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On Binary Structure of Supra Topological Spaces

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ABSTRACT: In this paper we introduce the new concept of binary supra topology and deals with concrete examples. Also we examine some binary supra topological properties. Further characterizations and properties of weak and strong forms binary supra continuity have been obtained.

Key Words: Supra topology, Binary topology, Binary continuity, Binary closure, Supra closure.

Contents

1. Introduction

Let X be a nonempty set. The subclass $\mu \subseteq$ P(X) where P(X) is a power set of X is called a supra topology on X if X, $\emptyset \in \mu$ and μ is closed under arbitrary union. The pair (X,μ) is called a supra topological space. The members of μ are called supraopen sets and some of the properties are discussed in [5]. Let (X,τ) be a topological space and μ be an supra topology on X. We call μ a supratopology associated with τ if $\tau \subset \mu$. Let (X,τ_1) and (Y,τ_2) be two topological space and μ be an associated supra topology with τ_1. A function f:X $\longrightarrow$ Y is supra continuous function if the inverse image of each open set in Y is supra open in X. A function f:(X,τ) $\longrightarrow$ (Y,σ) is said to be θ-continuous [7](weakly continuous [7]) if for each x $\in$ X and each V $\in \sigma$ containing f(x), there exist U $\in \tau$ containing x such that cl(U) $\subseteq f^{-1}(cl(V))$(U $\subseteq f^{-1}(cl(V))$). In 1965, O.Njastad [6] introduced a weak form of open sets called α-sets. A single structure which carries the subsets of X as well as the subsets of Y for studying the information about the ordered pair (A,B) of subsets of X and Y. Such a structure is called a binary structure from X to Y is

2000 *Mathematics Subject Classification:* 54C05 given in [11]. A point (x,y) $\in$ X $\times$ Y is called binary point of X and Y.

In this paper we introduce the concept of Binary supra topological space is nothing but a binary supra topology from X to Y is a binary structure $\mu \subseteq$P(X) $\times$ P(Y) that satisfies the following axioms. If (X,Y) $\in$ B_μ and ($\emptyset$, $\emptyset$) $\in$ B_μ and If $\{ (A_\alpha, B_\alpha) : \alpha \in \Delta\}$ is a family of members of B_μ, then $(\cup A_\alpha, \cup B_\alpha) \in B_\mu$. We also investigate binary supra α-continuous function and some of the relationship between them.

2. Preliminaries

In this section we have given the preliminaries and definition required in the sequel of our work.

Definition 2.1. *[7]:A function f:X $\longrightarrow$ Y is said to be η-continuous if for every regular open sets U,V of Y,*

(i) $f^{-1}(V) \subseteq int(cl(f^{-1}(V)))$.

(ii) $int(cl(f^{-1}(U \cap V))) = int(cl(f^{-1}(U))) \cap int(cl(f^{-1}(V)))$

Definition 2.2. *[3]: Let A be a subset of X. Then,*

(i) the supra closure of a set A is denoted by $cl_\mu(A)$ defined by $cl_\mu(A) = \cap$ {B: B is a supra closed and $A \subseteq B$ }.

(ii) the supra interior of a set A is denoted by $int_\mu(A)$ defined by $int_\mu(A) = \cup$ {G: G is a supra open and $A \supseteq G$}.

Definition 2.3. *[3]: A subset A of X is called,*

(i) supra semiopen if $A \subseteq cl_\mu(int_\mu(A))$.

(ii) supra α-open if $A \subseteq int_\mu(cl_\mu(int_\mu(A)))$.

(iii) supra preopen if $A \subseteq int_\mu(cl_\mu(A))$.

Lemma 2.1. *: Let A be a subset of a supra space (X,μ). Then A is supra α-open iff A is supra semiopen and supra preopen.*

Proof: :Necessity.Let A $\in$ α_μ. By the definition A $\subseteq$ $int_\mu(cl_\mu(A))$ and A $\subseteq$ $cl_\mu(int_\mu(A))$. Therefore we obtain A $\in$ $SO_\mu(X) \cap PO_\mu(X)$.
Sufficiency. A $\in$ $SO_\mu(X) \cap PO_\mu(X)$. Let A $\in$ $SO_\mu(X)$ and hence it follows from A $\in$ $PO_\mu(X)$ so that A $\subseteq$ $int_\mu(cl_\mu(A)) \subseteq int_\mu(cl_\mu(cl_\mu(int_\mu(A)))) = int_\mu(cl_\mu(int_\mu(A)))$. Therefore A $\in$ α_μ-open. □

Lemma 2.2. *: Let A and B be subsets of (X, μ). If either A $\in$ SO(X,μ) or B $\in$ SO(X,μ) then $int_\mu cl_\mu(A \cap B) = int_\mu(cl_\mu(A)) \cap int_\mu(cl_\mu(B))$.*

Proof: : For any subsets A $\subseteq$ X and B $\subseteq$ X, we generally have $int_\mu(cl_\mu(A \cap B)) \subseteq int_\mu(cl_\mu(A)) \cap int_\mu(cl_\mu(B))$. Assume A $\in$ SO(X,μ). Then we have $cl_\mu(A) \subseteq cl_\mu(int_\mu(A))$. Therefore $int_\mu(cl_\mu(A)) \cap int_\mu(cl_\mu(B)) = int_\mu(cl_\mu(int_\mu(cl_\mu(A)))) \cap int_\mu(cl_\mu(B)) \subseteq int_\mu(cl_\mu(cl_\mu(A))) \cap int_\mu(cl_\mu(B)) = int_\mu(cl_\mu(cl_\mu(int_\mu(A)))) \cap int_\mu(cl_\mu(B)) \subseteq int_\mu(cl_\mu(int_\mu(A) \cap cl_\mu(B))) \subseteq int_\mu(cl_\mu(int_\mu(A) \cap int_\mu(B))) \subseteq int_\mu(cl_\mu(A \cap B))$. This completes the proof. □

Definition 2.4. *[11]: Let X and Y be two nonempty sets and let (A,B) $\in$ P(X) $\times$ P(Y) and (C,D) $\in$ P(X) $\times$ P(Y) respectively. Then*

(i) *(A,B) $\subseteq$ (C,D) iff A $\subseteq$ C and B $\subseteq$ D.*

(ii) *(A,B) = (C,D) iff A = C and B = D.*

(iii) *(A,B) $\cup$ (C,D) iff (A $\cup$ C) and (B $\cup$ D)*

(iv) *(A,B) $\cap$ (C,D) iff (A $\cap$ C) and (B $\cap$ D)*

(v) $(A^c, B^c) = (X \setminus A, Y \setminus B)$

(vi) $(A, B) - (C, D) = (A, B) \cap (C, D)^c$.

Definition 2.5. *[11]: A binary topology from X to Y is a binary structure M$\subseteq$P(X) $\times$ P(Y) that satisfies the following axioms.*

(i) $(\emptyset, \emptyset)$ *and* $(X, Y) \in M$.

(ii) $(A_1 \cap A_2, B_1 \cap B_2) \in M$ *whenever* $(A_1, B_1) \in M$ *and* $(A_2, B_2) \in M$.

(iii) *If* $\{ (A_\alpha, B_\alpha) : \alpha \in \Delta\}$ *is a family of members of M, then* $(\cup A_\alpha, \cup B_\alpha : \alpha \in \Delta) \in M$.

If M is a binary topology from X to Y then the triplet (X,Y,M) is called binary topological space and the members of M are called binary open subsets of the binary topological space (X,Y,M). The elements of X $\times$ Y are called the binary points of the binary topological space (X,Y,M).

If Y = X then M is called a binary topology on X in which case we write (X,X,M) as a binary space.

Definition 2.6. *[11]: The ordered pair* $((A, B)^{1*}, (A, B)^{2*})$ *is called the binary closure of (A,B), denoted by B-cl(A,B) in the binary space (X,Y,M) where (A,B)* $\subseteq (X, Y)$ *and,*

(i) $(A, B)^{1^*} = \cap \{ A_\alpha : (A_\alpha, B_\alpha)$ *is binary closed and* $(A, B) \subseteq (A_\alpha, B_\alpha)\}$.

(ii) $(A, B)^{2^*} = \cap \{ B_\alpha : (A_\alpha, B_\alpha)$ *is binary closed and* $(A, B) \subseteq (A_\alpha, B_\alpha)\}$.

And also, (A,B) is binary closed in (X,Y,M) iff (A,B) = B-cl(A,B).

Definition 2.7. *[11]: Let (X,Y,M) be a binary topological space and (A,B)* $\subseteq (X, Y)$*. The ordered pair* $((A, B)^{1^o}, (A, B)^{2^o})$ *is called the binary interior of (A,B) denoted by B-int(A,B) where,*

(i) $(A,B)^{1^o} = \cup \{ A_\alpha : (A_\alpha, B_\alpha)$ *is binary open and* $(A_\alpha, B_\alpha) \subseteq$ *(A,B)*$\}$.

(ii) $(A,B)^{2^o} = \cup \{ B_\alpha : (A_\alpha, B_\alpha)$ *is binary open and* $(A_\alpha, B_\alpha) \subseteq$ *(A,B)*$\}$.

And also binary open in (X,Y,M) iff (A,B)= B-int(A,B).

Definition 2.8. *[11]: Let (X,Y,M) be a binary topological space.Let (A,B)* $\subseteq$ *(X,Y). Then (A,B) is called binary regular open if (A,B) = B-int(B-cl(A,B)).*

Definition 2.9. *[11]: If f:Z* $\longrightarrow X \times Y$ *be a function and* $A \subseteq X$ *and* $B \subseteq Y$*.We define* $f^{-1}(A,B) = \{ z \in Z : f(z) = (x,y) \in$ *(A,B)*$\}$.

3. Binary Supra Topology

In this section by merging binary and supra topological space we have formed a new topological structure called binary supra topological space and also paves way to some peculiar yields.

Definition 3.1. *: A Binary Supra Topology from X to Y is a binary structure* $B_\mu \subseteq$ *P(X)* $\times$ *P(Y) that satisfies the following axioms.*

(i) *If (X,Y)* $\in B_\mu$ *and* $(\emptyset,\emptyset) \in B_\mu$.

(ii) *If* $\{(A_\alpha, B_\alpha) : \alpha \in \Delta\}$ *is a family of members of* B_μ*, then* $(\cup A_\alpha, \cup B_\alpha) \in B_\mu$.

If B_μ is a binary supra topology from X to Y then the triplet (X,Y, B_μ) is called binary supra topological space. The elements of B_μ are called binary supra open sets. The complement of binary supra open sets are called Binary supra closed sets.

Definition 3.2. *: Let (X,Y,* B_μ*) be a binary supra topological space and let (x,y)*$\in X \times Y$*, then a subset (A,B) of (X,Y) is called a binary supra neighbourhood of (x,y) if there exist a binary supra open set (U,V) such that (x,y)* $\in$ *(U,V)* $\subseteq$ *(A,B).*

Example 3.3. *:Let X = {a,b,c} and Y = {1,2} with binary supra topology* $B_\mu = \{ (X,Y), (\emptyset,\emptyset), (\{b\},\{1\}), (\{a,b\},\{2\}), (\{a,b\},Y)\}$*.Hence* $(\{a,b\},Y), (\{a,b\}, \{2\}), (X,Y)$ *is binary supra neighbourhood of a point (a,2).*

Definition 3.4. *: Let (X,Y,* B_μ*) be a binary supra topological space and let (A,B) be a subset of (X,Y). Then a binary point (x,y)* $\in X \times Y$ *is called a limit point of (A,B) if [(U,V)-(x,y)]* $\cap$ *(A,B)* $\neq (\emptyset,\emptyset)$ *for all binary supra neighbourhood (U,V) of (x,y).*

Example 3.5. *:Let X = {a,b,c} with binary supra topology* $B_\mu = \{(X,X), (\emptyset,\emptyset), (\{b\},\{a\}), (\{a,b\},\{b\}), (\{a,b\},\{a,b\})\}$*. Let (A,B) = ({c},{a,c}). The set of all limit points of (A,B) are (a,c),(b,c),(c,a),(c,c),(c,b).*

Definition 3.6. *: The ordered pair* $((A,B)^{1^*}, (A,B)^{2^*})$ *is binary supra closure of (A,B), denoted by* $B_\mu cl(A,B)$ *in the binary supra space (X,Y,* B_μ*) where (A,B)* $\subseteq$ *(X,Y).*

(i) $(A,B)^{1^*} = \cap \{ A_\alpha : (A_\alpha, B_\alpha)$ *is binary supra closed and (A,B)* $\subseteq (A_\alpha, B_\alpha)\}$.

(ii) $(A,B)^{2^*} = \cap \{ B_\alpha : (A_\alpha, B_\alpha)$ *is binary supra closed and (A,B)* $\subseteq (A_\alpha, B_\alpha)\}$.

Definition 3.7. *: Let* (X,Y, B_μ) *be a binary supra topological space and (A,B)* $\subseteq (X,Y)$*. The ordered pair* $((A,B)^{1^o}, (A,B)^{2^o})$ *is called binary supra interior of (A,B), denoted by* $B_\mu int(A,B)$.

(i) $(A,B)^{1^o} = \cup \{ A_\alpha : (A_\alpha, B_\alpha)$ *is binary supra open and* $(A_\alpha, B_\alpha) \subseteq$ *(A,B)*$\}$.

(ii) $(A,B)^{2^o} = \cup \{ B_\alpha : (A_\alpha, B_\alpha)$ *is binary supra open and* $(A_\alpha, B_\alpha) \subseteq (A,B)\}$.

Example 3.8. *:Let X = {a,b,c} and Y = {1,2} with binary supra topology* $B_\mu = \{ (X,Y), (\emptyset,\emptyset), (\{a\}, Y), (\{a,b\}, \{2\}), (\emptyset, Y), (\{a,b\}, Y)\}$. B_μ *closed sets* $= \{ (X,Y), (\emptyset,\emptyset), (\{b,c\}, \emptyset), (\{c\}, \{1\}), (X, \emptyset), (\{c\}, \emptyset)\}$*. Let (A,B) = ({b},∅) then* $B_\mu cl(A,B)$ *= ({b,c},∅),* $B_\mu int(A,B)$ *= (∅,∅).*

4. Characterisations of binary supra interior and closure

Here we discussed the properties of binary supra interior and closure in binary supra space.

Theorem 4.1. *:In a binary supra topological space* (X,Y, B_μ) *if (A,B)* $\subseteq (X,Y)$ *then prove the following,*

(i) $B_\mu cl(A,B)$ *is the smallest* B_μ *closed set containing (A,B).*

(ii) (A,B) is binary supra closed in (X,Y, B_μ) *iff (A,B) =* $B_\mu cl(A,B)$.

Proof: : (i):Let $\{ (A_\alpha, B_\alpha) : \alpha \in \Delta\}$ be the collection of all binary supra closed sets containing (A,B). Then (C,D) = $\cap \{ (A_\alpha, B_\alpha) : \alpha \in \Delta\}$ is a binary supra closed set. Now each (A_α, B_α) is a superset of (A,B) that is (A,B) contained in their intersection. Therefore (A,B) $\subseteq$ (C,D) that is (C,D) $\subseteq$ (A_α, B_α) for each (x,y) $\in \Delta$ and hence (C,D) is the smallest binary supra closed set containing (A,B). Therefore $B_\mu cl(A,B)$ is the smallest binary closed set containing (A,B).
(ii): As we know that (A,B) be binary supra closed. Since (A,B) $\subseteq$ $B_\mu cl(A,B)$ that is $B_\mu cl(A,B)$ is the smallest binary supra closed set containing (A,B). Conversely, let $B_\mu cl(A,B)$ = (A,B) then (A,B) is binary supra closed because by definition of $B_\mu cl(A,B)$ is the smallest binary supra closed set containing (A,B) and $B_\mu cl(A,B)$ = (A,B) is given it follows that (A,B) is binary supra closed. □

Proposition 4.2. *:Let (A,B) and (C,D)* $\subseteq P(X) \times P(Y)$ *and* (X,Y, B_μ) *is a binary supra space. Then*

(i) $B_\mu cl(\emptyset,\emptyset) = (\emptyset,\emptyset)$ *and* $B_\mu cl(X,Y) = (X,Y)$.

(ii) (A,B) $\subseteq$ $B_\mu cl(A,B)$.

(iii) $B_\mu cl(B_\mu cl(A,B)) = B_\mu cl(A,B)$.

(iv) $B_\mu cl(A,B) \cup\ B_\mu cl(C,D) \subseteq\ B_\mu cl((A,B) \cup (C,D))$.

(v) $B_\mu cl((A,B) \cap (C,D)) \subseteq\ B_\mu cl(A,B) \cap\ B_\mu cl(C,D)$.

Proof: : (i): By the theorem 4.1 (A,B) is binary supra closed in (X,Y, B_μ) iff (A,B) = $B_\mu$cl(A,B) and since both $(\emptyset,\emptyset)$ and (X,Y) are binary supra closed sets then $B_\mu cl(\emptyset,\emptyset) =\ (\emptyset,\emptyset)$ and $B_\mu cl(X,Y) = (X,Y)$.
(ii): By the theorem 4.1 $B_\mu cl(A,B)$ is the smallest binary supra closed containing (A,B) so that (A,B) $\subseteq\ B_\mu cl(A,B)$.
(iii): By the theorem 4.1 (A,B) is binary supra closed in (X,Y, B_μ) iff (A,B) = $B_\mu$cl(A,B) and $B_\mu$cl(A,B) is also binary supra closed set. Hence $B_\mu cl(B_\mu cl(A,B))$ = $B_\mu cl(A,B)$.
(iv): (A,B) $\subseteq$ (A,B) $\cup$ (C,D) and (C,D) $\subseteq$ (A,B) $\cup$ (C,D). Therefore $B_\mu cl(A,B) \subseteq B_\mu cl((A,B)\cup(C,D))$ and $B_\mu cl(C,D) \subseteq\ B_\mu cl((A,B)\cup(C,D))$. Hence $B_\mu cl(A,B)$ $\cup\ B_\mu cl(C,D) \subseteq\ B_\mu cl((A,B) \cup (C,D))$.
(v): (A,B) $\cap$ (C,D) $\subseteq$ (A,B) and (A,B) $\cap$ (C,D) $\subseteq$ (C,D). Therefore $B_\mu$cl((A,B) $\cap$ (C,D)) $\subseteq\ B_\mu$cl(A,B). Therefore $B_\mu$cl((A,B) $\cap$ (C,D)) $\subseteq\ B_\mu cl(C,D)$. Hence $B_\mu$cl((A,B)$\cap$ (C,D)) $\subseteq\ B_\mu$cl(A,B) $\cap\ B_\mu$cl(C,D). □

Proposition 4.3. *: Let (X,Y, B_μ) be a binary supra topological space and let (A,B) be a subset of (X,Y, B_μ). Then*

(i) $B_\mu int(A,B)$ *is an binary supra open.*

(ii) $B_\mu int(A,B)$ *is the largest binary supra open set contained in (A,B).*

(ii) (A,B) is binary supra open if and only if $B_\mu int(A,B)$ = *(A,B).*

Proof: : (i): We know that every binary supra open set is a binary supra neighbourhood of each of its points. Let (x,y)$\in\ B_\mu int(A,B) \Longrightarrow$ (x,y) is an binary supra interior point of (A,B) so that there exist a binary supra open set (C,D) such that (x,y) $\in$ (C,D) $\subseteq$ (A,B). Now (C,D) is binary supra open, it is a binary supra neighbourhood of each of its points and hence (A,B) being a superset of (C,D) is also binary supra neighbourhood of each point of (C,D). Hence by definition every binary point of (C,D) is an binary supra interior point of (C,D). Therefore (C,D) $\subseteq\ B_\mu int(A,B)$. Now from (i)(x,y) $\in$ (C,D) $\subseteq\ B_\mu int(A,B)$. Since (x,y) chosen arbitrarily, it follows that each (x,y) $\in\ B_\mu int(A,B)$ which is contained in $B_\mu int(A,B)$. Hence $B_\mu int(A,B)$ is a neighbourhood of each of its points and consequently $B_\mu int(A,B)$ is binary supra open.
(ii): Let (C,D) be any open subset of (A,B) and let (x,y) $\in$ (C,D) that is (x,y) $\in$ (C,D) $\subseteq$ (A,B). Since (C,D) is binary supra open, (A,B) is a neighbourhood of (x,y) $\in$ (C,D) and consequently (x,y) is an binary supra interior of (A,B). Since (x,y) $\in$ (C,D) $\Longrightarrow$ (x,y) $\in\ B_\mu int(A,B)$. Therefore (C,D) $\subseteq\ B_\mu int(A,B)$ and by (i) is an binary supra open set. Thus $B_\mu int(A,B)$ contains every binary supra open subset (C,D) of (A,B) and as such $B_\mu int(A,B)$ is the largest subset of (A,B).
(iii):Let (A,B) = $B_\mu int(A,B)$,we know that $B_\mu int(A,B)$ is an binary supra open

then (A,B) is also binary supra open. Conversely (A,B) is binary supra open. Then by (i), $B_\mu int(A, B)$ is the largest binary supra open subset of (A,B). Hence $B_\mu int(A, B) =$ (A,B). □

Theorem 4.4. *:Let (X,Y, B_μ) be a binary supra topological space and (A,B),(C,D) be any subsets of (X,Y), then prove the following.*

(i) $B_\mu int(\emptyset, \emptyset) = (\emptyset, \emptyset)$.

(ii) $B_\mu int(X, Y) = (X, Y)$.

(iii) $B_\mu int((A, B) \cap (C, D)) \subseteq B_\mu int(A, B) \cap B_\mu int(C, D)$.

(iv) $B_\mu int(B_\mu int(A, B)) = B_\mu int(A, B)$.

(v) $B_\mu int(A, B) \cup B_\mu int(C, D) \subseteq B_\mu int((A, B) \cup (C, D))$

Proof: : (i) and (ii): By the proposition 4.3 (A,B) is binary supra open iff $B_\mu int(A, B) =$ (A,B). Since both $(\emptyset, \emptyset)$ and (X,Y) are binary supra open sets it follows that $B_\mu int(\emptyset, \emptyset) = (\emptyset, \emptyset)$, $B_\mu int(X, Y) =$ (X,Y).
(iii): (A,B) ∩ (C,D) ⊆ (A,B) and (C,D) ∩ (A,B) ⊆ (C,D). Then $B_\mu int((A, B) \cap (C, D)) \subseteq B_\mu int(A, B)$, $B_\mu int((A, B) \cap (C, D)) \subseteq B_\mu int(C, D)$. Hence $B_\mu int((A, B) \cap (C, D)) \subseteq B_\mu int(A, B) \cap B_\mu int(C, D)$.
(iv): As we know that $B_\mu int(A, B)$ is binary supra open and hence $B_\mu int(B_\mu int(A, B)) = B_\mu int(A, B)$.
(v): We know that $(A, B) \subseteq (A, B) \cup (C, D) and (C, D) \subseteq (A, B) \cup (C, D)$. Therefore $B_\mu int(A, B) \subseteq B_\mu int((A, B) \cup (C, D))$ and $B_\mu int(C, D) \subseteq B_\mu int((A, B) \cup (C, D))$. Hence $B_\mu int(A, B) \cup B_\mu int(C, D) \subseteq B_\mu int((A, B) \cup (C, D))$. □

5. Binary supra continuity

In this section we define a new form of continuity called binary supra continuity which is a map from a topological(single dimension)space to a binary space(2-D space).

Definition 5.1. *: Let (X,Y,M) be a binary topological space, let (Z,τ) be a topological space and μ be an supra topology associated with τ. Let f:Z ⟶ X × Y be a function. Then f is called binary supra continuous if* $f^{-1}(A, B)$ *is supra open in Z for every binary open set (A,B) in X × Y.*

Example 5.2. *:Let X = {a,b,c} and Z = {p,q,r,s} with binary topology M =* $\{ (X, X), (\emptyset, \emptyset), (\{b\}, \{a\}), (\{b, c\}, \{a\}), (\{b, c\}, \{a, b\})\}$. *Binary closed sets =* $\{ (X, Y), (\emptyset, \emptyset), (\{a, c\}, \{b, c\}), (\{a\}, \{b, c\}), (\{a\}, \{c\})\}$. $\tau = \{Z, \{p\}, \{q, r\}, \emptyset, \{p, q, r\}\}$, $\mu = \{Z, \{p\}, \{q, r\}, \{p, r\}, \emptyset, \{p, q, r\}, \{p, s\}, \{q, r, s\}, \{p, r, s\}\}$. *f(p) = (b,a),f(q) = (a,c),f(r) = (b,a),f(s) = (b,a). Hence f is binary supra continuous.*

Theorem 5.3. *: Let (X,Y,M) is a binary topological spaces, (Z,τ) is a topological space and μ be an associated supra topology with τ. Then a function f: Z ⟶ X × Y binary supra continuous iff if the inverse image under f of every binary open set (A,B) in (X,Y,M) is supra open in Z.*

Proof: : If $f^{-1}(U,V) = \emptyset$ then $\emptyset$ is supra open. But if $f^{-1}(U,V) \neq \emptyset$ then let x be an arbitrary element of $f^{-1}(U,V)$ so that f(x) ∈ (U,V). As f is binary supra continuous hence corresponding to binary open set (U,V) in X × Y there exists an supra open set G containing x such that f(G) ⊆ (U,V) or G ⊆ $f^{-1}(U,V)$. Hence x ∈ G ⊆ $f^{-1}(U,V)$. It is clear that $f^{-1}(U,V)$ is a supra neighbourhood of x. Therefore $f^{-1}(U,V)$ is supra open.
Conversely to prove that f:Z ⟶ X × Y is binary supra continuous, that is it is binary supra continuous at every point x of Z. Let (U,V) be any binary open set containing f(x) so that x ∈ $f^{-1}(U,V)$ where $f^{-1}(U,V)$ is supra open set. Put $f^{-1}(U,V)$ = A where A is an supra open set containing x. Also f(A) = $f(f^{-1}(U,V))$ ⊆ (U,V). Hence by definition, f is binary supra continuous at x, but x is arbitrary it follows that f is binary continuous at every point x of Z. Hence f is binary supra continuous. □

Theorem 5.4. *: Let (X,Y,M) is a binary topological spaces,(Z,τ) is a topological space and μ be an associated supra topology with τ. Let f:Z ⟶ X × Y be a function such that Z \ $f^{-1}(A,B)$ = $f^{-1}(X \setminus A, Y \setminus B)$ for all A ⊆ X and B ⊆ Y. Then f is binary supra continuous iff $f^{-1}(A,B)$ is supra closed in Z for all binary closed sets (A,B) in (X,Y,M).*

Proof: :Assume that f is binary supra continuous. Let (A,B) ∈ X × Y be binary closed. Therefore,(X \A, Y \B) is binary open set. Since f is binary supra continuous, we have f^{-1}(X \A,Y \B) is supra open in Z. Therefore Z \ f^{-1}(A,B) is supra open in Z. Hence $f^{-1}(A,B)$ is supra closed in Z. Conversely, assume that if $f^{-1}(A,B)$ is binary supra closed in Z for all binary closed set (A,B) in (X,Y, B_μ). Let (A,B) ∈ X×Y be a binary open set. To prove $f^{-1}(A,B)$ is supra open in Z. Since (A,B) ∈ B_μ, we have (X \A, Y \B) is binary closed set in X × Y. Therefore, by our assumption f^{-1}(X \A,Y \B) is supra closed in Z. Thus, Z \ f^{-1}(A,B) is supra closed in Z. Hence $f^{-1}(A,B)$ is supra closed in Z. Therefore f is binary continuous. □

Theorem 5.5. *: Let the function f:Z ⟶ X × Y is binary supra continuous iff for every subset (A,B) of X × Y , $int_\mu(f^{-1}(A,B)) \supseteq f^{-1}(B-int(A,B))$.*

Proof: :Let (A,B) be any subset of (X,Y) then $B-int(A,B)$ is binary open and f being binary supra continuous it follows that $f^{-1}(B-int(A,B))$ is a supra open set in Z. Therefore $int_\mu(f^{-1}(B-int(A,B))) = f^{-1}(B-int(A,B))$. Again B-int(A,B) ⊆ (A,B) $\Longrightarrow f^{-1}(B-int(A,B)) \subseteq f^{-1}(A,B) \Longrightarrow int_\mu(f^{-1}(B-int(A,B))) \subseteq int_\mu(f^{-1}(A,B))$. Hence $int_\mu(f^{-1}(A,B)) \supseteq f^{-1}(B-int(A,B))$. Conversely, Let (A,B) be any binary open set in (X,Y) so that B-int(A,B) = (A,B). Now $int_\mu(f^{-1}(A,B)) \supseteq f^{-1}(B-int(A,B)) = f^{-1}(A,B)$. Since B-int(A,B) = (A,B). Therefore $int_\mu(f^{-1}(A,B)) \supseteq f^{-1}(A,B)$. But $int_\mu(f^{-1}(A,B)) \subseteq f^{-1}(A,B)$, since B-int(A,B) = (A,B). Hence $int_\mu(f^{-1}(A,B)) = f^{-1}(A,B)$. Here $f^{-1}(A,B)$ is supra open where (A,B) is binary open. Therefore f is binary supra continuous. □

6. Distinct forms of continuity in binary supra space

In binary supra space we have explored various forms of continuity and its properties were investigated.

Definition 6.1. *: A subset (A,B) of $(X,Y,\ B_\mu)$ is called*

(i) *a binary supra α-open set if (A,B) $\subseteq\ B_\mu int(B_\mu cl(B_\mu int(A,B)))$.*

(ii) *a binary supra semiopen set if (A,B) $\subseteq B_\mu cl(B_\mu int(A,B))$.*

(iii) *a binary supra preopen set if (A,B) $\subseteq B_\mu int(B_\mu cl(A,B))$.*

(iv) *a binary supra regular open set if (A,B) $=\ B_\mu int(B_\mu cl(A,B))$*

(v) *a binary supra nowhere dense set if (A,B) $\neq B_\mu int(B_\mu cl(A,B))$*

The family of all binary supra α-open sets (binary supra semiopen sets, binary supra preopen sets, binary supra regular open sets) is denoted by $B_\mu\alpha$($B_\mu SO$, $B_\mu PO$, $B_\mu RO$).

Remark 6.2. *Binary supra α-open sets forms a binary supra topology. Every binary supra open sets is binary supra α-open sets but the converse is not true.*

Example 6.3. *:Let X = {a,b} and Y = {1,2} with binary supra topology $B_\mu = \{\ (X,Y),(\emptyset,\emptyset),(\{a\},\{2\}),(\{b\},Y)\}$. $B_\mu\alpha = \{\ (X,Y),(\emptyset,\emptyset),(X,\{2\}),(\{a\},Y),(\{a\},\{2\}),(\{b\},Y)\}$. Here $B_\mu\alpha$-open sets need not be binary supra open set.*

Example 6.4. *:Let X = {a,b} and Y = {1,2} with binary supra topology $B_\mu = \{\ (X,Y),(\emptyset,\emptyset),(X,\emptyset),(X,\{1\}),(X,\{2\}),(\{a\},\emptyset),(\{b\},\emptyset)\}$. $B_\mu RO = \{\ (X,Y),(\emptyset,\emptyset)\}$. Hence $B_\mu RO$ is binary supra open but the converse not true.*

Definition 6.5. *: Let (X,Y,M) be a binary topological space,let (Z,τ) be a topological space and μ be an supra topology associated with τ. Let f:Z $\longrightarrow$ X $\times$ Y be a function then:*

(i) *f is called binary supra α-continuous if $f^{-1}(A,B)$ is supra α-open in Z for every binary open set (A,B) in X $\times$ Y.*

(ii) *f is called binary supra semicontinuous if $f^{-1}(A,B)$ is supra semiopen in Z for every binary open set (A,B) in X $\times$ Y.*

(iii) *f is called binary supra precontinuous if $f^{-1}(A,B)$ is supra preopen in Z for every binary open set (A,B) in X $\times$ Y.*

Definition 6.6. *: Let (X,Y,M) is a binary topological spaces, (Z,τ) is a topological space and μ be an associated supra topology with τ. A function f:Z $\longrightarrow$ X $\times$ Y is said to be*
(i) Binary supra η-continuous function if for every binary regular open sets (A,B), (C,D) of X $\times$ Y,

(a) $f^{-1}(C, D) \subseteq int_\mu(cl_\mu(f^{-1}(C, D)))$ *and*

(b) $int_\mu(cl_\mu(f^{-1}((A, B) \cap (C, D)))) =$
$int_\mu(cl_\mu(f^{-1}(A, B))) \cap int_\mu(cl_\mu(f^{-1}(C, D)))$.

(ii) Binary supra θ *continuous if for each* $x \in Z$ *and each* $(U, V) \in B_\mu$ *containing* $f(x)$*, there exists* $A \in \mu$ *containing* x *where* $cl_\mu(A) \subseteq f^{-1}(B_\mu cl(U, V))$.
(iii)Binary supra weakly continuous if if for each $x \in Z$ *and each* $(U, V) \in B_\mu$ *containing* $f(x)$*, there exists* $A \in \mu$ *containing* x *where* $A \subseteq f^{-1}(B_\mu cl(U, V))$.

Theorem 6.7. *: Every binary continuous function is binary supra* α*-continuous function.*

Proof: : Let f:Z $\longrightarrow$ X $\times$ Y be a binary continuous function and (A,B) is binary open in (X,Y). Then $f^{-1}(A, B)$ is open in Z. Since μ is associated with τ then $\tau \subset \mu$. Therefore $f^{-1}(A, B)$ is supra open in Z and it is supra α open in Z. Hence f is binary supra α-continuous function. □

Remark 6.8. *: The converse of the above theorem is not true as shown in following example*

Example 6.9. *:Let X = {a,b}, Y = {1,2} and Z = {p,q,r} with binary topology* $M = \{(X, Y), (\emptyset, \emptyset), (\{a\}, \{2\}), (\{b\}, \{1\})\}$, $\tau = \{Z, \emptyset, \{p\}, \{p, q\}\}$. $\mu = \{Z, \emptyset, \{p\}, \{q\}, \{p, q\}\}$ *where* μ *is assosiated with* τ*. Supra* α*-open sets* $= \{Z, \emptyset, \{p\}, \{q\}, \{p, q\}, \{q, r\}, \{p, r\}\}$*. Define* f*:Z* $\longrightarrow$ *X* $\times$ *Y, where f(p)=(b,1), f(q)=(b,1), f(r)=(a,1), So* f *is binary supra* α*-continuous but not binary supra continuous.*

Theorem 6.10. *: Let* $f : Z \longrightarrow X \times Y$ *be a mapping, and if* $Z \setminus f^{-1}(A,B) = f^{-1}(X \setminus A, Y \setminus B)$ *for all* $A \subseteq X$ *and* $B \subseteq Y$ *then the following statements are equivalent.*

(i) f is binary supra α-continuous.

(ii) The inverse image of each binary closed set in X $\times$ Y is supra α-closed.

(iii) $\alpha_\mu cl(f^{-1}(A, B)) \subseteq f^{-1}(B - cl(A, B))$ for every binary set (A,B) of X $\times$ Y.

(iv) $f^{-1}(B - int(C, D)) \subseteq \alpha_\mu int(f^{-1}((C, D)))$ for every binary set (C,D) in X $\times$ Y.

Proof: : (i)$\Longrightarrow$(ii):trivial.
(ii)$\Longrightarrow$(iii): Since (A,B) is binary closed in X $\times$ Y, then it follows that $f^{-1}(B-cl(A, B))$ is supra α-closed in Z. Therefore, $f^{-1}(B - cl(A, B)) = \alpha_\mu cl(f^{-1}(B - cl(A, B))) \supseteq \alpha_\mu cl(f^{-1}(A, B))$.
(ii)$\Longrightarrow$(iv): Since (A,B) is binary closed in X $\times$ Y, then it follows that X \A and Y \B is binary open where (X,Y) \(A,B) = (C,D) it follows that $f^{-1}int(C, D)$ is supra open in Z. Therefore, $f^{-1}(B - int(C, D)) = \alpha_\mu int(f^{-1}(B - int(C, D))) \subseteq \alpha_\mu int(f^{-1}(C, D))$
(iv)$\Longrightarrow$(i): it is obvious. □

Proposition 6.11. *: Let f:Z $\longrightarrow$ X $\times$ Y be a mapping and if Z $\setminus$ f^{-1}(A,B)= f^{-1}(X $\setminus$A, Y $\setminus$B) where A $\subseteq$ X and B $\subseteq$ Y then the following are equivalent.*

(i) f is binary α_μ-continuous.

(ii) The inverse image of each binary closed set in X $\times$ Y is α_μ closed.

(iv) $cl_\mu(int_\mu(cl_\mu(f^{-1}(U,V)))) \subseteq f^{-1}(cl(U,V))$ for each (U,V) $\subseteq$ (X,Y).

Proof: : The proof is obvious. □

Theorem 6.12. *: A function f:Z $\longrightarrow$ X $\times$ Y is binary supra α- continuous function iff binary supra semicontinuous and binary supra precontinuous function.*

Proof: : Let f is binary supra α-continuous then inverse image of each binary open set in X $\times$ Y is supra α-open in Z. Since by using lemma 2.1 every binary supra α-continuous function is binary supra semicontinuous and binary supra precontinuous. Converse is obvious. □

Remark 6.13. *:The concept of binary supra precontinuous and binary supra semicontinuous are independent of each other.*

Example 6.14. *: Let X = {a,b}, Y = {1,2} and Z = {p,q,r,s} with binary topology M = {(X,Y), ($\emptyset$, $\emptyset$), ({a},{2}), ({a},Y), ({b},{1})},τ = {Z, $\emptyset$, {p}, {q,r,s}}, μ = {Z, $\emptyset$, {p}, {q,r}, {r,s}, {q,r,s}, {p,q,r}, {p,s}, {p,r,s}, {s}}, SO_μ = {Z, $\emptyset$, {p}, {s}, {p,s}, {q,r}, {r,s}, {p,q,r}, {q,r,s}, {p,r,s}}, PO_μ = {Z, $\emptyset$, {p}, {r}, {p,r}, {q,r}, {q,s}, {s}, {r,s}, {p,q,r}, {p,q,s}, {q,r,s}, {p,r,s}, {p,s}},Define f: Z $\longrightarrow$ X $\times$ Y, where f(p) = (a,2), f(q) = (a,1), f(r) = (a,2), f(s)=(b,2). Here $f^{-1}(\{a\},\{2\})$ = {p,r} which is supra preopen but not supra semiopen. Therefore f is binary supra precontinuous but not binary supra semicontinuous.*

Theorem 6.15. *: Every binary supra α-continuous mapping f:Z $\longrightarrow$ X $\times$ Y is binary supra θ-continuous.*

Proof: : Let x $\in$ Z and (A,B) $\subseteq$ (X,Y) be an binary open set containing f(x), By proposition $cl_\mu(int_\mu(cl_\mu(f^{-1}(A,B)))) \subseteq f^{-1}(B-cl(A,B))$. Since f is binary supra α continuous,then $f^{-1}(A,B) \subseteq int_\mu(cl_\mu(int_\mu(f^{-1}(A,B))))$
$\subseteq cl\mu(int_\mu(cl_\mu(int_\mu(f^{-1}(A,B))))) \subseteq cl_\mu(int_\mu(cl_\mu(f^{-1}(A,B)))) \subseteq f^{-1}(B-cl(A,B)$. Put $int_\mu(cl_\mu(int_\mu(f^{-1}(A,B))))$ = U, so U is a supra neighbourhood of x such that $cl_\mu(U) \subseteq f^{-1}(B-cl(A,B))$. Hence $cl_\mu(U) \subseteq f^{-1}(B-cl(A,B))$. Therefore, f is binary supra θ-continuous. □

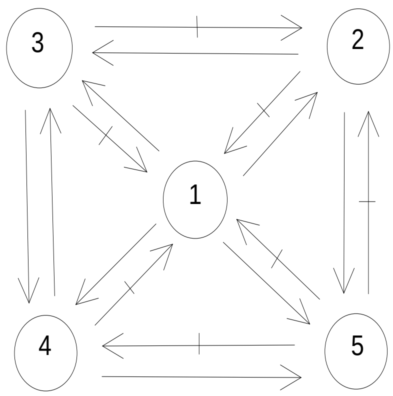
Example 6.16. *: Let X = {a,b}, Y = {1,2} and Z = {p,q,r} with binary topology M = $\{(X,Y),(\emptyset,\emptyset),(\{a\},\{2\}),(\{b\},\{1\})\}$. τ = { Z,$\emptyset$,{q,r}},μ = {Z,$\emptyset$,{p,q}, {q,r}}.$\mu = \alpha_\mu$, Define f:Z $\longrightarrow$ X $\times$ Y, where f(p)=(a,1), f(q)=(a,2), f(r)=(a,2), Here f is binary supra weakly continuous but not binary supra θ-continuous.*

Theorem 6.17. *: If a function f:Z $\longrightarrow X \times Y$ is binary supra α-continuous, then f is binary supra η-continuous.*

Proof: : Since f is binary supra α-continuous, by the lemma 2.1 $f^{-1}(U,V) \subseteq \alpha_\mu \in PO(Z,\mu)$ for any (U,V) $\in$ (X,Y,M) and hence $f^{-1}(U,V) \subseteq int_\mu(cl_\mu(f^{-1}(U,V)))$. Further, $f^{-1}(A,B)$, $f^{-1}(U,V) \in \alpha_\mu \subseteq SO(Z,\mu)$ for any (U,V), (A,B) $\in$ (X,Y,M) and hence by lemma 2.2 we have $int_\mu(cl_\mu f^{-1}(((U,V) \cap (A,B)))) = int_\mu(cl_\mu(f^{-1}(U,V))) \cap int_\mu(cl_\mu(f^{-1}(A,B)))$. It follows that f is binary supra η-continuous. □

Example 6.18. *: Let X = {a,b,c} and Z = {p,q,r} with binary topology M = $\{ (X,X), (\emptyset,\emptyset), (\{a\},\{b\})\}$, MRO = $\{ (X,X), (\emptyset,\emptyset)\}$,$\tau = \{Z,\emptyset,\{p\}\}$. $\mu = \{ Z,\emptyset,\{p\},\{p,r\},\{q,r\}\}$. $\mu_\alpha = \{ Z,\emptyset,\{p\},\{p,r\},\{q,r\}$, Define f:Z $\longrightarrow X{\times}X$ where f(p) = (b,c),f(q) = (a,b),f(r) = (b,c). Therefore f is binary supra η-continuous but not binary supra α- continuous.*

Remark 6.19. *:For a function f:Z $\longrightarrow X \times Y$, the following implication are known.*



1.Binary supra continuous, 2.Binary supra α-continuous, 3.Binary supra η-continuous,4.Binary supra θ-continuous, 5.Binary supra weakly continuous.

7. Conclusion

We introduce the basic concepts of binary supra topological spaces. We then studied some fundamental properties of binary supra topological space. Moreover we introduce the concepts of binary supra α-continuous maps by using binary supra α-open sets and investigated their behaviour. The interrelations among theta-continuous, η-continuous, weakly continuous in binary supra topological space were also studied. Further our concepts of binary supra topological space can be extended to stronger forms.

References

1. Devi.R, Sampathkumar.S and Caldas.M,*On Supra α-open sets and Supra α-continuous function.*, General Mathematics.,Vol 16, No.2, 77-84,(2008).

2. Dickman.R.F, Porter.J.R, Rubin.L.R,*Completely regular absolutes and projective objects.*, Pacific J.Math 94,277-295,(1981).

3. Kamaraj.M, Ramkumar.G, Ravi.O,*On Supra Quotient mappings*, International Journal of Mathematics Archive-3(1), 245-252,(2012).

4. Long.P.E and McGehee.E.E Jr.,*Properties of Almost Continuous function*, Proceedings of the American Mathematical Society,Vol.24, No 1, 175-180,(1970).

5. Mashhour.A.S, Allam.A.A, Mahmoud.F.S, and Khedr.F.H, *On Supra Topological Spaces., Indian J.Pure appl.Math, 14(4):April, 502-510,(1983).*

6. *Njastad.O,On some classes of nearly open sets.,Pacific J.Math. 15, 961-970,(1965).*

7. *Noiri.T, Yatsuhiro., On α-continuous function, Casopis pro pestovanimatematiky Vol.109, No 2, 118-126,(1984).*

8. *Popa.V,On some weaker forms of continuity, studi.Cerc.Mat. 33, 543-546,(1981).*

9. *Reilly.I.L and Vamanamurthy.M.K, On α-continuity In Topological Spaces, Acta Math.Hungar.45,27-32,(1985).*

10. *Singal.M.K, Singal.A.R,Almost continuous mappings,Yokohama Math.J 16, 63-73,(1968).*

11. *Thangavelu.P and Nithyanantha Jothi,Topology between two sets, Journal of Mathematical Sciences and Computer Applications 1(3):, 95-107,(2011).*

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