$, then $A = \vee_\mu(A) =^* \vee_\mu(A)$ but A is not μ-closed.*

Theorem 2.7. *Let A be a subset of a GTS (X, μ). Then the following hold:*
a). $^* \wedge_\mu (X - A) = X -^* \vee_\mu(A)$.
b). $^* \vee_\mu (X - A) = X -^* \wedge_\mu(A)$.

Proof: It follows from the definitions of ${}^*\wedge_\mu$ and ${}^*\vee_\mu$. □

Definition 2.8. *Let (X, μ) be a GTS. A subset A of X is called a ${}^*\wedge_\mu$-set if $A ={}^* \wedge_\mu(A)$and A is called a ${}^*\vee_\mu$-set if $A ={}^* \vee_\mu(A)$.*

Some properties of ${}^*\wedge_\mu$-sets are given in Theorem 2.9.

Theorem 2.9. *Let (X, μ) be a GTS. Then the following hold:*
a). ϕ, X are ${}^*\wedge_\mu$-sets.
b). A is a ${}^*\wedge_\mu$-set if and only if $X - A$ is a ${}^*\vee_\mu$-set.
c). Any union of ${}^*\wedge_\mu$-sets is again a ${}^*\wedge_\mu$-set.
d). Any intersection of ${}^*\wedge_\mu$-sets is again a ${}^*\wedge_\mu$-set.
e). Any union of ${}^*\vee_\mu$-sets is again a ${}^*\vee_\mu$-set.
f). Any intersection of ${}^*\vee_\mu$-sets is again a ${}^*\vee_\mu$-set.

Proof: a). Since $\phi, X \in \lambda_\mu O(X, \mu)$, the result follows from (g) of Theorem 2.2.
b). Suppose $A$is a ${}^*\wedge_\mu$-set. Then $A ={}^* \wedge_\mu(A)$. Now $X - A = X -{}^* \wedge_\mu(A) =$
${}^* \vee_\mu (X - A)$, by Theorem 2.7(b). Therefore $X - A$ is a ${}^*\vee_\mu$-set.
The converse is similar.
c). Let $\{A_i|i \in I\}$ be a family of ${}^*\wedge_\mu$-sets. Therefore, $A_i ={}^* \wedge_\mu(A_i)$ for every $i \in I$. Now, ${}^* \wedge_\mu (\cup\{A_i|i \in I\}) = \cup\{{}^*\wedge_\mu(A_i)|i \in I\}$, by Theorem 2.2$(d)$ and hence ${}^* \wedge_\mu (\cup\{A_i|i \in I\}) = \cup\{A_i|i \in I\}$.
d). Let $\{A_i|i \in I\}$ be a family of ${}^*\wedge_\mu$-sets. Therefore, $A_i ={}^* \wedge_\mu(A_i)$ for every $i \in I$. Now, $\cap\{A_i|i \in I\} = \cap\{{}^*\wedge_\mu(A_i|i \in I)\} \supset^* \wedge_\mu(\cap\{A_i|i \in I\})$ by Theorem 2.2(e) and therefore ${}^* \wedge_\mu (\cap\{A_i|i \in I\}) = \cap\{A_i|i \in I\}$ by Theorem 2.2(a).
e). Let $\{B_i|i \in I\}$ be a family of ${}^* \vee \mu$-sets. Then $\{X - B_i|i \in I\}$ is a family of ${}^*\wedge_\mu$-sets by (b). Hence $\cap\{X - B_i|i \in I\}$ is a ${}^*\wedge_\mu$-sets by (d). This implies $X - \cup\{B_i|i \in I\}$ is a ${}^*\wedge_\mu$-set. Thus $\cup\{B_i|i \in I\}$ is a ${}^*\vee_\mu$-set.
f). The proof is similar to (e). □

Corollary 2.10. *Let ${}^*\wedge_\mu$=$\{A \subset X|A ={}^* \wedge_\mu(A)\}$and ${}^*\vee_\mu = \{A \subset X|A ={}^* \vee_\mu(A)\}$. Then $(X,{}^* \wedge_\mu)$ and $(X,{}^* \vee_\mu)$ are Alexandroff spaces.*

Proof: By Theorem 2.9, ${}^*\wedge_\mu$ and ${}^*\vee_\mu$ are topologies. Further, arbitrary intersection of ${}^*\wedge_\mu$- open sets (resp. ${}^*\vee_\mu$- open sets) are also ${}^*\wedge_\mu$-open (resp. ${}^*\vee_\mu$- open). Hence $(X,{}^* \wedge_\mu)$ and $(X,{}^* \vee_\mu)$ are Alexandroff spaces. □

Theorem 2.11. *Let (X, μ) be a GTS. Then*
a). Every $\wedge_\mu$-set is a ${}^*\wedge_\mu$-set.
b). Every $\vee_\mu$-set is a ${}^*\vee_\mu$-set.

Proof: a). Let A be a $\wedge_\mu$-set. Then $A = \wedge_\mu(A)$ and therefore, by Theorem 2.2(a), $A ={}^* \wedge_\mu(A)$. Hence A is ${}^* \wedge_\mu (A)$-set.
b). Let A be a $\vee_\mu$-set. Then $A = \vee_\mu(A)$ and therefore, by Theorem 2.3(a), $A ={}^* \vee_\mu(A)$. Hence A is a ${}^*\vee_\mu$-set. □

Example 2.12 shows that the converse of Theorem 2.11 is not true.

Example 2.12. *Let $X = \{a, b, c, d\}$ and $\mu = \{\phi, \{a, b, c\}, \{b, c, d\}, X\}$. If $A = \{a\}$, then A is a $^*\wedge_\mu$-set but it is not a $\wedge_\mu$-set. If $A = \{d\}$, then A is a $^*\vee_\mu$-set but it is not a $\vee_\mu$-set.*

Theorem 2.13. *Let (X, μ) be a GTS. Then the following hold:*
a). For any subset A of X, $^* \wedge_\mu (A)$ is a $^*\wedge_\mu$-set.
b). If A is μ-open, then A is λ_μ-open.
c). If A is λ_μ-open, then A is a $^*\wedge_\mu$-set.
d). If A is μ-closed, then A is a $^*\wedge_\mu$-set.

Proof: Proof follows from Theorem 2.2. □

3. $^*\lambda_\mu$-closed sets

In this section, we define $^*\lambda_\mu$-closed sets and study some of their properties.

Definition 3.1. *A subset A of a GTS (X, μ) is called a $^*\lambda_\mu$-closed set if $A = T \cap C$, where T is a $^*\wedge_\mu$-set and C is λ_μ-closed. The complement of a $^*\lambda_\mu$-closed set is called a $^*\lambda_\mu$-open set.*

We denote the collection of all $^*\lambda_\mu$-open (resp. $^*\lambda_\mu$-closed) sets of X by $^*\lambda_\mu O(X, \mu)$ (resp. $^*\lambda_\mu C(X, \mu)$).
The following theorem gives characterizations of $^*\lambda_\mu$-closed sets.

Theorem 3.2. *Let A be a subset of a GTS (X, μ). Then the following are equivalent.*
a). A is $^*\lambda_\mu$-closed.
b). $A = T \cap c_{\lambda_\mu}(A)$, where T is a $^*\wedge_\mu$-set.
c). $A =^* \wedge_\mu(A) \cap c_{\lambda_\mu}(A)$.

Proof: a) $\Rightarrow$ b). Let $A = T \cap F$, where T is a $^*\wedge_\mu$-set and F is a λ_μ-closed set in X. Since $A \subset F$, $c_{\lambda_\mu}(A) \subset c_{\lambda_\mu}(F) = F$. Thus $A = T \cap F \supset T \cap c_{\lambda_\mu}(A) \supset A$. Therefore, we have $A = T \cap c_{\lambda_\mu}(A)$.
b) $\Rightarrow$ c). Let $A = T \cap c_{\lambda_\mu}(A)$, where T is a $^*\wedge_\mu$-set. Since $A \subset T$, we have $^* \wedge_\mu (A) \subset^* \wedge_\mu(T) = T$. Hence $A \subset^* \wedge_\mu(A) \cap c_{\lambda_\mu}(A) \subset T \cap c_{\lambda_\mu}(A) = A$. Therefore A $= {}^* \wedge_\mu (A) \cap c_{\lambda_\mu}(A)$.
c) $\Rightarrow$ a). Since $^* \wedge_\mu (A)$ is a $^*\wedge_\mu$-set and $c_{\lambda_\mu}(A)$ is a λ_μ-closed set, A is $^*\lambda_\mu$-closed set. □

Theorem 3.3. *For a GTS (X, μ), the following properties hold:*
a). λ_μ-open sets imply $^*\wedge_\mu$-sets and are implied by μ-open sets.
b). $\wedge_\mu$-sets imply $^*\wedge_\mu$-sets and are implied by μ-open sets.
c). $^*\wedge_\mu$-sets imply $^*\lambda_\mu$-closed sets and are implied by $\wedge_\mu$-sets.
d). λ_μ-closed sets imply $^*\lambda_\mu$-closed sets and are implied by $\wedge_\mu$-sets.

Proof: a). The proof follows immediately from Theorem 2.13.
b). This is an immediate consequence of Theorem 2.11.
c). This is obvious by Definition 3.1 and Theorem 2.11.
d). This is obvious by Definitions 1.3 and 3.1. □

Remark 3.4. *For a subset of a GTS (X, μ), the following implications hold:*

$$\begin{array}{ccccc} & & \mu\text{-open} & \Rightarrow & \lambda_\mu\text{-open} \\ & & \Downarrow & & \Downarrow \\ & & \wedge_\mu\text{-set} & \Rightarrow & {}^*\wedge_\mu\text{-set} \\ & & \Downarrow & & \Downarrow \\ \mu\text{-closed} & \Rightarrow & \lambda_\mu\text{-closed} & \Rightarrow & {}^*\lambda_\mu\text{-closed} \end{array}$$

The converse implications are not always true by Examples 2.5, 2.12, 3.5 and also [11, Example 2.8].

Example 3.5. *Let $X = \{a, b, c, d\}$ and $\mu = \{\phi, \{c\}, \{a, b, c\}, \{b, c, d\}, X\}$. If $A = \{b\}$ then, A is a ${}^*\lambda_\mu$-closed set but it is neither ${}^*\wedge_\mu$-set nor $\wedge_\mu$-set.*
If $A = \{a, c\}$ then A is a ${}^*\lambda_\mu$-closed set but it is neither λ_μ-closed nor λ_μ-open.
If $A = \{a, d\}$ then A is a ${}^*\lambda_\mu$-closed set but it is neither μ-closed nor μ-open.

Theorem 3.6. *For a GTS (X, μ), the following hold:*
a). Arbitrary intersection of ${}^*\lambda_\mu$-closed sets is a ${}^*\lambda_\mu$-closed set.
b). Arbitrary union ${}^*\lambda_\mu$-open sets is a ${}^*\lambda_\mu$-open set.

Proof: a). Let $\{A_i | i \in I\}$ be a collection of ${}^*\lambda_\mu$-closed sets. Then $A_i = T_i \cap C_i$, where T_i is a ${}^*\wedge_\mu$-set and C_i is a λ_μ-closed set. Also $\cap A_i = (\cap T_i) \cap (\cap C_i)$, where $\cap T_i$ is a ${}^*\wedge_\mu$-set and $\cap C_i$ is a λ_μ-closed set. Therefore $\cap A_i$ is a ${}^*\lambda_\mu$-closed set.
b). Let $\{B_i | i \in I\}$ be a collection of ${}^*\lambda_\mu$-open sets. Then $\{X - B_i\}$ is a collection of ${}^*\lambda_\mu$-closed sets. By (a), $\cap\{X - B_i\}$ is a ${}^*\lambda_\mu$-closed set. That is $X - \cup B_i$ is a ${}^*\lambda_\mu$-closed set. Therefore $\cup B_i$ is a ${}^*\lambda_\mu$-open set. □

4. Applications

In this section we define and discuss some new separation axioms.

Definition 4.1. [11]. *Let (X, μ) be a GTS. A subset $A \subseteq X$ is called a $\lambda_\mu - D$-set if there are two λ_μ-open sets U, V in X such that $U \neq X$ and $A = U - V$.*

It is obvious that every λ_μ-open set $U \neq X$ is a $\lambda_\mu - D$-set since $U = U - \phi$.

Definition 4.2. [11]. *A GTS (X, μ) is said to be*
i).$\lambda_\mu - D_0$ if for $x, y \in X$ such that $x \neq y$ there exists a $\lambda_\mu - D$-set containing one but not the other.
ii).$\lambda_\mu - D_1$ if for $x, y \in X$ such that $x \neq y$ there exists a $\lambda_\mu - D$-set containing x but not y and a $\lambda_\mu - D$-set containing y but not x.
iii).$\lambda_\mu - D_2$ if for $x, y \in X$ such that $x \neq y$ there exists disjoint $\lambda_\mu - D$-sets (resp.λ_μ-open sets) U and V such that $x \in U$ and $y \in V$.

Definition 4.3. *A GTS* (X, μ) *is said to be*
i).$\lambda_\mu - T_0$ *if for* $x, y \in X$ *such that* $x \neq y$ *there exists a* λ_μ*-open set containing one but not the other.*
ii).$\lambda_\mu - T_1$ *if for* $x, y \in X$ *such that* $x \neq y$ *there exists a* λ_μ*-open set containing* x *but not* y *and a* λ_μ*-open set containing* y *but not* x*.*
iii).$\lambda_\mu - T_2$ *if for* $x, y \in X$ *such that* $x \neq y$ *there exists disjoint* λ_μ*-open sets* U *and* V *such that* $x \in U$ *and* $y \in V$*.*
iv) $\lambda_\mu - T_{1/2}$ *if every singleton is* λ_μ*-open or* λ_μ*-closed.*

Remark 4.4. *For a GTS* (X, μ)*,the following hold.*
i). If X is $\lambda_\mu - T_i$*, then it is* $\lambda_\mu - T_{i-1}$*,* $i = 1, 2$*.*
ii). If X is $\lambda_\mu - T_i$*, then it is* $\lambda_\mu - D_i$*,* $i = 0, 1, 2$*.*

Theorem 4.5. *For a GTS* (X, μ)*, the following statements are equivalent.*
i). X is $\lambda_\mu - D_0$*.*
ii).X is $\lambda_\mu - T_0$*.*
iii). Every singleton is ${}^*\lambda_\mu$*-closed.*

Proof: *ii)* $\Rightarrow$ *i)* It follows from Remark 4.4(*ii*).
i) $\Rightarrow$ *ii)* Let (X, μ) be $\lambda_\mu - D_0$. Then for any distinct pair of points x and y of X at least one belongs to a $\lambda_\mu - D$-set O. Therefore we choose $x \in O$ and $y \notin O$. Suppose $O = U - V$ for which $U \neq X$ and U and V are λ_μ-open sets in X. This implies that $x \in U$. For the case that $y \notin O$ we have (i) $y \notin U$, (ii) $y \in U$ and $y \in V$. For (i), the space X is $\lambda_\mu - T_0$ since $x \in U$ and $y \notin U$. For (ii), the space X is also $\lambda_\mu - T_0$ since $y \in V$ but $x \notin V$.
ii) $\Rightarrow$ *iii)* Let $x \in X$. Since X is $\lambda_\mu - T_0$, then for every point $x \neq y$ there exists a set A_y containing x and is disjoint form $\{y\}$ such that A_y is either λ_μ-open or λ_μ-closed. Let L be the intersection of all λ_μ-open sets A_y and F be the intersection of all λ_μ-closed sets A_y. Clearly L is a ${}^*\wedge_\mu$-set and F is λ_μ-closed. Note that $\{x\} = L \cap F$. This shows that $\{x\}$ is ${}^*\lambda_\mu$-closed.
iii) $\Rightarrow$ *ii)* Let x and y be two different points of X. Then by (iii), $\{x\} = L \cap F$, where L is a ${}^*\wedge_\mu$-set and F is λ_μ-closed. If F does not contain y, then $X - F$ is a λ_μ-open set containing y and we are done. If F contains y, then $y \notin L$ and thus for some λ_μ-open set U containing L, we have $y \notin U$. Hence X is $\lambda_\mu - T_0$. □

Theorem 4.6. *Let* (X, μ) *be a GTS. Then X is* $\lambda_\mu - T_1$ *iff every subset of X is a* ${}^*\wedge_\mu$*- set.*

Proof: Suppose (X, μ) is a $\lambda_\mu - T_1$. Let A be a subset of X. By Theorem 2.2(a), $A \subset^* \wedge_\mu(A)$. Suppose $x \notin A$. Then $X - \{x\}$ is a λ_μ-open set such that $A \subset X - \{x\}$ and so ${}^* \wedge_\mu (A) \subset X - \{x\}$. Therefore, $x \notin^* \wedge_\mu(A)$ and hence $\wedge_\mu(A) \subset A$. Hence every subset of X is a ${}^*\wedge_\mu$-set. Conversely, suppose every subset of X is a ${}^*\wedge_\mu$-set and so ${}^* \wedge_\mu (\{x\}) = \{x\}$ for every $x \in X$. Let x,y$\in$ X such that $x \neq y$. Then $y \notin^* \wedge_\mu(\{x\})$ and $x \notin^* \wedge_\mu(\{y\})$. Since $y \notin^* \wedge_\mu(\{x\})$,there is a λ_μ-open set U such that $x \in U$ and $y \notin U$. Similarly, since $x \notin^* \wedge_\mu(\{y\})$,there is a λ_μ-open set V such that $y \in V$ and $x \notin V$. Therefore, X is a $\lambda_\mu - T_1$ space. □

Corollary 4.7. *For a GTS (X, μ), the following are equivalent.*
i). X is $\lambda_\mu - T_1$.
ii). Every subset of X is a $^*\wedge_\mu$- set.
iii). Every subset of X is a $^*\vee_\mu$- set.

Proof: (i) and (ii) are equivalent by Theorem 4.6.
(ii) and (iii) are equivalent by Theorem 2.9(b). □

Theorem 4.8. *If (X, μ) is $\lambda_\mu - D_1$, then it is $\lambda_\mu - T_0$.*

Proof: Proof follows from Remark 3.3(ii) of [11] and Theorem 4.4. □

Theorem 4.9. *For a GTS (X, μ), the following conditions are equivalent:*
(a) X is a $\lambda_\mu - T_{1/2}$.
(b) Every subset of X is $^*\lambda_\mu$-closed.

Proof: $(a) \Rightarrow (b)$: Let $A \subseteq X$. Let A_1 be the set of all $\lambda_\mu-$open singetons of $X - A$ and $A_2 = X - (A_1 \cup A)$. Set $F = \bigcap_{x\in A_1}(X - \{x\})$ and $L = \bigcap_{x\in A_2}(X - \{x\})$. Note that F is $\lambda_\mu-$closed and L is a $^* \wedge_\mu$ $-$set. Moreover, $A = F \cap L$. Thus A is $^*\lambda_\mu-$closed.
$(b) \Rightarrow (a)$: Let $x \in X$. Assume that $\{x\}$ is not $\lambda_\mu-$open. Then $A = X - \{x\}$ is not $\lambda_\mu-$closed. Since A is $^*\lambda_\mu$-closed, $A = T \cap F$, where T is a $^*\wedge_\mu$- set and F is λ_μ- closed. Then the only possibility for $F = X$ and $T = X - \{x\}$, then A is a $^*\wedge_\mu$- set, i.e., $A =^* \wedge_\mu(A)$. Since X is the only superset of A, then A is λ_μ-open. Hence $\{x\}$ is $^*\lambda_\mu$-closed. □

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