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# Almost Slightly $\nu g-$open and Almost Slightly $\nu g-$closed Mappings

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ABSTRACT: The aim of this paper is to introduce and study the concepts of almost slightly $\nu g$-open and almost slightly $\nu g$-closed mappings and the interrelationship between other such maps.

Key Words: $\nu g$-open set, $\nu g$-open map, $\nu g$-closed map, Aalmost slightly-closed map, Almost slightly-pre closed map, Almost slightly $\nu g$-open, Almost slightly $\nu g$-closed map, Almost slightly $\nu g$-open and Almost slightly $\nu g$-closed map.

## Contents



## 1. Introduction

T.M.Nour introduced slightly semi-continuous functions during the year 1995. After him T.Noiri and G.I.Chae further studied slightly semi-continuous functions on 2000. During 2001 T.Noiri individually studied slightly $\beta-$continuous functions. C.W.Baker introduced slightly precontinuous functions. Erdal Ekici and M. Caldas studied slightly $\gamma-$continuous functions. Arse Nagli Uresin and others studied slightly $\delta-$precontinuous functions. The Author of the present paper studied slightly $\nu-$continuous functions, Almost Slightly Continuity, Slightly open and Slightly closed mappings, Almost Slightly semi-Continuity, Slightly semi-open and Slightly semi-closed mappings, Almost Slightly pre-continuity, Slightly pre-open and Slightly pre-closed mappings in the year 2013. S. Balasubramanian, C. Sandhya and P.A.S. Vyjayanthi studied Slightly $\nu$-open mappings in the year 2013. S. Balasubramanian and C. Sandhya studied Almost Slightly $\beta$-continuity, Slightly $\beta$-open and Slightly $\beta$-closed mappings in the year 2013. Recently in the year 2014 S. Balasubramanian, P.A.S. Vyjaanthi and C. Sandhya studied Slightly $\nu$-closed mappings. Inspired with these developements we introduce in this paper a new variety of slightly open and closed functions called slightly $\nu g-$open and slightly $\nu g-$closed function and study its basic properties; interrelation with other type of such functions available in the literature. Throughout the paper a space X means a topological space (X,$\tau$).

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## 2. Preliminaries

**Definition 2.1.** *$A \subset X$ is said to be*
*(i)$\nu$-open [resp: regular $\alpha-$open or r$\alpha-$open] if there exists a regular open set $O$ such that $O \subset A \subset \overline{(O)}$ [resp: $O \subset A \subset \alpha\overline{(O)}$].*
*(ii)Regular open [resp: semi-open, $\alpha-$open; pre-open; $\beta-$open] if $A = (\overline{A})^o$ [resp: $A \subseteq \overline{(A^o)}$; $A \subseteq ((\overline{A^o}))^o$; $A \subseteq (\overline{A})^o$; $A \subseteq \overline{(\overline{A})^o}$].*
*(iii)Regular closed [resp: semi-closed; $\alpha-$closed; pre-closed; $\beta-$closed] if $A = \overline{A^o}$[resp: $(\overline{A})^o \subseteq A$; $((\overline{A^o}))^o \subseteq A$; $\overline{(A^o)} \subseteq A$; $\overline{(\overline{A})^o} \subseteq A$].*
*(iv) resp:$\nu$-closed if its complement is $\nu-$open.*
*(v)g-closed [resp: rg-closed] if $\overline{A} \subseteq U$ whenever $A \subseteq U$ and $U$ is [resp: regular] open in $X$.*
*(vi)sg-closed [gs-closed] if $s(\overline{A}) \subseteq U$ whenever $A \subseteq U$ and $U$ is semi-open [open] in $X$.*
*(vii) pg-closed [gp-closed; gpr-closed] if $p(\overline{A}) \subseteq U$ whenever $A \subseteq U$ and $U$ is pre-open [open; regular-open] in $X$.*
*(viii) $\alpha g$-closed [g$\alpha-$closed; rg$\alpha$-closed] if $\alpha(\overline{A}) \subseteq U$ whenever $A \subseteq U$ and $U$ is$\{\alpha-$open [open r$\alpha-$open] in $X$.*
*(ix)$\nu g$-closed if $\nu(\overline{A}) \subseteq U$ whenever $A \subseteq U$ and $U$ is $\nu-$open in $X$.*
*(x)$\beta g$-closed if $\beta(\overline{A}) \subseteq U$ whenever $A \subseteq U$ and $U$ is $\beta-$open in $X$.*

**Definition 2.2.** *A function $f: X \to Y$ is said to be*
*(i) continuous[resp: semi-continuous; pre-continuous; nearly-continuous; $\nu-$continuous; $\alpha-$continuous; r$\alpha-$continuous; $\beta-$continuous] if the inverse image of every open set is open[resp: semi-open; pre-open; rgular-open; $\nu-$open; $\alpha-$open; r$\alpha-$open; $\beta-$open]*
*(ii) irresolute [resp:pre-irresolute; nearly-irresolute; $\nu-$irresolute; $\alpha-$irresolute; r$\alpha-$irresolute; $\beta-$irresolute] if the inverse image of every semi-open [resp:pre-open; regular-open; $\nu-$open; $\alpha-$open; r$\alpha-$open; $\beta-$open] set is semi-open[resp:pre-open; regular-open; $\nu-$open; $\alpha-$open; r$\alpha-$open; $\beta-$open]*
*(iii) g-continuous [resp: sg-continuous; pg-continuous; rg-continuous; $\nu g-$continuous; $\alpha g-$continuous; rg$\alpha-$continuous; $\beta g-$continuous] if the inverse image of every closed set is g-closed [resp: sg-closed; pg-closed; rg-closed; $\nu g-$closed; $\alpha g-$closed; rg$\alpha-$closed; $\beta g-$closed]*
*(iv) g-irresolute [resp: sg-irresolute; pg-irresolute; rg-irresolute; $\nu g-$irresolute; $\alpha g-$irresolute; rg$\alpha-$irresolute; $\beta g-$irresolute] if the image of every g-closed [resp: sg-closed; pg-closed; rg-closed; $\nu g-$closed; $\alpha g-$closed; rg$\alpha-$closed; $\beta g-$closed] set is g-closed [resp: sg-closed; pg-closed; rg-closed; $\nu g-$closed; $\alpha g-$closed; rg$\alpha-$closed; $\beta g-$closed]*

**Note 1.** *From the definition 2.1 we have the following implication diagram.*

*g-closed* *gs-closed*
$\downarrow$ $\downarrow$ $\nwarrow$
$rg\alpha-closed \rightarrow rg\text{-}closed \rightarrow \nu g-$**closed** $\leftarrow sg\text{-}closed \leftarrow \beta g\text{-}closed$
$\uparrow$ $\uparrow$ $\uparrow$ $\uparrow$ $\uparrow$
$\nearrow r\alpha-closed \rightarrow \nu-closed \searrow$ $\uparrow$ $\uparrow$
$Regular\ closed \rightarrow \pi-closed \rightarrow closed \rightarrow \alpha-closed \rightarrow semi\ closed \rightarrow \beta-closed$
$\swarrow$ $\downarrow \searrow$ $\searrow$
$\pi g\text{-}closed$ $pre\text{-}closed \rightarrow \omega-closed \not\leftrightarrow g\alpha-closed$
$\searrow$ $\searrow$
$gpr\text{-}closed \leftarrow gp\text{-}closed \leftarrow pg\text{-}closed$ $r\omega-closed$

**Definition 2.3.** *$X$ is said to be $T_{\frac{1}{2}}$[resp: $s-T_{\frac{1}{2}}$; $p-T_{\frac{1}{2}}$; $\alpha-T_{\frac{1}{2}}$; $r-T_{\frac{1}{2}}$; $\nu-T_{\frac{1}{2}}$] if every generalized[resp: semi-generalized; pre-generalized; $\alpha-$generalized; regular-generalized; $\nu-$generalized] closed set is closed[resp: semi-closed; pre-closed; $\alpha-$closed; regular-closed; $\nu$-closed]*

## 3. Almost Slightly $\nu$g-open mappings

**Definition 3.1.** *A function $f\colon X \rightarrow Y$ is said to be almost slightly $\nu g$-open if the image of every r-clopen set in $X$ is $\nu g$-open in $Y$.*

**Example 3.2.** *Let $X = Y = \{a,b,c\}$; $\tau = \{\phi,\{a\},\{b,c\},X\}$; $\sigma = \{\phi,\{a\},\{b\},\{a,b\},Y\}$. Let $f : X \rightarrow Y$ be defined f(a) = c, f(b) = a and f(c) = b. Then f is almost slightly $\nu g$-open, almost slightly rg-open and almost slightly rg$\alpha$-open but not almost slightly open, almost slightly semi-open, almost slightly pre-open, almost slightly $\alpha$-open, almost slightly r$\alpha$-open, almost slightly $\nu$-open, almost slightly $\pi$-open, almost slightly $\beta$-open, almost slightly g-open, almost slightly sg-open, almost slightly gs-open, almost slightly pg-open, almost slightly gp-open and almost slightly $\beta g$-open.*

**Example 3.3.** *Let $X = Y = \{a,b,c,d\}$; $\tau = \{\phi,\{a,b\},\{c,d\},X\}$; $\sigma = \{\phi,\{a\},\{b\},\{d\},\{a,b\},\{a,d\},\{b,d\},\{a,b,c\},\{a,b,d\},X\}$. Let $f : X \rightarrow Y$ be defined f(a) = b, f(b) = a, f(c) = d and f(d) = c. Then f is not almost slightly $\nu g$-open.*

**Theorem 3.4.** *We have the following interrelation among the following almost slightly open mappings*

(i) al.sl.g.open al.sl.gs.open

$\downarrow$ $\downarrow$ $\nwarrow$

al.sl.rg$\alpha$.open $\rightarrow$ al.sl.rg.open $\rightarrow$ **al.sl.$\nu g$.open** $\leftarrow$ al.sl.sg.open $\leftarrow$ al.sl.$\beta$g.open

$\uparrow$ $\uparrow$ $\uparrow$ $\uparrow$ $\uparrow$

$\nearrow$ al.sl.$r\alpha$.open $\rightarrow$ al.sl.$\nu$.open $\searrow$ $\uparrow$ $\uparrow$

al.sl.r.open $\rightarrow$ al.sl.$\pi$.open $\rightarrow$ al.sl.open $\rightarrow$ al.sl.$\alpha$.open $\rightarrow$ al.sl.s.open $\rightarrow$ al.sl.$\beta$.open

$\swarrow$ $\downarrow\searrow$ $\searrow$

al.sl.$\pi$g.open al.sl.p.open $\rightarrow$ al.sl.$\omega$.open $\not\leftrightarrow$ al.sl.g$\alpha$.open

$\searrow$ $\searrow$

al.sl.gpr.open $\leftarrow$ al.sl.gp.open $\leftarrow$ al.sl.pg.open al.sl.r$\omega$.open None is reversible.

(ii) sl.g.open sl.gs.open

$\downarrow$ $\downarrow$ $\nwarrow$

sl.rg$\alpha$.open $\rightarrow$ sl.rg.open $\rightarrow$ **al.sl.$\nu g$.open** $\leftarrow$ sl.sg.open $\leftarrow$ sl.$\beta$g.open

$\uparrow$ $\uparrow$ $\uparrow$ $\uparrow$ $\uparrow$

$\nearrow$ sl.$r\alpha$.open $\rightarrow$ sl.$\nu$.open $\searrow$ $\uparrow$ $\uparrow$

sl.r.open $\rightarrow$ sl.$\pi$.open $\rightarrow$ sl.open $\rightarrow$ sl.$\alpha$.open $\rightarrow$ sl.s.open $\rightarrow$ sl.$\beta$.open

$\swarrow$ $\downarrow\searrow$ $\searrow$

sl.$\pi$g.open sl.p.open $\rightarrow$ sl.$\omega$.open $\not\leftrightarrow$ sl.g$\alpha$.open

$\searrow$ $\searrow$

sl.gpr.open $\leftarrow$ sl.gp.open $\leftarrow$ sl.pg.open sl.r$\omega$.open None is reversible.

(iii) If $\nu$GO(Y) = RO(Y), then the reverse relations hold for all almost slightly open maps.

al.sl.g.open al.sl.gs.open

$\updownarrow$ $\updownarrow$ $\nearrow\nwarrow$

al.sl.rg$\alpha$.open $\leftrightarrow$ al.sl.rg.open $\leftrightarrow$ **al.sl.$\nu g$.open** $\leftrightarrow$ al.sl.sg.open $\leftrightarrow$ al.sl.$\beta$g.open

$\updownarrow$ $\updownarrow$ $\updownarrow$ $\updownarrow$ $\updownarrow$

$\nwarrow\nearrow$ al.sl.$r\alpha$.open $\leftrightarrow$ al.sl.$\nu$.open $\swarrow\searrow$ $\updownarrow$ $\updownarrow$

al.sl.r.open $\leftrightarrow$ al.sl.$\pi$.open $\leftrightarrow$ al.sl.open $\leftrightarrow$ al.sl.$\alpha$.open $\leftrightarrow$ al.sl.s.open $\leftrightarrow$ al.sl.$\beta$.open

**Theorem 3.5.** *(i) If $(Y, \sigma)$ is discrete, then f is almost slightly open of all types. (ii) If f is almost slightly open and g is $\nu g$-open then $g \circ f$ is almost slightly $\nu g$-open. (iii)If f is open and g is almost slightly $\nu g$-open then $g \circ f$ is almost slightly $\nu g$-open.*

**Corollary 3.6.** *If f is almost slightly open and g is $g - [rg-; sg-; gs-; \beta g-; r\alpha g-; rg\alpha-; r\alpha-; \alpha-; s-; p-; \beta-; \nu-; \pi-; r-]$ open then $g \circ f$ is almost slightly $\nu g$-open.*

**Corollary 3.7.** *If f is open[r-open] and g is $al - sl - g - [al - sl - rg-; al - sl - sg-; al - sl - gs-; al - sl - \beta g-; al - sl - r\alpha g-; al - sl - rg\alpha-; al - sl - r\alpha-; al - sl - \alpha-; al - sl - s-; al - sl - p-; al - sl - \beta-; al - sl - \pi-]$ open then $g \circ f$ is almost slightly $\nu g$-open.*

**Theorem 3.8.** *If $f\colon X \to Y$ is almost slightly $\nu g$-open, then $f(A^o) \subset \nu g(f(A))^o$*

*Proof.* Let $A \subseteq X$ be r-clopen and $f\colon X \to Y$ is almost slightly $\nu g$-open gives $f(A^o)$ is $\nu g$-open in Y and $f(A^o) \subset f(A)$ which in turn gives $\nu g(f(A^o))^o \subset \nu g(f(A))^o$ - - -

(1) Since $f(A^o)$ is $\nu g$-open in Y, $\nu g(f(A^o))^o = f(A^o)$ - - - - - - - - - - -(2) combining (1) and (2) we have $f(A^o) \subset \nu g(f(A))^o$ for every subset A of X. □

**Remark 3.9.** *Converse is not true in general.*

**Corollary 3.10.** *If* $f: X \to Y$ *is* $al-sl-g-[al-sl-rg-; al-sl-sg-; al-sl-gs-; al-sl-\beta g-; al-sl-r\alpha g-; al-sl-rg\alpha-; al-sl-r\alpha-; al-sl-\alpha-; al-sl-s-; al-sl-p-; al-sl-\beta-; al-sl-\pi-]$ *open, then* $f(A^o) \subset \nu g(f(A))^o$

**Theorem 3.11.** *If* $f: X \to Y$ *is almost slightly* $\nu g$*-open and* $A \subseteq X$ *is open, f(A) is* $\tau_{\nu g}$*-open in Y.*

*Proof.* Let $A \subset X$ be r-clopen and $f: X \to Y$ is almost slightly $\nu g$-open $\Rightarrow$ $f(A^o) \subset \nu g(f(A))^o \Rightarrow f(A) \subset \nu g(f(A))^o$, since $f(A) = f(A^o)$. But $\nu g(f(A))^o \subset f(A)$. Combining we get $f(A) = \nu g(f(A))^o$. Hence $f$(A) is $\nu g$-open in Y.
□

**Corollary 3.12.** *If* $f: X \to Y$ *is* $al-sl-g-[al-sl-rg-; al-sl-sg-; al-sl-gs-; al-sl-\beta g-; al-sl-r\alpha g-; al-sl-rg\alpha-; al-sl-r-; al-sl-r\alpha-; al-sl-\alpha-; al-sl-s-; al-sl-p-; al-sl-\beta-; al-sl-\nu-; al-sl-\pi-]$*open, then f(A) is* $\tau_{\nu g}$ *open in Y if A is open set in X.*

**Theorem 3.13.** *If* $\nu g(A)^o = r(A)^o$ *for every* $A \subset Y$*, then the following are equivalent:*
*a)* $f: X \to Y$ *is almost slightly* $\nu g$*-open map*
*b)* $f(A^o) \subset \nu g(f(A))^o$

*Proof.* (a) ⇒(b) follows from Theorem 3.8.
(b) ⇒ (a) Let A be any r-clopen set in X, then $f(A) = f(A^o) \subset \nu g(f(A))^o$ by hypothesis. We have $f(A) \subset \nu g(f(A))^o$, which implies $f$(A) is $\nu g$-open. Therefore $f$ is almost slightly $\nu g$-open.
□

**Theorem 3.14.** *If* $\nu(A)^o = r(A)^o$ *for every* $A \subset Y$*, then the following are equivalent:*
*a)* $f: X \to Y$ *is almost slightly* $\nu g$*-open map*
*b)* $f(A^o) \subset \nu g(f(A))^o$

*Proof.* (a) ⇒(b) follows from Theorem 3.8.
(b) ⇒ (a) Let A be any r-clopen set in X, then $f(A) = f(A^o) \subset \nu g(f(A))^o$ by hypothesis. We have $f(A) \subset \nu g(f(A))^o$, which implies $f$(A) is $\nu g$-open. Therefore $f$

is almost slightly $\nu g$-open.
□

**Theorem 3.15.** *$f : X \to Y$ is almost slightly $\nu g$-open iff for each subset $S$ of $Y$ and each r-clopen set $U$ containing $f^{-1}(S)$, there is an $\nu g$-open set $V$ of $Y$ such that $S \subseteq V$ and $f^{-1}(V) \subseteq U$.*

*Proof.* Assume $f : X \to Y$ is almost slightly $\nu g$-open. Let $S \subseteq Y$ and U be r-clopen set U containing $f^{-1}(S)$. Then X-U is r-clopen in X and $f$(X-U) is $\nu g$-open in Y as $f$ is almost slightly $\nu g$-open and V = Y - $f$(X-U) is $\nu g$-open in Y. $f^{-1}(S) \subseteq U \Rightarrow S \subseteq f(U) \Rightarrow S \subseteq V$ and $f^{-1}(V) = f^{-1}(Y - f(X - U)) = f^{-1}(Y) - f^{-1}(f(X - U)) = f^{-1}(Y) - (X - U) = X - (X - U) = U$.

Conversely Let F be r-clopen in X $\Rightarrow F^c$ is r-clopen. Then $f^{-1}(f(F^c)) \subseteq F^c$. By hypothesis there exists a $\nu g$-open set V of Y, such that $f(F^c) \subseteq V$ and $f^{-1}(V) \supset F^c$ and so $F \subseteq [f^{-1}(V)]^c$. Hence $V^c \subseteq f(F) \subseteq f[f^{-1}(V)^c] \subseteq V^c \Rightarrow f(F) \subseteq V^c \Rightarrow f(F) = V^c$. Thus $f(F)$ is $\nu g$-open in Y. Therefore $f$ is almost slightly $\nu g$-open.
□

**Remark 3.16.** *Composition of two almost slightly $\nu g$-open maps is not almost slightly $\nu g$-open in general.*

**Theorem 3.17.** *Let X, Y, Z be topological spaces and every $\nu g$-open set is r-clopen in Y. Then the composition of two almost slightly $\nu g$-open maps is almost slightly $\nu g$-open.*

*Proof.* (a) Let $f$ and $g$ be almost slightly $\nu g$-open maps. Let A be any r-clopen set in X $\Rightarrow f$(A) is r-clopen in Y (by assumption) $\Rightarrow g(f(A)) = g \circ f(A)$ is $\nu g$-open in Z. Therefore $g \circ f$ is almost slightly $\nu g$-open.
□

**Corollary 3.18.** *Let X, Y, Z be topological spaces and every $g - [rg-; sg-; gs-; \beta g-; r\alpha g-; rg\alpha-; r\alpha-; \alpha-; s-; p-; \beta-; \pi-]$open set is r-clopen [r-clopen] in Y. Then the composition of two $al - sl - g - [al - sl - rg-; al - sl - sg-; al - sl - gs-; al - sl - \beta g-; al - sl - r\alpha g-; al - sl - rg\alpha-; al - sl - r\alpha-; al - sl - \alpha-; al - sl - s-; al - sl - p-; al - sl - \beta-; al - sl - \nu-; al - sl - \pi-; al - sl - r-]$ open maps is almost slightly $\nu g$-open.*

**Example 3.19.** *Let $X = Y = Z = \{a, b, c\}$; $\tau = \{\phi, \{a\}, \{a, b\}, X\}$; $\sigma = \{\phi, \{a, c\}, Y\}$ and $\eta = \{\phi, \{a\}, \{b, c\}, Z\}$. $f : X \to Y$ be defined $f(a) = c, f(b) = b$ and $f(c) = a$ and $g : Y \to Z$ be defined $g(a) = b, g(b) = a$ and $g(c) = c$, then $g$, $f$ and $g \circ f$ are almost slightly $\nu g$-open.*

**Theorem 3.20.** *If $f : X \to Y$ is almost slightly g-open[almost slightly rg-open], $g : Y \to Z$ is $\nu g$-open and Y is $T_{\frac{1}{2}}[r - T_{\frac{1}{2}}]$ then $g \circ f$ is almost slightly $\nu g$-open.*

*Proof.* (a) Let A be r-clopen in X. Then $f$(A) is g-open and so open in Y as Y is $T_{\frac{1}{2}} \Rightarrow g(f\ (A)) = g \circ f\,(A)$ is $\nu g$-open in Z (since $g$ is $\nu g$-open). Hence $g \circ f$ is almost slightly $\nu g$-open.
□

**Corollary 3.21.** *If $f : X \to Y$ is almost slightly g-open [almost slightly rg-open], $g : Y \to Z$ is $g - [rg-; sg-; gs-; \beta g-; r\alpha g-; rg\alpha-; r\alpha-; \alpha-; s-; p-; \beta-; \nu-; \pi-; r-]$ and Y is $T_{\frac{1}{2}}[r - T_{\frac{1}{2}}]$ then $g \circ f$ is almost slightly $\nu g$-open.*

**Theorem 3.22.** *If $f : X \to Y$ is g-open[rg-open], $g : Y \to Z$ is almost slightly $\nu g$-open and Y is $T_{\frac{1}{2}}[r - T_{\frac{1}{2}}]$ then $g \circ f$ is almost slightly $\nu g$-open.*

*Proof.* (a) Let A be r-clopen in X. Then $f$(A) is g-open and so open in Y as Y is $T_{\frac{1}{2}} \Rightarrow g(f\ (A)) = g \circ f\,(A)$ is $\nu g$-open in Z (since $g$ is almost slightly $\nu g$-open). Hence $g \circ f$ is almost slightly $\nu g$-open.
□

**Corollary 3.23.** *If $f : X \to Y$ is g-open[rg-open], $g : Y \to Z$ is $al - sl - g - [al - sl - rg-; al - sl - sg-; al - sl - gs-; al - sl - \beta g-; al - sl - r\alpha g-; al - sl - rg\alpha-; al - sl - r\alpha-; al - sl - \alpha-; al - sl - s-; al - sl - p-; al - sl - \beta-; al - sl - \pi-]$ open and Y is $T_{\frac{1}{2}}[rT_{\frac{1}{2}}]$], then $g \circ f$ is almost slightly $\nu g$-open.*

**Theorem 3.24.** *If $f : X \to Y$, $g : Y \to Z$ be two mappings such that $g \circ f$ is almost slightly $\nu g$-open then the following statements are true.*
*a) If f is continuous [r-continuous] and surjective then g is almost slightly $\nu g$-open.*
*b) If f is g-continuous[resp: rg-continuous], surjective and X is $T_{\frac{1}{2}}$ [resp: $rT_{\frac{1}{2}}$]] then g is almost slightly $\nu g$-open.*

*Proof.* For A r-clopen in Y, $f^{-1}(A)$ open in X $\Rightarrow (g \circ f)(f^{-1}(A)) = g(A)$ $\nu g$-open in Z. Hence $g$ is almost slightly $\nu g$-open.

Similarly one can prove the remaining parts and hence omitted.
□

**Corollary 3.25.** *If $f : X \to Y$, $g : Y \to Z$ be two mappings such that $g \circ f$ is $al - sl - g - [al - sl - rg-; al - sl - sg-; al - sl - gs-; al - sl - \beta g-; al - sl - r\alpha g-; al - sl - rg\alpha-; al - sl - r\alpha-; al - sl - \alpha-; al - sl - s-; al - sl - p-; al - sl - \beta-; al - sl - \nu-; al - sl - \pi-; al - sl - r-]$open then the following statements*

*are true. a) If f is continuous [r-continuous] and surjective then g is almost slightly $\nu g$-open.*
*b) If f is g-continuous[rg-continuous], surjective and X is $T_{\frac{1}{2}}[rT_{\frac{1}{2}}]$] then g is almost slightly $\nu g$-open.*

**Theorem 3.26.** *If $f : X \to Y$, $g : Y \to Z$ be two mappings such that $g \circ f$ is $\nu g$-open then the following statements are true.*
*a) If f is almost slightly-continuous [almost slightly-r-continuous] and surjective then g is almost slightly $\nu g$-open.*
*b) If f is almost slightly-g-continuous[almost slightly-rg-continuous], surjective and X is $T_{\frac{1}{2}}$[resp: $rT_{\frac{1}{2}}$]] then g is almost slightly $\nu g$-open.*

*Proof.* For A r-clopen in Y, $f^{-1}(A)$ open in X $\Rightarrow (g \circ f)(f^{-1}(A)) = g(A)$ $\nu g$-open in Z. Hence $g$ is almost slightly $\nu g$-open.
□

**Corollary 3.27.** *If $f : X \to Y$, $g : Y \to Z$ be two mappings such that $g \circ f$ is $g - [rg-; sg-; gs-; \beta g-; r\alpha g-; rg\alpha-; r\alpha-; \alpha-; s-; p-; \beta-; \nu-; \pi-; r-]$open then the following statements are true.*
*a) If f is almost slightly-continuous [almost slightly-r-continuous] and surjective then g is almost slightly $\nu g$-open.*
*b) If f is almost slightly-g-continuous[almost slightly-rg-continuous], surjective and X is $T_{\frac{1}{2}}[rT_{\frac{1}{2}}]$] then g is almost slightly $\nu g$-open.*

**Theorem 3.28.** *If X is $\nu g$-regular, $f : X \to Y$ is r-open, r-continuous, almost slightly $\nu g$-open surjective and $A^o = A$ for every $\nu g$-open set in Y then Y is $\nu g$-regular.*

**Corollary 3.29.** *If X is $\nu g$-regular, $f : X \to Y$ is r-open, r-continuous, almost slightly $\nu g$-open, surjective and $A^o = A$ for every open set in Y then Y is $\nu g$-regular.*

**Theorem 3.30.** *If $f : X \to Y$ is almost slightly $\nu g$-open and A open in X, then $f_A : (X, \tau_A) \to (Y, \sigma)$ is almost slightly $\nu g$-open.*

*Proof.* Let F be a r-clopen set in A. Then $F = A \cap E$ for some open set E of X and so F is open in X $\Rightarrow f$(A) is $\nu g$-open in Y. But $f(F) = f_A(F)$. Therefore $f_A$ is almost slightly $\nu g$-open.
□

**Theorem 3.31.** *If $f : X \to Y$ is almost slightly $\nu g$-open, X is $T_{\frac{1}{2}}[rT_{\frac{1}{2}}]$ and A is g-open [rg-open] set of X then $f_A : (X, \tau_A) \to (Y, \sigma)$ is almost slightly $\nu g$-open.*

*Proof.* Let F be a r-clopen set in A. Then $F = A \cap E$ for some open set E of X and so F is open in X $\Rightarrow f$(A) is $\nu g$-open in Y. But $f(F) = f_A(F)$. Therefore $f_A$ is almost slightly $\nu g$-open.
□

**Corollary 3.32.** *If $f: X \to Y$ is $al-sl-g-[al-sl-rg-; al-sl-sg-; al-sl-gs-; al-sl-\beta g-; al-sl-r\alpha g-; al-sl-rg\alpha-; al-sl-r-; al-sl-r\alpha-; al-sl-\alpha-; al-sl-s-; al-sl-p-; al-sl-\beta-; al-sl-\nu-; al-sl-\pi-]$ open and A open in X, then $f_A : (X, \tau_A) \to (Y, \sigma)$ is almost slightly $\nu g$-open.*

**Theorem 3.33.** *If $f_i : X_i \to Y_i$ be almost slightly $\nu g$-open for i = 1, 2. Let $f: X_1 \times X_2 \to Y_1 \times Y_2$ be defined as $f(x_1, x_2) = (f_1(x_1), f_2(x_2))$. Then $f: X_1 \times X_2 \to Y_1 \times Y_2$ is almost slightly $\nu g$-open.*

*Proof.* Let $U_1 \times X_2 \subseteq X_1 \times X_2$ where $U_i$ is r-clopen in $X_i$ for i = 1, 2. Then $f(U_1 \times U_2) = f_1(U_1) \times f_2(U_2)$ is $\nu g$-open set in $Y_1 \times Y_2$ . Hence $f$ is almost slightly $\nu g$-open.
□

**Corollary 3.34.** *If $f_i : X_i \to Y_i$ be $al-sl-g-[al-sl-rg-; al-sl-sg-; al-sl-gs-; al-sl-\beta g-; al-sl-r\alpha g-; al-sl-rg\alpha-; al-sl-r-; al-sl-r\alpha-; al-sl-\alpha-; al-sl-s-; al-sl-p-; al-sl-\beta-; al-sl-\nu-; al-sl-\pi-]$open for i = 1, 2. Let $f: X_1 \times X_2 \to Y_1 \times Y_2$ be defined as $f(x_1, x_2) = (f_1(x_1), f_2(x_2))$, then $f: X_1 \times X_2 \to Y_1 \times Y_2$ is almost slightly $\nu g$-open.*

**Theorem 3.35.** *Every $\nu g$-open and contra $\nu g$-closed is almost slightly $\nu g$-open map but not conversely.*

*Proof.* Let A be any r-clopen set in X, then A is both open and closed in X. For, $f$ is $\nu$g-open and contra $\nu$g-closed, $f$(A) is $\nu$g-open. Hence $f$ is almost slightly $\nu$g-open. □

**Example 3.36.** *Let $X = Y = \{a, b, c, d\}$; $\tau = \{\phi, \{a\}, \{b\}, \{d\}, \{a, b\}, \{a, d\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, X\}$. Let $f: X \to Y$ be defined f(a) = b, f(b) = a, f(c) = d and f(d) = c. Then f is almost slightly $\nu g$-open but not contra $\nu g$-closed and almost contra $\nu g$-closed.*

**Example 3.37.** *Let $X = Y = \{a, b, c, d\}$; $\tau = \{\phi, \{a\}, \{b\}, \{d\}, \{a, b\}, \{a, d\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, X\}$. Let $f: X \to Y$ be defined f(a) = c, f(b) = d, f(c) = a and f(d) = b. Then f is almost slightly $\nu g$-open but not $\nu g$-open and almost $\nu g$-open.*

**Corollary 3.38.** *If f is* $g-[rg-;sg-;gs-;\beta g-;r\alpha g-;rg\alpha-;r-;r\alpha-;\alpha-;s-;p-;\beta-;\nu-;\pi-]$ *open and* $c-g-[c-rg-;c-sg-;c-gs-;c-\beta g-;c-r\alpha g-;c-rg\alpha-;c-r-;c-r\alpha-;c-\alpha-;c-s-;c-p-;c-\beta-;c-\nu-;c-\pi-]$ *closed then f is almost slightly* $\nu g$*-open.*

*Proof.* Let A be any r-clopen set in X, then A is both open and closed in X. For, $f$ is g-open and contra g-closed, $f$(A) is g-open and so $\nu$g-open[by remark 1]. Hence $f$ is almost slightly $\nu$g-open. □

**Corollary 3.39.** *If f is open and g is* $al-sl-g-[al-sl-rg-;al-sl-sg-;al-sl-gs-;al-sl-\beta g-;al-sl-r\alpha g-;al-sl-rg\alpha-;al-sl-r-;al-sl-r\alpha-;al-sl-\alpha-;al-sl-s-;al-sl-p-;al-sl-\beta-;al-sl-\nu-;al-sl-\pi-]$ *open then* $g\circ f$ *is almost slightly* $\nu g$*-open.*

## 4. Almost Slightly $\nu$g-closed mappings

**Definition 4.1.** *A function* $f\colon X\to Y$ *is said to be almost slightly* $\nu g$*-closed if the image of every r-clopen set in X is* $\nu g$*-closed in Y.*

**Example 4.2.** *Let* $X=Y=\{a,b,c\}$*;* $\tau=\{\phi,\{a\},\{b,c\},X\}$; $\sigma=\{\phi,\{a\},\{b\},\{a,b\},Y\}$. *Let* $f:X\to Y$ *be defined f(a) = c, f(b) = a and f(c) = b. Then f is almost slightly* $\nu g$*-closed, almost slightly rg-closed and almost slightly* $rg\alpha$*-closed but not almost slightly closed, almost slightly semi-closed, almost slightly pre-closed, almost slightly-*$\alpha$*-closed, almost slightly* $r\alpha$*-closed, almost slightly* $\nu$*-closed, almost slightly* $\pi$*-closed, almost slightly* $\beta$*-closed, almost slightly g-closed, almost slightly sg-closed, almost slightly gs-closed, almost slightly pg-closed, almost slightly gp-closed and almost slightly* $\beta g$*-closed.*

**Example 4.3.** *Let* $X=Y=\{a,b,c,d\}$*;* $\tau=\{\phi,\{a,b\},\{c,d\},X\}$*;* $\sigma=\{\phi,\{a\},\{b\},\{d\},\{a,b\},\{a,d\},\{b,d\},\{a,b,c\},\{a,b,d\},X\}$. *Let* $f:X\to Y$ *be defined f(a) = b, f(b) = a, f(c) = d and f(d) = c. Then f is not almost slightly* $\nu g$*-closed.*

**Theorem 4.4.** *We have the following interrelation among the following almost slightly closed mappings*

(i) al.sl.g.closed al.sl.gs.closed
$\downarrow$ $\downarrow$ $\nwarrow$
al.sl.rg$\alpha$.closed $\rightarrow$ al.sl.rg.closed $\rightarrow$ **al.sl.$\nu g$.closed** $\leftarrow$ al.sl.sg.closed $\leftarrow$ al.sl.$\beta$g.closed
$\uparrow$ $\uparrow$ $\uparrow$ $\uparrow$ $\uparrow$
$\nearrow$ al.sl.$r\alpha$.closed $\rightarrow$ al.sl.$\nu$.closed $\searrow$ $\uparrow$ $\uparrow$
al.sl.r.closed $\rightarrow$ al.sl.$\pi$.closed $\rightarrow$ al.sl.closed $\rightarrow$ al.sl.$\alpha$.closed $\rightarrow$ al.sl.s.closed $\rightarrow$ al.sl.$\beta$.closed
$\swarrow$ $\downarrow$ $\searrow$ $\searrow$
al.sl.$\pi$g.closed al.sl.p.closed $\rightarrow$ al.sl.$\omega$.closed $\not\leftrightarrow$ al.sl.g$\alpha$.closed
$\searrow$ $\searrow$
al.sl.gpr.closed $\leftarrow$ al.sl.gp.closed $\leftarrow$ al.sl.pg.closed al.sl.r$\omega$.closed None is reversible.

(ii) sl.g.closed sl.gs.closed
$\downarrow$ $\downarrow$ $\nwarrow$
sl.rg$\alpha$.closed $\rightarrow$ sl.rg.closed $\rightarrow$ **al.sl.$\nu g$.closed** $\leftarrow$ sl.sg.closed $\leftarrow$ sl.$\beta$g.closed
$\uparrow$ $\uparrow$ $\uparrow$ $\uparrow$ $\uparrow$
$\nearrow$ sl.$r\alpha$.closed $\rightarrow$ sl.$\nu$.closed $\searrow$ $\uparrow$ $\uparrow$
sl.r.closed $\rightarrow$ sl.$\pi$.closed $\rightarrow$ sl.closed $\rightarrow$ sl.$\alpha$.closed $\rightarrow$ sl.s.closed $\rightarrow$ sl.$\beta$.closed
$\swarrow$ $\downarrow$ $\searrow$ $\searrow$
sl.$\pi$g.closed sl.p.closed $\rightarrow$ sl.$\omega$.closed $\not\leftrightarrow$ sl.g$\alpha$.closed
$\searrow$ $\searrow$
sl.gpr.closed $\leftarrow$ sl.gp.closed $\leftarrow$ sl.pg.closed sl.r$\omega$.closed None is reversible.

(iii) If $\nu GC(Y) = RC(Y)$, then the reverse relations hold for all almost slightly closed maps.
al.sl.g.closed al.sl.gs.closed
$\updownarrow$ $\updownarrow$ $\nearrow\nwarrow$
al.sl.rg$\alpha$.closed $\leftrightarrow$ al.sl.rg.closed $\leftrightarrow$ **al.sl.$\nu g$.closed** $\leftrightarrow$ al.sl.sg.closed $\leftrightarrow$ al.sl.$\beta$g.closed
$\updownarrow$ $\updownarrow$ $\updownarrow$ $\updownarrow$ $\updownarrow$
$\nwarrow\nearrow$ al.sl.$r\alpha$.closed $\leftrightarrow$ al.sl.$\nu$.closed $\swarrow\searrow$ $\updownarrow$ $\updownarrow$
al.sl.r.closed $\leftrightarrow$ al.sl.$\pi$.closed $\leftrightarrow$ al.sl.closed $\leftrightarrow$ al.sl.$\alpha$.closed $\leftrightarrow$ al.sl.s.closed $\leftrightarrow$ al.sl.$\beta$.closed

**Theorem 4.5.** *(i) If $(Y, \sigma)$ is discrete, then f is almost slightly closed of all types.*
*(ii) If f is almost slightly closed and g is $\nu g$-closed then $g \circ f$ is almost slightly $\nu g$-closed.*
*(iii)If f is closed and g is almost slightly $\nu g$-closed then $g \circ f$ is almost slightly $\nu g$-closed.*

**Corollary 4.6.** *If f is almost slightly closed and g is $g-[rg-; sg-; gs-; \beta g-; r\alpha g-; rg\alpha-; r\alpha-; \alpha-; s-; p-; \beta-; \nu-; \pi-; r-]$ closed then g∘f is almost slightly $\nu g$-closed.*

**Corollary 4.7.** *If f is closed[r-closed] and g is $al-sl-g-[al-sl-rg-; al-sl-sg-; al-sl-gs-; al-sl-\beta g-; al-sl-r\alpha g-; al-sl-rg\alpha-; al-sl-r\alpha-; al-sl-\alpha-; al-sl-s-; al-sl-p-; al-sl-\beta-; al-sl-\pi-]$ closed then $g \circ f$ is almost slightly $\nu g$-closed.*

**Theorem 4.8.** *If $f\colon X \to Y$ is almost slightly $\nu g$-closed, then $\nu g(\overline{(f(A))}) \subset f(\overline{(A)})$.*

*Proof.* Let $A \subset X$ be r-clopen and $f\colon X \to Y$ is almost slightly $\nu g$-closed $\underline{\text{gives}}$ $f(\overline{A})$ is $\nu g$-closed in Y and $f(A) \subset f(\overline{(A)})$ which in $\underline{\text{turn gives}}$ $\nu g(\overline{(f(A))}) \subset \nu g((\overline{f(\overline{(A)})}))$------ (1) Since $f(\overline{(A)})$ is $\nu g$-closed in Y, $\nu g((\overline{f(\overline{(A)})})) = f(\overline{(A)})$ ------ (2) From

(1) and (2) we have $\nu g(\overline{(f(A))}) \subset f(\overline{(A)})$ for every subset A of X.
□

**Remark 4.9.** *Converse is not true in general.*

**Corollary 4.10.** *If $f: X \to Y$ is $al-sl-g-[al-sl-rg-; al-sl-sg-; al-sl-gs-; al-sl-\beta g-; al-sl-r\alpha g-; al-sl-rg\alpha-; al-sl-r\alpha-; al-sl-\alpha-; al-sl-s-; al-sl-p-; al-sl-\beta-; al-sl-\pi-]$ closed, then $\nu g(\overline{(f(A))}) \subset f(\overline{(A)})$*

**Theorem 4.11.** *If $f: X \to Y$ is almost slightly $\nu g$-closed and $A \subseteq X$ is closed, f(A) is $\tau_{\nu g}$-closed in Y.*

*Proof.* Let $A \subset X$ be r-clopen and $f: X \to Y$ is almost slightly $\nu g$-closed implies $\nu g(\overline{(f(A))}) \subset f(\overline{(A)})$ which in turn implies $\nu g(\overline{(f(A))}) \subset f(A)$, since $f(A) = f(\overline{(A)})$. But $f(A) \subset \nu g(\overline{(f(A))})$. Combining we get $f(A) = \nu g(\overline{(f(A))})$. Hence $f$(A) is $\tau_{\nu g}$-closed in Y.
□

**Corollary 4.12.** *If $f: X \to Y$ is $al-sl-g-[al-sl-rg-; al-sl-sg-; al-sl-gs-; al-sl-\beta g-; al-sl-r\alpha g-; al-sl-rg\alpha-; al-sl-r-; al-sl-r\alpha-; al-sl-\alpha-; al-sl-s-; al-sl-p-; al-sl-\beta-; al-sl-\nu-; al-sl-\pi-]$closed, then f(A) is $\tau_{\nu g}$ closed in Y if A is closed set in X.*

**Theorem 4.13.** *If $\nu g(\overline{(A)}) = r(\overline{(A)})$ for every $A \subset Y$ and X is discrete space, then the following are equivalent:*
*a) $f: X \to Y$ is almost slightly $\nu g$-closed map*
*b) $\nu g(\overline{(f(A))}) \subset f(\overline{(A)})$*

*Proof.* (a) ⇒ (b) follows from Theorem 4.8
(b) ⇒ (a) Let A be any r-clopen set in X, then $f(A) = f(\overline{(A)}) \supset \nu g(\overline{(f(A))})$ by hypothesis. We have $f(A) \subset \nu g(\overline{(f(A))})$. Combining we get $f(A) = \nu g(\overline{(f(A))}) = r(\overline{(f(A))})$[by given condition] which implies $f$(A) is r-closed and hence $\nu g$-closed. Thus $f$ is almost slightly $\nu g$-closed.
□

**Theorem 4.14.** *If $\nu(\overline{(A)}) = r(\overline{(A)})$ for every $A \subset Y$ and X is discrete space, then the following are equivalent:*
*a) $f: X \to Y$ is almost slightly $\nu g$-closed map*
*b) $\nu g(\overline{(f(A))}) \subset f(\overline{(A)})$*

*Proof.* (a) $\Rightarrow$ (b) follows from Theorem 4.8
(b) $\Rightarrow$ (a) Let A be any r-clopen set in X, then $f(A) = f(\overline{(A)}) \supset \nu g(\overline{(f(A))})$ by hypothesis. We have $f(A) \subset \nu g(\overline{(f(A))})$. Combining we get $f(A) = \nu g(\overline{(f(A))}) = r(\overline{(f(A))})$[by given condition] which implies $f$(A) is r-closed and hence $\nu g$-closed. Thus $f$ is almost slightly $\nu g$-closed.
□

**Theorem 4.15.** *$f : X \to Y$ is almost slightly $\nu g$-closed iff for each subset S of Y and each r-clopen set U containing $f^{-1}(S)$, there is an $\nu g$-closed set V of Y such that $S \subseteq V$ and $f^{-1}(V) \subseteq U$.*

*Proof.* Assume $f : X \to Y$ is almost slightly $\nu g$-closed. Let $S \subseteq Y$ and U be r-clopen set containing $f^{-1}(S)$. Then X-U is r-clopen in X and $f(X - U)$ is $\nu g$-closed in Y as $f$ is almost slightly $\nu g$-closed and $V = Y - f(X - U)$ is $\nu g$-closed in Y. $f^{-1}(S) \subseteq U \Rightarrow S \subseteq f(U) \Rightarrow S \subseteq V$ and $f^{-1}(V) = f^{-1}(Y - f(X - U)) = f^{-1}(Y) - f^{-1}(f(X - U)) = f^{-1}(Y) - (X - U) = X - (X - U) = U$.

Conversely Let F be r-clopen in X $\Rightarrow F^c$ is r-clopen. Then $f^{-1}(f(F^c)) \subseteq F^c$. By hypothesis there exists an $\nu g$-closed set V of Y, such that $f(F^c) \subseteq V$ and $f^{-1}(V) \supset F^c$ and so $F \subseteq [f^{-1}(V)]^c$. Hence $V^c \subseteq f(F) \subseteq f[f^{-1}(V)^c] \subseteq V^c \Rightarrow f(F) \subseteq V^c \Rightarrow f(F) = V^c$. Thus $f$(F) is $\nu g$-closed in Y. Therefore $f$ is almost slightly $\nu g$-closed.
□

**Remark 4.16.** *Composition of two almost slightly $\nu g$-closed maps is not almost slightly $\nu g$-closed in general.*

**Theorem 4.17.** *Let X, Y, Z be topological spaces and every $\nu g$-closed set is r-clopen in Y. Then the composition of two almost slightly $\nu g$-closed maps is almost slightly $\nu g$-closed.*

*Proof.* (a) Let $f$ and $g$ be almost slightly $\nu g$-closed maps. Let A be any r-clopen set in X $\Rightarrow f$(A) is r-clopen in Y (by assumption) $\Rightarrow g(f$ (A)) $= g \circ f$(A) is $\nu g$-closed in Z. Therefore $g \circ f$ is almost slightly $\nu g$-closed.
□

**Corollary 4.18.** *Let X, Y, Z be topological spaces and every $g - [rg-; sg-; gs-; \beta g-; r\alpha g-; rg\alpha-; r\alpha-; \alpha-; s-; p-; \beta-; \pi-]$closed set is r-clopen in Y. Then the composition of two $al - sl - g - [al - sl - rg-; al - sl - sg-; al - sl - gs-; al - sl - \beta g-; al - sl - r\alpha g-; al - sl - rg\alpha-; al - sl - r\alpha-; al - sl - \alpha-; al - sl - s-; al - sl - p-; al - sl - \beta-; al - sl - \nu-; al - sl - \pi-; al - sl - r-]$closed maps is almost slightly $\nu g$-closed.*

**Example 4.19.** *Let* $X = Y = Z = \{a, b, c\}$*;* $\tau = \{\phi, \{a\}, \{a, b\}, X\}; \sigma = \{\phi, \{a, c\}, Y\}$ *and* $\eta = \{\phi, \{a\}, \{b, c\}, Z\}$*.* $f : X \to Y$ *be defined* $f(a) = c, f(b) = b and f(c) = a$ *and* $g : Y \to Z$ *be defined* $g(a) = b, g(b) = a and g(c) = c$*, then* $g$*,* $f$ *and* $g \circ f$ *are almost slightly* $\nu g$*-closed.*

**Theorem 4.20.** *If* $f : X \to Y$ *is almost slightly g-closed[almost slightly rg-closed],* $g : Y \to Z$ *is* $\nu g$*-closed and* $Y$ *is* $T_{\frac{1}{2}}[r - T_{\frac{1}{2}}]$ *then* $g \circ f$ *is almost slightly* $\nu g$*-closed.*

*Proof.* (a) Let A be r-clopen in X. Then $f$(A) is g-closed and so closed in Y as Y is $T_{\frac{1}{2}} \Rightarrow g(f\ (\text{A})) = g \circ f\,(\text{A})$ is $\nu g$-closed in Z (since $g$ is $\nu g$-closed). Hence $g \circ f$ is almost slightly $\nu g$-closed.
□

**Corollary 4.21.** *If* $f : X \to Y$ *is almost slightly g-closed [almost slightly rg-closed],* $g : Y \to Z$ *is* $g - [rg-; sg-; gs-; \beta g-; r\alpha g-; rg\alpha-; r\alpha-; \alpha-; s-; p-; \beta-; \nu-; \pi-; r-]$ *closed and* $Y$ *is* $T_{\frac{1}{2}}[r - T_{\frac{1}{2}}]$ *then* $g \circ f$ *is almost slightly* $\nu g$*-closed.*

**Theorem 4.22.** *If* $f : X \to Y$ *is g-closed[rg-closed],* $g : Y \to Z$ *is almost slightly* $\nu g$*-closed and* $Y$ *is* $T_{\frac{1}{2}}[r - T_{\frac{1}{2}}]$ *then* $g \circ f$ *is almost slightly* $\nu g$*-closed.*

*Proof.* (a) Let A be r-clopen in X. Then $f$(A) is g-closed and so closed in Y as Y is $T_{\frac{1}{2}} \Rightarrow g(f\ (\text{A})) = g \circ f(\text{A})$ is $\nu g$-closed in Z (since $g$ is almost slightly $\nu g$-closed). Hence $g \circ f$ is almost slightly $\nu g$-closed.
□

**Corollary 4.23.** *If* $f : X \to Y$ *is g-closed [rg-closed],* $g : Y \to Z$ *is* $al - sl - g - [al - sl - rg-; al - sl - sg-; al - sl - gs-; al - sl - \beta g-; al - sl - r\alpha g-; al - sl - rg\alpha-; al - sl - r\alpha-; al - sl - \alpha-; al - sl - s-; al - sl - p-; al - sl - \beta-; al - sl - \pi-]$ *closed and* $Y$ *is* $T_{\frac{1}{2}}[r - T_{\frac{1}{2}}]$ *then* $g \circ f$ *is almost slightly* $\nu g$*-closed.*

**Theorem 4.24.** *If* $f : X \to Y$*,* $g : Y \to Z$ *be two mappings such that* $g \circ f$ *is almost slightly* $\nu g$*-closed then the following statements are true.*
*a) If f is continuous [r-continuous] and surjective then g is almost slightly* $\nu g$*-closed.*
*b) If f is g-continuous[resp: rg-continuous], surjective and X is* $T_{\frac{1}{2}}$*[resp:* $rT_{\frac{1}{2}}$*]] then g is almost slightly* $\nu g$*-closed.*

*Proof.* For A r-clopen in Y, $f^{-1}(A)$ closed in X $\Rightarrow (g \circ f)(f^{-1}(A)) = g(A)$ $\nu g$-closed in Z. Hence $g$ is almost slightly $\nu g$-closed.

Similarly one can prove the remaining parts and hence omitted.
□

**Corollary 4.25.** *If $f: X \to Y,\ g: Y \to Z$ be two mappings such that $g \circ f$ is $al-sl-g-[al-sl-rg-; al-sl-sg-; al-sl-gs-; al-sl-\beta g-; al-sl-r\alpha g-; al-sl-rg\alpha-; al-sl-r\alpha-; al-sl-\alpha-; al-sl-s-; al-sl-p-; al-sl-\beta-; al-sl-\nu-; al-sl-\pi-; al-sl-r-]$closed then the following statements are true.*
*a) If f is continuous [r-continuous] and surjective then g is almost slightly $\nu g$-closed.*
*b) If f is g-continuous[rg-continuous], surjective and X is $T_{\frac{1}{2}}[rT_{\frac{1}{2}}]$ then g is almost slightly $\nu g$-closed.*

**Theorem 4.26.** *If $f: X \to Y,\ g: Y \to Z$ be two mappings such that $g \circ f$ is $\nu g$-closed then the following statements are true.*
*a) If f is almost slightly-continuous [almost slightly-r-continuous] and surjective then g is almost slightly $\nu g$-closed.*
*b) If f is almost slightly-g-continuous[almost slightly-rg-continuous], surjective and X is $T_{\frac{1}{2}}$[resp: $rT_{\frac{1}{2}}$]] then g is almost slightly $\nu g$-closed.*

*Proof.* For A r-clopen in Y, $f^{-1}(A)$ closed in X $\Rightarrow (g \circ f)(f^{-1}(A)) = g(A)$ $\nu g$-closed in Z. Hence $g$ is almost slightly $\nu g$-closed.
□

**Corollary 4.27.** *If $f: X \to Y,\ g: Y \to Z$ be two mappings such that $g \circ f$ is $g-[rg-; sg-; gs-; \beta g-; r\alpha g-; rg\alpha-; r\alpha-; \alpha-; s-; p-; \beta-; \nu-; \pi-; r-]$ closed then the following statements are true.*
*a) If f is almost slightly-continuous [almost slightly-r-continuous] and surjective then g is almost slightly $\nu g$-closed.*
*b) If f is almost slightly-g-continuous[almost slightly-rg-continuous], surjective and X is $T_{\frac{1}{2}}[rT_{\frac{1}{2}}]$] then g is almost slightly $\nu g$-closed.*

**Theorem 4.28.** *If X is $\nu g$-regular, $f: X \to Y$ is r-closed, nearly-continuous, almost slightly $\nu g$-closed surjection and $\overline{A} = A$ for every $\nu g$-closed set in Y, then Y is $\nu g$-regular.*

**Corollary 4.29.** *If X is $\nu g$-regular, $f: X \to Y$ is r-closed, nearly-continuous, almost slightly $\nu g$-closed surjection and $\overline{A} = A$ for every closed set in Y then Y is $\nu g$-regular.*

**Theorem 4.30.** *If $f: X \to Y$ is almost slightly $\nu g$-closed and A closed in X, then $f_A: (X, \tau_A) \to (Y, \sigma)$ is almost slightly $\nu g$-closed.*

*Proof.* Let F be an r-clopen set in A. Then $F = A \cap E$ for some closed set E of X and so F is closed in X $\Rightarrow f$(A) is $\nu g$-closed in Y. But $f(F) = f_A(F)$. Therefore $f_A$

is almost slightly $\nu g$-closed.
□

**Theorem 4.31.** *If $f\colon X \to Y$ is almost slightly $\nu g$-closed, X is $rT_{\frac{1}{2}}$ and A is rg-closed set of X then $f_A : (X, \tau_A) \to (Y, \sigma)$ is almost slightly $\nu g$-closed.*

*Proof.* Let F be a r-clopen set in A. Then $F = A \cap E$ for some closed set E of X and so F is closed in X $\Rightarrow f$(A) is $\nu g$-closed in Y. But $f(F) = f_A(F)$. Therefore $f_A$ is almost slightly $\nu g$-closed. □

**Corollary 4.32.** *If $f\colon X \to Y$ is $al - sl - g - [al - sl - rg-; al - sl - sg-; al - sl - gs-; al - sl - \beta g-; al - sl - r\alpha g-; al - sl - rg\alpha-; al - sl - r-; al - sl - r\alpha-; al - sl - \alpha-; al - sl - s-; al - sl - p-; al - sl - \beta-; al - sl - \nu-; al - sl - \pi-]$ closed and A closed in X, then $f_A : (X, \tau_A) \to (Y, \sigma)$ is almost slightly $\nu g$-closed.*

**Theorem 4.33.** *If $f_i : X_i \to Y_i$ be almost slightly $\nu g$-closed for i = 1, 2. Let $f\colon X_1 \times X_2 \to Y_1 \times Y_2$ be defined as $f(x_1, x_2) = (f_1(x_1), f_2(x_2))$. Then $f\colon X_1 \times X_2 \to Y_1 \times Y_2$ is almost slightly $\nu g$-closed.*

*Proof.* Let $U_1 \times X_2 \subseteq X_1 \times X_2$ where $U_i$ is r-clopen in $X_i$ for i = 1, 2. Then $f(U_1 \times U_2) = f_1(U_1) \times f_2(U_2)$ is $\nu g$-closed set in $Y_1 \times Y_2$. Hence $f$ is almost slightly $\nu g$-closed.
□

**Corollary 4.34.** *If $f_i : X_i \to Y_i$ be $al - sl - g - [al - sl - rg-; al - sl - sg-; al - sl - gs-; al - sl - \beta g-; al - sl - r\alpha g-; al - sl - rg\alpha-; al - sl - r-; al - sl - r\alpha-; al - sl - \alpha-; al - sl - s-; al - sl - p-; al - sl - \beta-; al - sl - \nu-; al - sl - \pi-]$closed for i = 1, 2. Let $f\colon X_1 \times X_2 \to Y_1 \times Y_2$ be defined as $f(x_1, x_2) = (f_1(x_1), f_2(x_2))$, then $f\colon X_1 \times X_2 \to Y_1 \times Y_2$ is almost slightly $\nu g$-closed.*

**Theorem 4.35.** *Every $\nu g$-closed and contra $\nu g$-open map is almost slightly $\nu g$-closed map but not conversely.*

*Proof.* Let A be any r-clopen set in X, then A is both open and closed in X. For, $f$ is $\nu$g-closed and contra $\nu$g-open, $f$(A) is $\nu$g-open. Hence $f$ is almost slightly $\nu$g-closed. □

**Example 4.36.** *Let $X = Y = \{a, b, c, d\}$; $\tau = \{\phi, \{a\}, \{b\}, \{d\}, \{a, b\}, \{a, d\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, X\}$. Let $f\colon X \to Y$ be defined f(a) = b, f(b) = a, f(c) = d and f(d) = c. Then $f$ is almost slightly $\nu g$-closed but not contra $\nu g$-open and almost contra $\nu g$-open.*

**Example 4.37.** *Let* $X = Y = \{a, b, c, d\}$; $\tau = \{\phi, \{a\}, \{b\}, \{d\}, \{a, b\}, \{a, d\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, X\}$. *Let* $f : X \to Y$ *be defined f(a) = c, f(b) = d, f(c) = a and f(d) = b. Then f is almost slightly* $\nu g$*-closed but not* $\nu g$*-closed and almost* $\nu g$*-closed.*

**Corollary 4.38.** *If f is* $g-[rg-; sg-; gs-; \beta g-; r\alpha g-; rg\alpha-; r-; r\alpha-; \alpha-; s-; p-; \beta-; \nu-; \pi-]$*closed and* $c - g - [c - rg-; c - sg-; c - gs-; c - \beta g-; c - r\alpha g-; c - rg\alpha-; c - r-; c - r\alpha-; c - \alpha-; c - s-; c - p-; c - \beta-; c - \nu-; c - \pi-]$*open then f is almost slightly* $\nu g$*-closed.*

*Proof.* Let A be any r-clopen set in X, then A is both open and closed in X. For, $f$ is g-closed and contra g-open, $f$(A) is g-closed and so $\nu$g-closed[by remark 1]. Hence $f$ is almost slightly $\nu$g-closed. □

**Corollary 4.39.** *If f is closed and g is* $al - sl - g - [al - sl - rg-; al - sl - sg-; al - sl - gs-; al - sl - \beta g-; al - sl - r\alpha g-; al - sl - rg\alpha-; al - sl - r-; al - sl - r\alpha-; al - sl - \alpha-; al - sl - s-; al - sl - p-; al - sl - \beta-; al - sl - \nu-; al - sl - \pi-]$*closed then* $g \circ f$ *is almost slightly* $\nu g$*-closed.*

## Conclusion

In this paper author defined new open and closed mappings called almost slightly $\nu g$-open and almost slightly $\nu g$-Closed mappings and studied their interrelations with other types of almost slightly-continuous functions.

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## References

1. Arse Nagli Uresin, Aynur kerkin, T.Noiri, slightly $\delta-$precontinuous functions, Commen, Fac. Sci. Univ. Ark. Series 41.56(2)(2007)1-9.
2. C.W.Baker, Slightly precontinuous funtions, Acta Math Hung, 94(1-6)(2002) 45-52.
3. Balasubramanian.S., Almost Slightly Continuity, Slightly open and Slightly closed mappings - Indian Journal of Science, Vol.5, No.13 (Oct 2013)29 - 36.
4. Balasubramanian.S., Almost Slightly semi-Continuity, Slightly semi-open and Slightly semi-closed mappings - Indian Journal of Engineering, Vol.5, No.13 (Oct 2013)44 - 52.
5. Balasubramanian.S., Sandhya.C., and Aruna Swathi Vyjayanthi.P., Slightly $\nu$-open mappings - Aryabhatta Journal of Mathematics and Informatics, Vol.5, No.02(2013), 313 - 320.
6. Balasubramanian.S., Almost Slightly pre-continuity, Slightly pre-open and Slightly pre-closed mappings - International Journal of Mathematical Archive, Vol.4, No.11(2013)45-57.
7. Balasubramanian.S., and Sandhya.C., Almost Slightly $\beta$-continuity, Slightly $\beta$-open and Slightly $\beta$-closed mappings - International Journal of Mathematical Archive, Vol.4, No.11(2013)58-70.
8. Balasubramanian.S., Aruna Swathi Vyjayanthi.P., and Sandhya.C., Slightly $\nu$-closed mappings -General Mathematics Notes, Vol.20,No.1 (2014)1-11.

9. Balasubramanian.S., Slightly $\nu g$-open and Slightly $\nu g$-closed mappings-(Communicated)

10. Erdal Ekici and M. Caldas, slightly $\gamma-$continuous functions, Bol.Sac.Paran.Mat(38)V.22.2,(2004)63-74.

11. T.Noiri and G.I.Chae, A Note on slightly semi continuous functions, Bull.Cal.Math.Soc 92(2)(2000) 87-92.

12. T.Noiri, slightly $\beta-$continuous functions, Internat. J. Math. & Math. Sci. 28(8) (2001) 469-478.

13. T.M.Nour, Slightly semi continuous functions Bull.Cal.Math.Soc 87, (1995) 187-190.

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