



Weakly b -Open Functions in Bitopological Spaces

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ABSTRACT: The aim of this paper is to introduce the notion of weakly b -open functions in a bitopological spaces. Some properties of this function are established and the relationships with some other types of spaces are also investigated.

Key Words: Bitopological spaces; (i, j) - b -open; (i, j) - b -closed; (i, j) - θ -closed; (i, j) -regular open.

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1. Introduction

The notion of bitopological spaces (X, τ_1, τ_2) , where X is a non-empty set and τ_1, τ_2 are different topologies on X was introduced by Kelly [9]. In 1996, Andrijevic [2] introduced the concept of b -open sets in topological spaces. After that Al-Hawary and Al-Omari [1] defined the notion of b -open sets in bitopological spaces and established several fundamental properties. Noiri et al. [11] defined the notion of weakly b -open functions in topological spaces and established several properties of this notion.

The purpose of this paper is to present the concept of weakly b -open functions in bitopological spaces and to obtain several characterizations and properties of this concept.

2. Preliminaries

Throughout this paper, X and Y represents bitopological spaces (X, τ_1, τ_2) and (Y, σ_1, σ_2) on which no separation axioms are assumed and (i, j) means the topologies τ_i, τ_j where $i, j \in \{1, 2\}, i \neq j$. For a subset A of (X, τ_1, τ_2) , $i\text{-int}(A)$ (respectively, $i\text{-cl}(A)$) denotes the interior (respectively, closure) of A with respect to the topology τ_i , where $i \in \{1, 2\}$.

Now, we list some definitions and results those will be used throughout this article.

Definition 2.1. A subset A of a bitopological space (X, τ_1, τ_2) is said to be

- (a) (i, j) -b-open ([1]) if $A \subset i\text{-int}(j\text{-cl}(A)) \cup j\text{-cl}(i\text{-int}(A))$.
- (b) (i, j) -regular open ([3]) if $A = i\text{-int}(j\text{-cl}(A))$.
- (c) (i, j) -regular closed ([4]) if $A = i\text{-cl}(j\text{-int}(A))$.
- (d) (i, j) -preopen ([6]) if $A \subset i\text{-int}(j\text{-cl}(A))$
- (e) (i, j) - α -open ([7]) if $A \subset i\text{-int}(j\text{-cl}(i\text{-int}(A)))$

The complement of (i, j) -b-open set is (i, j) -b-closed.

Definition 2.2. [2] Let A be a subset of a bitopological space (X, τ_1, τ_2) . Then,

(a) the (i, j) -b-closure of A denoted by $(i, j)\text{-bcl}(A)$, is defined by the intersection of all (i, j) -b-closed sets containing A .

(b) the (i, j) -b-interior of A denoted by $(i, j)\text{-bint}(A)$, is defined by the union of all (i, j) -b-open sets contained in A .

Lemma 2.1. [1] Let (X, τ_1, τ_2) be a bitopological space and A be a subset of X . Then,

- (a) $(i, j)\text{-bint}(A)$ is (i, j) -b-open.
- (b) $(i, j)\text{-bcl}(A)$ is (i, j) -b-closed.
- (c) A is (i, j) -b-open if and only if $A = (i, j)\text{-bint}(A)$.
- (d) A is (i, j) -b-closed if and only if $A = (i, j)\text{-bcl}(A)$.

Lemma 2.2. [14] Let A be a subset of a bitopological space (X, τ_1, τ_2) . Then,

- (a) $X \setminus (i, j)\text{-bcl}(A) = (i, j)\text{-bint}(X \setminus A)$
- (b) $X \setminus (i, j)\text{-bint}(A) = (i, j)\text{-bcl}(X \setminus A)$

Lemma 2.3. [1] Let (X, τ_1, τ_2) be a bitopological space and A be a subset of X . Then $x \in (i, j)\text{-bcl}(A)$ if and only if for every (i, j) -b-open set U containing x , $U \cap A \neq \emptyset$.

Definition 2.3. [8] Let (X, τ_1, τ_2) be a bitopological space and A be a subset of X . A point x of X is said to be in (i, j) - θ -closure of A , denoted by $(i, j)\text{-cl}_\theta(A)$, if $A \cap j\text{-cl}(U) \neq \emptyset$ for every τ_i -open set U containing x , where $i, j \in \{1, 2\}$ and $i \neq j$.

A subset A of X is said to be (i, j) - θ -closed if $A = (i, j)\text{-cl}_\theta(A)$. A subset A of X is said to be (i, j) - θ -open if $X \setminus A$ is (i, j) - θ -closed. The (i, j) - θ -interior of A , denoted by $(i, j)\text{-int}_\theta(A)$ is defined as the union of all (i, j) - θ -open sets contained in A . Therefore $x \in (i, j)\text{-int}_\theta(A)$ if and only if there exists a τ_i -open set U containing x such that $x \in U \subset j\text{-cl}(U) \subset A$.

Lemma 2.4. [8] For a subset A of a bitopological space (X, τ_1, τ_2) , the following properties hold :

$$(a) \quad X \setminus (i, j)\text{-cl}_\theta(A) = (i, j)\text{-int}_\theta(X \setminus A)$$

$$(b) \quad X \setminus (i, j)\text{-int}_\theta(A) = (i, j)\text{-cl}_\theta(X \setminus A)$$

Lemma 2.5. [8] Let (X, τ_1, τ_2) be a bitopological space. If U is a τ_j -open set of X , then $(i, j)\text{-cl}_\theta(U) = i\text{-cl}(U)$.

3. (i, j) -Weakly b -Open Functions

Definition 3.1. A function $f : (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$ is said to be (i, j) -weakly b -open if $f(U) \subset (i, j)\text{-bint}(f(j\text{-cl}(U)))$, for every τ_i -open set U of X .

Theorem 3.1. The following statements are equivalent for a function $f : (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$.

(a) f is (i, j) -weakly b -open.

(b) $f((i, j)\text{-int}_\theta(A)) \subset (i, j)\text{-bint}(f(A))$, for every subset A of X .

(c) $(i, j)\text{-int}_\theta(f^{-1}(B)) \subset f^{-1}((i, j)\text{-bint}(B))$, for every subset B of Y .

(d) $f^{-1}((i, j)\text{-bcl}(B)) \subset (i, j)\text{-cl}_\theta(f^{-1}(B))$, for every subset B of Y .

(e) For every $x \in X$ and every τ_i -open set U of X containing x , there exists an (i, j) - b -open set V containing $f(x)$ such that $V \subset f(j\text{-cl}(U))$.

Proof: (a) \Rightarrow (b) Let $A \subset X$ and $x \in (i, j)\text{-int}_\theta(A)$. Then there exists a τ_i -open set U of X such that $x \in U \subset j\text{-cl}(U) \subset A$. Thus $f(x) \in f(U) \subset f(j\text{-cl}(U)) \subset f(A)$. Since f is (i, j) -weakly b -open function, therefore $f(U) \subset (i, j)\text{-bint}(f(j\text{-cl}(U))) \subset (i, j)\text{-bint}(f(A))$. Thus $f(x) \in (i, j)\text{-bint}(f(A))$. This implies that $x \in f^{-1}((i, j)\text{-bint}(f(A)))$. So, $(i, j)\text{-int}_\theta(A) \subset f^{-1}((i, j)\text{-bint}(f(A)))$. Hence $f((i, j)\text{-int}_\theta(A)) \subset (i, j)\text{-bint}(f(A))$.

(b) \Rightarrow (c) Let $B \subset Y$. Then $f^{-1}(B)$ is a subset of X . Next by (b), $f((i, j)\text{-int}_\theta(f^{-1}(B))) \subset (i, j)\text{-bint}(f(f^{-1}(B))) \subset (i, j)\text{-bint}(B)$. This implies $(i, j)\text{-int}_\theta(f^{-1}(B)) \subset f^{-1}((i, j)\text{-bint}(B))$.

(c) \Rightarrow (d) Let $B \subset Y$ and $x \notin (i, j)\text{-cl}_\theta(f^{-1}(B))$. Then $x \in X \setminus (i, j)\text{-cl}_\theta(f^{-1}(B)) = (i, j)\text{-int}_\theta(X \setminus f^{-1}(B)) = (i, j)\text{-int}_\theta(f^{-1}(Y \setminus B)) \subset f^{-1}((i, j)\text{-bint}(Y \setminus B)) = f^{-1}(Y \setminus (i, j)\text{-bcl}(B)) = X \setminus f^{-1}((i, j)\text{-bcl}(B))$. So, $x \notin f^{-1}((i, j)\text{-bcl}(B))$. Hence $f^{-1}((i, j)\text{-bcl}(B)) \subset (i, j)\text{-cl}_\theta(f^{-1}(B))$.

(d) \Rightarrow (e) Let $x \in X$ and U be a τ_i -open set of X containing x . Let $B = Y \setminus f(j\text{-cl}(U))$. By (d), then we have

$$\begin{aligned} f^{-1}((i, j)\text{-bcl}(Y \setminus f(j\text{-cl}(U)))) &\subset (i, j)\text{-cl}_\theta(f^{-1}(Y \setminus f(j\text{-cl}(U)))) \\ &= (i, j)\text{-cl}_\theta(X \setminus f^{-1}(f(j\text{-cl}(U)))) \\ &\subset (i, j)\text{-cl}_\theta(X \setminus j\text{-cl}(U)) \\ &= i\text{-cl}(X \setminus j\text{-cl}(U)), \text{ by Lemma 2.4.} \\ &= X \setminus i\text{-int}(j\text{-cl}(U)) \\ &\subset X \setminus i\text{-int}(U) = X \setminus U. \end{aligned}$$

Thus $f^{-1}((i, j)\text{-bcl}(Y \setminus f(j\text{-cl}(U)))) \subset X \setminus U$.

$$\Rightarrow f^{-1}(Y \setminus (i, j)\text{-bint}(f(j\text{-cl}(U)))) \subset X \setminus U.$$

$$\Rightarrow X \setminus f^{-1}((i, j)\text{-bint}(f(j\text{-cl}(U)))) \subset X \setminus U.$$

Hence $U \subset f^{-1}((i, j)\text{-bint}(f(j\text{-cl}(U))))$. This implies that $f(x) \in f(U) \subset (i, j)\text{-bint}(f(j\text{-cl}(U))) \subset f(j\text{-cl}(U))$. Let $V = (i, j)\text{-bint}(f(j\text{-cl}(U)))$. Then V is (i, j) - b -open and $f(x) \in V \subset f(j\text{-cl}(U))$.

(e) \Rightarrow (a) Let U be a τ_i -open set of X containing x . By (e), there exists an (i, j) - b -open set V of Y containing $f(x)$ such that $V \subset f(j\text{-cl}(U))$. Thus $f(x) \in V \subset (i, j)\text{-bint}(f(j\text{-cl}(U)))$. Hence $f(U) \subset (i, j)\text{-bint}(f(j\text{-cl}(U)))$ and so f is (i, j) -weakly b -open. \square

Theorem 3.2. *The following statements are equivalent for a function $f : (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$.*

(a) f is (i, j) -weakly b -open.

(b) $(i, j)\text{-bcl}(f(j\text{-int}(i\text{-cl}(U)))) \subset f(i\text{-cl}(U))$, for every subset U of X .

(c) $(i, j)\text{-bcl}(f(j\text{-int}(F))) \subset f(F)$, for every (i, j) -regular closed set F of X .

(d) $(i, j)\text{-bcl}(f(U)) \subset f(i\text{-cl}(U))$, for every τ_j -open set U of X .

Proof: (a) \Rightarrow (b) Let $x \in X$ and $U \subset X$ such that $x \in U$. Let $f(x) \in Y \setminus f(i\text{-cl}(U))$. Then $x \in X \setminus i\text{-cl}(U)$. This implies that, there exists a τ_i -open set V containing x such that $V \cap U = \emptyset$. Thus $j\text{-cl}(V) \cap j\text{-int}(i\text{-cl}(U)) = \emptyset$. Since f is (i, j) -weakly b -open, therefore by theorem 3.1, there exists an (i, j) - b -open set W containing $f(x)$ such that $W \subset f(j\text{-cl}(V))$. So, $W \cap f(j\text{-int}(i\text{-cl}(U))) = \emptyset$ and therefore $f(x) \in X \setminus (i, j)\text{-bcl}(f(j\text{-int}(i\text{-cl}(U))))$. Hence $(i, j)\text{-bcl}(f(j\text{-int}(i\text{-cl}(U)))) \subset f(i\text{-cl}(U))$.

(b) \Rightarrow (c) Let F be (i, j) -regular closed set in X . Therefore $F = i\text{-cl}(j\text{-int}(F))$. Now $(i, j)\text{-bcl}(f(j\text{-int}(F))) = (i, j)\text{-bcl}(f(j\text{-int}(i\text{-cl}(j\text{-int}(F))))) \subset f(i\text{-cl}(j\text{-int}(F))) = f(F)$. Hence $(i, j)\text{-bcl}(f(j\text{-int}(F))) \subset f(F)$.

(c) \Rightarrow (d) Let U be a τ_j -open subset of X . Then $i\text{-cl}(U)$ is (i, j) -regular closed in X . Now $(i, j)\text{-bcl}(f(U)) \subset (i, j)\text{-bcl}(f(j\text{-int}(i\text{-cl}(U)))) \subset f(i\text{-cl}(U))$.

(d) \Rightarrow (a) Let U be a τ_i -open subset of X . Then $j\text{-cl}(U)$ is τ_j -closed. Now $Y \setminus (i, j)\text{-bint}(f(j\text{-cl}(U))) = (i, j)\text{-bcl}(Y \setminus f(j\text{-cl}(U))) = (i, j)\text{-bcl}(f(X \setminus j\text{-cl}(U))) \subset f(i\text{-cl}(X \setminus j\text{-cl}(U))) = f(X \setminus i\text{-int}(j\text{-cl}(U))) \subset f(X \setminus i\text{-int}(U)) = f(X \setminus U) = Y \setminus f(U)$. Thus $f(U) \subset (i, j)\text{-bint}(f(j\text{-cl}(U)))$ and hence f is (i, j) -weakly b -open. \square

Theorem 3.3. *The following statements are equivalent for a function $f : (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$.*

- (a) f is (i, j) -weakly b -open.
- (b) $f(i\text{-int}(F)) \subset (i, j)\text{-bint}(f(F))$, for every τ_j -closed set F of X .
- (c) $f(U) \subset (i, j)\text{-bint}(f(j\text{-cl}(U)))$, for every (i, j) -preopen set U of X .
- (d) $f(U) \subset (i, j)\text{-bint}(f(j\text{-cl}(U)))$, for every (i, j) - α -open set U of X .

Proof: (a) \Rightarrow (b) Let F be a τ_j -closed subset of X . Then we have $i\text{-int}(F)$ is τ_i -open. Since f is (i, j) -weakly b -open, therefore $f(i\text{-int}(F)) \subset (i, j)\text{-bint}(f(j\text{-cl}(i\text{-int}(F)))) \subset (i, j)\text{-bint}(f(F))$.

(b) \Rightarrow (c) Let U be a (i, j) -preopen set in X . Then by (b), we have $f(U) \subset f(i\text{-int}(j\text{-cl}(U))) \subset (i, j)\text{-bint}(f(j\text{-cl}(U)))$.

(c) \Rightarrow (d) Since every (i, j) - α -open set is (i, j) -preopen, so the result follows immediately.

(d) \Rightarrow (a) Let U be a τ_i -open set in X . Then U is (i, j) - α -open in X . Therefore by (d), we have $f(U) \subset (i, j)\text{-bint}(f(j\text{-cl}(U)))$. Hence f is (i, j) -weakly b -open. \square

Theorem 3.4. *The following statements are equivalent for a function $f : (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$.*

- (a) f is (i, j) -weakly b -open.
- (b) $f^{-1}((i, j)\text{-bcl}(B)) \subset (i, j)\text{-cl}_\theta(f^{-1}(B))$, for every subset B of Y .
- (c) $(i, j)\text{-bcl}(f(A)) \subset f((i, j)\text{-cl}_\theta(A))$, for every subset A of X .
- (d) $(i, j)\text{-bcl}(f(i\text{-int}((i, j)\text{-cl}_\theta(A)))) \subset f((i, j)\text{-cl}_\theta(A))$, for every subset A of X .

Proof: (a) \Rightarrow (b) Assume that f is (i, j) -weakly b -open. Let B be any subset of Y and $x \in f^{-1}((i, j)\text{-}bcl(B))$. Then $f(x) \in (i, j)\text{-}bcl(B)$. Let V be a τ_i -open set of X containing x . Since f is (i, j) -weakly b -open, therefore by theorem 3.1, there exists an (i, j) - b -open set U containing $f(x)$ such that $U \subset f(j\text{-}cl(V))$. Also, $f(x) \in (i, j)\text{-}bcl(B)$, therefore we get $U \cap A \neq \emptyset$ and hence $\emptyset \neq f^{-1}(U) \cap f^{-1}(B) \subset j\text{-}cl(V) \cap f^{-1}(B)$. Therefore we get $x \in (i, j)\text{-}cl_\theta(f^{-1}(B))$. Thus $f^{-1}((i, j)\text{-}bcl(B)) \subset (i, j)\text{-}cl_\theta(f^{-1}(B))$.

(b) \Rightarrow (c) Let A be any subset of X . Then we have $f^{-1}((i, j)\text{-}bcl(f(A))) \subset (i, j)\text{-}cl_\theta(f^{-1}(f(A))) \subset (i, j)\text{-}cl_\theta(A)$. Hence $(i, j)\text{-}bcl(f(A)) \subset f((i, j)\text{-}cl_\theta(A))$.

(c) \Rightarrow (d) Let A be any subset of X . Since $(i, j)\text{-}cl_\theta(A)$ is τ_i -closed in X , therefore by (b) and Lemma 2.4, we have $(i, j)\text{-}bcl(f(j\text{-}int((i, j)\text{-}cl_\theta(A)))) \subset f((i, j)\text{-}cl_\theta(j\text{-}int((i, j)\text{-}cl_\theta(A)))) = f(i\text{-}cl(j\text{-}int((i, j)\text{-}cl_\theta(A)))) \subset f(i\text{-}cl((i, j)\text{-}cl_\theta(A))) = f((i, j)\text{-}cl_\theta(A))$.

(d) \Rightarrow (a) Let V be any τ_j -open sub-set of X . Then by Lemma 2.4, $V \subset j\text{-}int(i\text{-}cl(V)) = j\text{-}int((i, j)\text{-}cl_\theta(V))$. Now, by (d) and Lemma 2.4, $(i, j)\text{-}bcl(f(V)) \subset (i, j)\text{-}bcl(f(j\text{-}int((i, j)\text{-}cl_\theta(V)))) \subset f((i, j)\text{-}cl_\theta(V)) = f(i\text{-}cl(V))$. Thus we obtain $(i, j)\text{-}bcl(f(V)) \subset f(i\text{-}cl(V))$ and hence by Theorem 3.2, we have f is (i, j) -weakly b -open. \square

Definition 3.2. [9] A bitopological space (X, τ_1, τ_2) is said to be (i, j) -regular if for each $x \in X$ and each τ_i -open set U containing x , there exists a τ_i -open set V such that $x \in V \subset j\text{-}cl(V) \subset U$.

Theorem 3.5. If X is (i, j) -regular, then for a bijective function $f : (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$, the following statements are equivalent :

- (a) f is (i, j) -weakly b -open.
- (b) $f(A)$ is (i, j) - b -closed in Y for every (i, j) - θ -closed set A of X .
- (c) $f(B)$ is (i, j) - b -open in Y for every (i, j) - θ -open set B of X .

(d) For every subset C of Y and for every (i, j) - θ -closed sub-set A of X such that $f^{-1}(C) \subset A$, there exists an (i, j) - b -closed sub-set F in Y containing C such that $f^{-1}(F) \subset A$.

Proof: (a) \Rightarrow (b) Let A be any (i, j) - θ -closed set of X . Since f is (i, j) -weakly b -open, therefore by theorem 3.4, we have $(i, j)\text{-}bcl(f(A)) \subset f((i, j)\text{-}cl_\theta(A)) = f(A)$. Hence $f(A)$ is (i, j) - b -closed subset in Y .

(b) \Rightarrow (c) Let B be any (i, j) - θ -open sub-set of X . Then $X \setminus B$ is (i, j) - θ -closed sub-set in X . By (b), $f(X \setminus B) = Y \setminus f(B)$ is (i, j) - b -closed in Y . Hence $f(B)$ is

(i, j) - b -open in Y .

(c) \Rightarrow (d) Let C be any subset of Y and A be an (i, j) - θ -closed set in X such that $f^{-1}(C) \subset A$. Since $X \setminus A$ is (i, j) - θ -open in X , therefore by (c), $f(X \setminus A)$ is (i, j) - b -open in Y . Let $F = Y \setminus f(X \setminus A)$. Then F is (i, j) - b -closed and $C \subset F$. Now, $f^{-1}(F) = f^{-1}(Y \setminus f(X \setminus A)) = f^{-1}(f(A)) \subset A$. Thus there exists an (i, j) - b -closed set F containing C such that $f^{-1}(F) \subset A$.

(d) \Rightarrow (a) Let C be any subset of Y . Let $A = (i, j)\text{-cl}_\theta(f^{-1}(C))$. Since X is (i, j) -regular, then A is (i, j) - θ -closed set in X and $f^{-1}(C) \subset A$. By (d), there exists an (i, j) - b -closed set F in Y containing C such that $f^{-1}(F) \subset A$. Since F is (i, j) - b -closed, we have $f^{-1}((i, j)\text{-bcl}(C)) \subset f^{-1}(F) \subset A = (i, j)\text{-cl}_\theta(f^{-1}(C))$. Hence by theorem 3.4, f is (i, j) -weakly b -open. \square

Definition 3.3. [10] A function $f : (X, \tau) \longrightarrow (Y, \sigma)$ is said to be strongly continuous if $f(\text{cl}(A)) \subset f(A)$, for every subset A of X .

Theorem 3.6. If a function $f : (X_1, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$ is (i, j) -weakly b -open and strongly j -continuous, then $f(A)$ is (i, j) - b -open in Y , for every τ_i -open sub-set A in X .

Proof: Let A be a τ_i -open set in X . Since f is (i, j) -weakly b -open and strongly j -continuous, therefore $f(A) \subset (i, j)\text{-bint}(f(j\text{-cl}(A))) \subset (i, j)\text{-bint}(f(A))$. Thus $f(A) = (i, j)\text{-bint}(f(A))$ and so $f(A)$ is (i, j) - b -open in Y . \square

Theorem 3.7. Let $f : (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$ be a bijective function. If $f((i, j)\text{-cl}_\theta(A))$ is (i, j) - b -closed in Y for every subset A of X , then f is (i, j) -weakly b -open.

Proof: Let A be any subset of X . Since $f((i, j)\text{-cl}_\theta(A))$ is (i, j) - b -closed, therefore $(i, j)\text{-bcl}(f(A)) \subset (i, j)\text{-bcl}(f((i, j)\text{-cl}_\theta(A))) = f((i, j)\text{-cl}_\theta(A))$. Hence by theorem 3.4, f is (i, j) -weakly b -open. \square

Definition 3.4. A function $f : (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$ is said to be (i, j) -contra b -open (respectively, (i, j) -contra b -closed) if $f(U)$ is (i, j) - b -closed (respectively, (i, j) - b -open) in Y for every τ_j -open (respectively, τ_j -closed) subset U of X .

Theorem 3.8. If a function $f : (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$ is (i, j) -contra b -closed, then f is (i, j) -weakly b -open.

Proof: Let A be any τ_i -open subset of X . Then $j\text{-cl}(A)$ is τ_j -closed in X . Therefore $f(A) \subset f(j\text{-cl}(A)) = (i, j)\text{-bint}(f(j\text{-cl}(A)))$. Hence f is (i, j) -weakly b -open. \square

Theorem 3.9. If a bijective function $f : (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$ is (i, j) -contra b -open, then f is (i, j) -weakly b -open.

Proof: Let A be any τ_j -open subset of X . Then $i\text{-cl}(A)$ is τ_i -closed in X . Since f is (i, j) -contra b -open, therefore $f(A)$ is (i, j) - b -closed. Now $(i, j)\text{-}bcl(f(A)) = f(A) \subset f(i\text{-cl}(A))$. By Theorem 3. 2, f is (i, j) -weakly b -open. \square

Definition 3.5. [13] A bitopological space (X, τ_1, τ_2) is said to be pairwise connected if it cannot be expressed as the union of two non-empty disjoint sub-sets A and B such that A is τ_i -open and B is τ_j -open.

Definition 3.6. A bitopological space (X, τ_1, τ_2) is said to be pairwise b -connected if it cannot be expressed as the union of two non-empty disjoint sets A and B such that A is (i, j) - b -open and B is (j, i) - b -open.

Theorem 3.10. If $f : (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$ is a bijective (i, j) -weakly b -open function of a space (X, τ_1, τ_2) onto a pairwise b -connected space (Y, σ_1, σ_2) , then (X, τ_1, τ_2) is pairwise connected.

Proof: Suppose that (X, τ_1, τ_2) is not pairwise connected. Then there exists a non-empty τ_i -open set A and a non-empty τ_j -open set B such that $A \cap B = \emptyset$ and $A \cup B = X$. This implies that $f(A) \cap f(B) = \emptyset$ and $f(A) \cup f(B) = Y$. Also, $f(A) \neq \emptyset$ and $f(B) \neq \emptyset$. Since f is (i, j) -weakly b -open, therefore $f(A) \subset (i, j)\text{-}bint(f(j\text{-cl}(A)))$ and $f(B) \subset (j, i)\text{-}bint(f(i\text{-cl}(B)))$. Again since A and B are τ_j -closed and τ_i -closed respectively, therefore we have $f(A) \subset (i, j)\text{-}bint(f(A))$ and $f(B) \subset (j, i)\text{-}bint(f(B))$. Hence $f(A) = (i, j)\text{-}bint(f(A))$ and $f(B) = (j, i)\text{-}bint(f(B))$. Consequently, $f(A)$ and $f(B)$ are (i, j) - b -open and (j, i) - b -open respectively. Which is a contradiction to the hypothesis that Y is pairwise b -connected. Hence (X, τ_1, τ_2) is pairwise connected. \square

Definition 3.7. [12] A bitopological space (X, τ_1, τ_2) is said to be (i, j) -hyperconnected if $j\text{-cl}(A) = X$, for every τ_i -open sub-set A of X .

Theorem 3.11. Let (X, τ_1, τ_2) be an (i, j) -hyperconnected space. Then a function $f : (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$ is (i, j) -weakly b -open if and only if $f(X)$ is (i, j) - b -open in Y .

Proof: Suppose that f is (i, j) -weakly b -open. Since X is τ_i -open, therefore we have $f(X) \subset (i, j)\text{-}bint(f(j\text{-cl}(X))) = (i, j)\text{-}bint(f(X))$. Hence $f(X)$ is (i, j) - b -open in Y .

Conversely, let $f(X)$ be (i, j) - b -open in Y and A be a τ_i -open set in X . Then $f(A) \subset f(X) = (i, j)\text{-}bint(f(X)) = (i, j)\text{-}bint(f(j\text{-cl}(A)))$. Hence f is (i, j) -weakly b -open. \square

Definition 3.8. [3] A subset A of a bitopological space (X, τ_1, τ_2) is said to be (i, j) -quasi H -closed relative to X if for each cover $\{B_\alpha : \alpha \in \Lambda\}$ of A by τ_i -open sets of X , there exists a finite subset Λ_0 of Λ such that $A \subset \bigcup\{j\text{-cl}(B_\alpha) : \alpha \in \Lambda_0\}$.

Definition 3.9. A subset A of a bitopological space (X, τ_1, τ_2) is said to be (i, j) - b -compact relative to X if every cover of A by (i, j) - b -open sub-sets of X has a finite subcover.

Theorem 3.12. If a bijective function $f : (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$ is (i, j) -weakly b -open and A is (i, j) - b -compact relative to Y , then $f^{-1}(A)$ is (i, j) -quasi H -closed relative to X .

Proof: Let A be (i, j) - b -compact relative to Y and $\{B_\alpha : \alpha \in \Lambda\}$ be an open cover of $f^{-1}(A)$ by τ_i -open sets of X . Therefore $f^{-1}(A) \subset \bigcup\{B_\alpha : \alpha \in \Lambda\}$ and so $A \subset \bigcup\{f(B_\alpha) : \alpha \in \Lambda\}$. Since f is (i, j) -weakly b -open, therefore $f(B_\alpha) \subset (i, j)$ - $bint(f(j-cl(B_\alpha)))$. Then $A \subset \bigcup\{(i, j)$ - $bint(f(j-cl(B_\alpha))) : \alpha \in \Lambda\}$. Also, A is (i, j) - b -compact relative to Y and (i, j) - $bint(f(j-cl(B_\alpha)))$ is (i, j) - b -open for each $\alpha \in \Lambda$, therefore there exists a finite subset Λ_0 of Λ such that $A \subset \bigcup\{(i, j)$ - $bint(f(j-cl(B_\alpha))) : \alpha \in \Lambda_0\}$. This implies that $f^{-1}(A) \subset \bigcup\{f^{-1}((i, j)$ - $bint(f(j-cl(B_\alpha)))) : \alpha \in \Lambda_0\} \subset \bigcup\{f^{-1}(f(j-cl(B_\alpha))) : \alpha \in \Lambda_0\} \subset \bigcup\{j-cl(B_\alpha) : \alpha \in \Lambda_0\}$. Hence $f^{-1}(A)$ is (i, j) -quasi H -closed relative to X . \square

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