



## Some results on soft bitopology

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**ABSTRACT:** In this paper we study some properties under soft bitopology. We have studied properties introducing soft nowhere dense set, soft boundary of a soft set, soft boundary with respect to a point, first category etc. from the point of view of soft bitopological space. In this paper we contribute some results in “soft bitopology” which was initially introduced by Şenel and Çağman in the year 2014.

**Key Words:** Soft set, soft topology, soft bitopology.

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### 1. Introduction and motivation

It seems easy to understand that a mathematical theory is based on various abstract thoughts. In these cases one has full freedom to establish certain environment depending on many neglecting facts; for example in physics we often neglect the frictional effect of air on a free falling body, but this fact is fully impossible in real life. Similarly other branches like medicine, economics, engineering, social sciences etc are full of uncertainties. Before 1999, we had only four mathematical tools to deal with uncertainties, namely probability theory, fuzzy set theory, rough set theory and the theory of interval mathematics. To overcome the choice of degree of membership in fuzzy set theory when the facts are concerned with uncertainties, Molodtsov [14] introduced the concept of soft set theory in the year 1999 and investigated various applications in game theory, smoothness of functions, operation researches, Perron integration, probability theory, theory of measurement and so on.

Later Maji et al. [25] defined various operations on soft sets to study some of the fundamental properties. Pei and Miao [13], Chen [12] pointed out errors in some of the results of the paper of Maji et al. [25] and introduced some new notions and properties. At present; investigations of different properties and applications of soft set theory have attracted many researchers from various backgrounds. It

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has been used in fuzzy set theory too (one may refer to [10,11]).

In the year 2011, Shabir and Naz [22] introduced soft topology and studied some introductory results. In the same year Çağman et al. [23] introduced soft topology in a different approach. Till then various researchers have studied various foundational results in soft topology (one may refer to [22,27,28,32]). Kandil et al. [2,3,4] introduced concept of soft ideal and studied some other weaker notations and properties. Recently Yuksel et al. [30] applied soft set theory to determine prostate cancer risk. Recently various forms of fuzzy soft topologies have come in existence from the view point of fuzzy set theory viz intuitionistic fuzzy soft topology, fuzzy soft topology etc. More recently Şenel and Çağman [18] introduced soft topological subspace and studied some properties.

It is well known to us that both general topology and fuzzy topology play crucial roles in mathematics, economics, data reduction, image processing, information sciences, genotype-phenotype mapping of DNA etc. One may refer to [7,24,9] for applications of topology in biology, where modified fundamental topological results play crucial role to resolve various difficulties with DNA, mRNA etc. Hence in this paper we are interested to contribute some results in soft bitopology which is a newly developed area by Şenel and Çağman in the year 2014 [17]. In this paper we mainly focus on soft finer topology via soft bitopology as it is well known that coarser topology ( resp. bitopology), coarser fuzzy topology (resp. fuzzy bitopology) etc play crucial role in various information sciences related to computer, biology, remote sensing, signalling etc.

In this paper we will follow Çağman's notion of soft topology.

## 2. Basic definitions

In this section we discuss some basic definitions and notions those are defined by various authors.

**Definition 2.1.** (see [23]) A soft set  $F_A$  on the universe  $U$  is defined by the set of ordered pairs  $F_A = \{(x, f_A(x)) : x \in E, f_A(x) \in P(U)\}$  where  $f_A : E \rightarrow P(U)$  such that  $f_A(x) = \emptyset$  if  $x \notin A$ .

Here  $f_A$  is called an *approximate function* of the soft set  $F_A$ . The value of  $f_A(x)$  may be arbitrary. Some of them may be empty, some may have nonempty intersection. we will denote the set of all soft sets over  $U$  as  $S(U)$ .

**Definition 2.2.** (see [23]) Let  $F_A \in S(U)$ . If  $f_A(x) = \emptyset$  for all  $x \in E$ , then  $F_A$  is called a *soft empty set*, denoted by  $F_\emptyset$ .

$f_A(x) = \emptyset$  means there is no element in  $U$  related to the parameter  $x \in E$ . Therefore, we do not display such elements in the soft sets, as it is meaningless to consider such parameters.

**Definition 2.3.** (see [23]) Let  $F_A \in S(U)$ . If  $f_A(x) = U$  for all  $x \in A$ , then  $F_A$  is called an  $A$ -universal soft set, denoted by  $F_{\tilde{A}}$ .

If  $A = E$  then the  $A$ -universal soft set is denoted by  $F_{\tilde{E}}$ .

**Definition 2.4.** (see [23]) Let  $F_A, F_B \in S(U)$ . Then  $F_A$  is a soft subset of  $F_B$ , denoted by  $F_A \subseteq F_B$ , if  $f_A(x) \subseteq f_B(x)$  for all  $x \in E$ .

**Definition 2.5.** (see [23]) Let  $F_A, F_B \in S(U)$ . Then  $F_A$  and  $F_B$  are soft equal, denoted by  $F_A = F_B$ , if and only if  $f_A(x) = f_B(x)$  for all  $x \in E$ .

**Definition 2.6.** (see [23]) Let  $F_A, F_B \in S(U)$ . Then, the soft union  $F_A \cup F_B$ , the soft intersection  $F_A \cap F_B$  and the soft difference  $F_A \setminus F_B$  of  $F_A$  and  $F_B$  are defined by the approximation functions

$f_{A \cup B}(x) = f_A(x) \cup f_B(x)$ ,  $f_{A \cap B}(x) = f_A(x) \cap f_B(x)$  and  $f_{A \setminus B}(x) = f_A(x) \setminus f_B(x)$  respectively and the soft complement  $F_A^c$  of  $F_A$  is defined by the approximate function,  $f_A^c(x) = f_A^c(x)$ , where  $f_A^c(x)$  is the complement of the set  $f_A(x)$ ; that is  $f_A^c(x) = U \setminus f_A(x)$  for all  $x \in E$ .

It is easy to see that  $(F_A^c)^c = F_A$  and  $F_{\emptyset}^c = F_{\tilde{E}}$ .

**Definition 2.7.** (see [23]) Let  $F_A \in S(U)$ . Then, the soft power set of  $F_A$  is defined by  $\tilde{P}(F_A) = \{F_{A_i} : F_{A_i} \subseteq F_A, i \in I \subseteq \mathbb{N}\}$  and its cardinality is defined by  $|\tilde{P}(F_A)| = 2^{\sum_{x \in E} |f_A(x)|}$  where  $|f_A(x)|$  is the cardinality of  $f_A(x)$

**Definition 2.8.** (see [23, 17]) Let  $F_A \in S(X)$ . A soft topology on  $F_A$ , denoted by  $\tilde{\tau}$ , is a collection of soft subsets of  $F_A$  having the following properties:

- (i)  $F_{\emptyset}, F_A \in \tilde{\tau}$ .
- (ii) Union of any number of soft sets in  $\tilde{\tau}$  belongs to  $\tilde{\tau}$ .
- (iii) Intersection of two soft sets in  $\tilde{\tau}$  belongs to  $\tilde{\tau}$ .

Then the pair  $(F_A, \tilde{\tau})$  is called a soft topological space.

**Definition 2.9.** (see [23]) Let  $(F_A, \tilde{\tau}_1)$  and  $(F_A, \tilde{\tau}_2)$  be soft topological spaces. Then, the following hold.

If  $\tilde{\tau}_2 \supseteq \tilde{\tau}_1$ , then  $\tilde{\tau}_2$  is soft finer than  $\tilde{\tau}_1$ .

If  $\tilde{\tau}_2 \supset \tilde{\tau}_1$ , then  $\tilde{\tau}_2$  is soft strictly finer than  $\tilde{\tau}_1$ .

If either  $\tilde{\tau}_2 \supseteq \tilde{\tau}_1$  or  $\tilde{\tau}_2 \subseteq \tilde{\tau}_1$ , then  $\tilde{\tau}_1$  is comparable with  $\tilde{\tau}_2$ .

**Definition 2.10.** (see [23]) Let  $(F_A, \tilde{\tau})$  be a soft topological space, then every element of  $\tilde{\tau}$  is called a soft open set. Clearly,  $F_\emptyset$  and  $F_A$  are soft open sets.

**Definition 2.11.** (see [23]) Let  $(F_A, \tilde{\tau})$  be a soft topological space and  $F_B \subseteq F_A$ . Then  $F_B$  is said to be soft closed if the soft set  $F_B^c$  is soft open.

**Definition 2.12.** (see [17]) Let  $F_A$  be a non-empty soft set on the universe  $U$ .  $\tilde{\tau}_1$  and  $\tilde{\tau}_2$  be two different soft topologies on  $F_A$ . Then,  $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$  is called a soft bitopological space.

The following definition of soft closed subset in a soft topological space (in Çağman's sense) is due to Renukadevi and Shanthi [33]. They modified the definition of soft closed subset based on some notional errors in [23]. Throughout this paper we follow their definitions.

**Definition 2.13.** (see [33]) Let  $(F_A, \tilde{\tau})$  be a soft topological space and  $F_B \subseteq F_A$ . Then  $F_B$  is said to be a soft closed set in  $F_A$  as the soft set  $F_{B/A}^c$  is soft open in  $F_A$  where  $F_{B/A}^c = F_B^c \cap F_A$ .

### 3. On soft bitopological space

In this section we define nowhere dense set and boundary etc. with respect to a soft point and proved some results.

At first let us recall the following definition:

**Definition 3.1.** (see [17]) Let  $F_A$  be a non-empty soft set on the universe  $U$ .  $\tilde{\tau}_1$  and  $\tilde{\tau}_2$  be two different soft topologies on  $F_A$ , Then  $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$  is called a soft bitopological space.

It is to be noted that  $\tau_i$ - $F_B$  indicates the  $\tau_i$ -soft open set  $F_B$ ,  $\tau_i$ - $F_B^o$  indicates the soft interior of  $F_B$  with respect to  $\tau_i$  and  $\tau_i$ - $\overline{F_B}$  indicates the soft closure of  $F_B$  with respect to  $\tau_i$  where  $i \in \{1, 2\}$ .

**Note 1:** Throughout this paper we write  $F_B, F_C$  etc and  $F_{A_i}$  etc and these mean that all are soft subsets of  $F_A$  where  $F_A \in S(U)$ .

**Definition 3.2.** Let  $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$  be a soft bitopological space. Then a soft subset  $F_B$  is said to be an  $(i, j)$ -soft nowhere dense set in  $F_A$  if and only if  $\tau_i$ -( $\tau_j$ - $\overline{F_B}$ ) $^o = F_\emptyset$  where  $i, j \in \{1, 2\}$ . The family of all  $(i, j)$ -soft nowhere dense sets of  $F_A$  is denoted by  $(i, j)$ - $\text{SND}(F_A)$ .

**Lemma 3.1.** *Let  $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$  be a soft bitopological space. Then  $F_B \in (i, j)\text{-}\mathcal{SND}(F_A) \Leftrightarrow \tau_j\text{-}\overline{F_B} \subseteq \tau_i\text{-}((\tau_j - \overline{F_B})^\circ_A)$ .*

**Proof:** Necessity.

Let  $F_B \in (i, j)\text{-}\mathcal{SND}(F_A)$  then  $\tau_i\text{-}(\tau_j\text{-}\overline{F_B})^\circ = F_\emptyset$ . Clearly  $\tau_i\text{-}(\tau_j\text{-}\overline{F_B})^\circ \tilde{\cap} \tau_j\text{-}\overline{F_B} = F_\emptyset$ . Thus  $\tau_j\text{-}\overline{F_B} \subseteq F_A \setminus \tau_i\text{-}(\tau_j\text{-}\overline{F_B})^\circ$ . Hence  $\tau_j\text{-}\overline{F_B} \subseteq \tau_i\text{-}(F_A \setminus \tau_j\text{-}\overline{F_B})$ .

Sufficiency.

Let  $\tau_j\text{-}\overline{F_B} \subseteq \tau_i\text{-}(F_A \setminus \tau_j\text{-}\overline{F_B})$ . Then  $\tau_i\text{-}(\tau_j\text{-}\overline{F_B})^\circ \tilde{\cap} \tau_j\text{-}\overline{F_B} = F_\emptyset$  implies  $\tau_i\text{-}(\tau_j\text{-}\overline{F_B})^\circ = F_\emptyset$ . Hence the result.  $\square$

**Theorem 3.1.** *Let  $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$  be a soft bitopological space and  $F_B \in (i, j)\text{-}\mathcal{SND}(F_A)$ , then for every soft subset  $F_C \in (\tilde{\tau}_1 \cap \tilde{\tau}_2) \setminus \{F_\emptyset\}$ ; there is a soft set  $F_D \in \tilde{\tau}_j \setminus \{F_\emptyset\}$  such that  $F_D \subseteq F_C$  and  $F_D \tilde{\cap} F_B = F_\emptyset$ . Conversely, if for every soft set  $F_C \in \tilde{\tau}_i \setminus \{F_\emptyset\}$ ; there is a soft subset  $F_D \in \tilde{\tau}_j \setminus \{F_\emptyset\}$  such that  $F_D \subseteq F_C$  and  $F_D \tilde{\cap} F_B = F_\emptyset$ , then  $F_B \in (i, j)\text{-}\mathcal{SND}(F_A)$  where  $i, j \in \{1, 2\}$ .*

**Proof:** Let  $F_B \in (i, j)\text{-}\mathcal{SND}(F_A)$  and  $F_C \in (\tilde{\tau}_1 \cap \tilde{\tau}_2) \setminus \{F_\emptyset\}$  be any soft set. Let  $F_D = F_C \setminus \tau_j\text{-}\overline{F_B}$ . Then  $F_D \in \tilde{\tau}_j$ .

If  $F_D = F_\emptyset$  then  $F_\emptyset = F_C \tilde{\cap} \tau_i\text{-}(\tau_j\text{-}\overline{F_B})^\circ = F_C \tilde{\cap} (F_A \setminus \tau_i\text{-}(\tau_j\text{-}\overline{F_B})^\circ) = F_C$  where  $F_K = F_A \setminus F_B$ . This contradicts to the fact  $F_C \neq F_\emptyset$ . Hence  $F_D \in \tilde{\tau}_j \setminus \{F_\emptyset\}$  i.e.  $F_D \neq F_\emptyset$ .

Conversely, suppose that  $F_B \notin (i, j)\text{-}\mathcal{SND}(F_A)$  and all other conditions in the statement of converse part hold. Then  $\tau_i\text{-}(\tau_j\text{-}\overline{F_B})^\circ \neq F_\emptyset$ . Consider an arbitrary soft set  $F_D \in \tilde{\tau}_j \setminus \{F_\emptyset\}$  satisfying  $F_D \subseteq F_C$  and  $F_D \tilde{\cap} F_B = F_\emptyset$ . Then  $F_D \subseteq \tau_j\text{-}\overline{F_B}$ . Hence  $F_D \tilde{\cap} \tau_j\text{-}\overline{F_B} \neq F_\emptyset$ ; which implies  $F_D \tilde{\cap} F_B \neq F_\emptyset$ . Thus we arrive at a contradiction. So,  $F_B \in (i, j)\text{-}\mathcal{SND}(F_A)$ .  $\square$

**Corollary 3.1.** *Let  $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$  be a soft bitopological space with  $\tilde{\tau}_1 \subseteq \tilde{\tau}_2$ , then  $F_B \in (1, 2)\text{-}\mathcal{SND}(F_A)$  if and only if for every  $F_C \in \tilde{\tau}_1 \setminus \{F_\emptyset\}$ , there is a soft set  $F_D \in \tilde{\tau}_2 \setminus \{F_\emptyset\}$  such that  $F_D \subseteq F_C$  and  $F_D \tilde{\cap} F_B = F_\emptyset$ .*

**Proposition 3.1.** *Let  $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$  be a soft bitopological space with  $\tilde{\tau}_1 \subseteq \tilde{\tau}_2$ . Then the following results hold:*

$$(i) (2, 1)\text{-}\mathcal{SND}(F_A) \subseteq 1\text{-}\mathcal{SND}(F_A) \subseteq (1, 2)\text{-}\mathcal{SND}(F_A).$$

$$(ii) (2, 1)\text{-}\mathcal{SND}(F_A) \subseteq 2\text{-}\mathcal{SND}(F_A) \subseteq (1, 2)\text{-}\mathcal{SND}(F_A).$$

**Proof:** (i) Let  $F_B \in (2, 1)\text{-}\mathcal{SND}(F_A)$  then by Lemma 3.1; we have  $\tau_1\text{-}\overline{F_B} \subseteq \tau_2\text{-}(F_A \setminus \tau_1\text{-}\overline{F_B}) \Rightarrow \tau_1\text{-}\overline{F_B} \subseteq F_A \setminus \tau_2\text{-}(\tau_1\text{-}\overline{F_B})^\circ \subseteq F_A \setminus \tau_1\text{-}(\tau_1\text{-}\overline{F_B})^\circ$ . This implies

$\tilde{\tau}_1 \cdot \overline{F_B} \cap \tilde{\tau}_1 \cdot (\tilde{\tau}_1 \cdot \overline{F_B})^o = F_\emptyset \Rightarrow \tilde{\tau}_1 \cdot (\tilde{\tau}_1 \cdot \overline{F_B})^o = F_\emptyset$ . Thus  $F_B \in 1\text{-}\mathcal{SND}(F_A)$ .

Since  $\tilde{\tau}_1 \subseteq \tilde{\tau}_2$ , we have  $\tilde{\tau}_2 \cdot \overline{F_B} \subseteq \tilde{\tau}_1 \cdot \overline{F_B}$ . Thus  $\tilde{\tau}_1 \cdot (\tilde{\tau}_2 \cdot \overline{F_B})^o \subseteq \tilde{\tau}_1 \cdot (\tilde{\tau}_1 \cdot \overline{F_B})^o = F_\emptyset$ . This implies  $\tilde{\tau}_1 \cdot (\tilde{\tau}_2 \cdot \overline{F_B})^o = F_\emptyset$ . Hence  $F_B \in (1, 2)\text{-}\mathcal{SND}(F_A)$ .

(ii) Let  $F_B \in (2, 1)\text{-}\mathcal{SND}(F_A)$ . This implies  $\tilde{\tau}_2 \cdot (\tilde{\tau}_1 \cdot \overline{F_B})^o = F_\emptyset$ . Since  $\tilde{\tau}_1 \subseteq \tilde{\tau}_2$ , it can be easily established that  $\tilde{\tau}_2 \cdot (\tilde{\tau}_2 \cdot \overline{F_B})^o = F_\emptyset$ . Thus  $F_B \in 2\text{-}\mathcal{SND}(F_A)$ .

Further more  $\tilde{\tau}_1 \subseteq \tilde{\tau}_2$ , we have  $\tilde{\tau}_2 \cdot (\tilde{\tau}_2 \cdot \overline{F_B})^o = F_\emptyset \Rightarrow \tilde{\tau}_1 \cdot (\tilde{\tau}_2 \cdot \overline{F_B})^o = F_\emptyset$ . Thus  $F_B \in (1, 2)\text{-}\mathcal{SND}(F_A)$ .  $\square$

**Corollary 3.2.** *Let  $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$  be a soft bitopological space with  $\tilde{\tau}_1 \subseteq \tilde{\tau}_2$ . Then  $F_B \in (2, 1)\text{-}\mathcal{SND}(F_A)$  if for every  $F_C \in \tilde{\tau}_2$  such that  $\tilde{\tau}_1 \cdot F_B^o \neq F_\emptyset$ ; there is a soft set  $F_D \in \tilde{\tau}_1 \setminus \{F_\emptyset\}$  such that  $F_D \subseteq F_C$  and  $F_D \cap F_B = F_\emptyset$ .*

*Conversely; if for every soft subset  $F_C \in \tilde{\tau}_2 \setminus \{F_\emptyset\}$  there is a soft set  $F_D \in \tilde{\tau}_1 \setminus \{F_\emptyset\}$  such that  $F_D \subseteq F_C$  and  $F_D \cap F_B = F_\emptyset$ , then  $F_B \in (2, 1)\text{-}\mathcal{SND}(F_A)$ .*

**Proof:** Using Proposition 3.1. and previous results we can easily prove this theorem.  $\square$

**Definition 3.3.** *Let  $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$  be a soft bitopological space. A soft subset  $F_B$  of  $F_A$  is said to be an  $(i, j)$ -soft dense in  $F_A$  if and only if  $\tilde{\tau}_i \cdot (\tilde{\tau}_j \cdot \overline{F_B}) = F_A$ . The set of  $(i, j)$ -soft dense sets in  $F_A$  is denoted by  $(i, j)\text{-}\mathcal{SD}(F_A)$ .*

**Definition 3.4.** *Let  $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$  be a soft bitopological space. A soft subset  $F_B$  of  $F_A$  is said to be an  $(i, j)$ -soft boundary in  $F_A$  if and only if  $\tilde{\tau}_i \cdot (\tilde{\tau}_j \cdot F_B^o) = F_\emptyset$ . The set of  $(i, j)$ -soft boundary sets in  $F_A$  is denoted by  $(i, j)\text{-}\mathcal{SBD}(F_A)$ .*

**Theorem 3.2.** *Let  $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$  be a soft bitopological space. If  $F_B \in j\text{-}\mathcal{SBD}(F_A)$  and  $F_C \in (i, j)\text{-}\mathcal{SND}(F_A)$  then  $F_D \in (i, j)\text{-}\mathcal{SBD}(F_A)$  where  $F_D = F_B \cup F_C$ .*

**Proof:** We have to show that  $\tilde{\tau}_i \cdot (\tilde{\tau}_j \cdot F_D^o) = F_\emptyset$ .

If  $F_B \in j\text{-}\mathcal{SBD}(F_A) \Leftrightarrow \tilde{\tau}_j \cdot (\tilde{\tau}_j \cdot F_B^o) = F_\emptyset \Leftrightarrow \tilde{\tau}_j \cdot F_B^o = F_\emptyset \Leftrightarrow \tilde{\tau}_j \cdot \overline{F_R} = F_A$ ; where  $F_R = F_A \setminus F_B$ .

Now  $F_A \setminus \tilde{\tau}_j \cdot \overline{F_C} = \tilde{\tau}_j \cdot \overline{F_R} \setminus \tilde{\tau}_j \cdot \overline{F_C} \subseteq \tilde{\tau}_j \cdot \overline{F_R \setminus F_C} = \tilde{\tau}_j \cdot \overline{F_K}$ , where  $F_K = F_A \setminus (F_B \cup F_C)$ .

Now  $F_A = \tilde{\tau}_i \cdot \overline{F_S}$ , where  $F_S = F_A \setminus \tilde{\tau}_j \cdot \overline{F_C}$ . Thus  $F_A = \tilde{\tau}_i \cdot (\overline{F_S}) \subseteq \tilde{\tau}_i \cdot (\overline{\tilde{\tau}_j \cdot \overline{F_K}})$ . Thus  $F_A = \tau_i \cdot (\tau_j \cdot \overline{F_K})$ , which implies  $\tau_i \cdot (\tau_j \cdot F_D^o) = F_\emptyset$ , where  $F_D = F_B \cup F_C$ . Hence the proof.  $\square$

**Theorem 3.3.** *Let  $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$  be a soft bitopological space with  $\tilde{\tau}_1 \subseteq \tilde{\tau}_2$ . If  $F_B, F_C \in (2, 1)\text{-}\mathcal{SND}(F_A)$ . Then  $F_D \in (2, 1)\text{-}\mathcal{SND}(F_A)$  where  $F_D = F_B \cup F_C$ .*

**Proof:** If  $F_B, F_C \in (2, 1)\text{-SND}(F_A)$ , then  $\tilde{\tau}_2-(\tilde{\tau}_1-\overline{F_B})^o = F_\emptyset$  and  $\tilde{\tau}_2-(\tilde{\tau}_1-\overline{F_C})^o = F_\emptyset$ .

So,  $\tilde{\tau}_2-(\tilde{\tau}_2-(\tilde{\tau}_1-\overline{F_B})^o)^o = F_\emptyset$ . Hence  $\tilde{\tau}_1-\overline{F_B} \in 2\text{-SBD}(F_A)$ . Also  $\tilde{\tau}_2-(\tilde{\tau}_2-(\tilde{\tau}_1-\overline{F_C})^o)^o \subseteq \tilde{\tau}_2-(\tilde{\tau}_1-\overline{F_C})^o = F_\emptyset$ . Thus  $\tilde{\tau}_1-\overline{F_C} \in 2\text{-SND}(F_A)$ . Hence by Theorem 3.2,  $\tilde{\tau}_1-\overline{F_D} \in 2\text{-SBD}(F_A)$ , where  $F_D = F_B \tilde{\cap} F_C$ .

Then  $\tilde{\tau}_2-(\tilde{\tau}_2-(\tilde{\tau}_1-\overline{F_D})^o)^o = F_\emptyset$ , Thus  $\tilde{\tau}_2-(\tilde{\tau}_1-\overline{F_D})^o = F_\emptyset \Rightarrow F_D \in (2, 1)\text{-SND}(F_A)$ .  $\square$

**Definition 3.5.** Let  $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$  be a soft bitopological space. A soft subset  $F_B$  of  $F_A$  is said to be an  $(i, j)$ -soft nowhere dense set at a point  $x$  if and only if there exists a  $\tilde{\tau}_i$ -soft open neighborhood  $F_C$  of  $x$  in  $F_A$  such that  $\tilde{\tau}_i-(\tilde{\tau}_j-\overline{F_D})^o = F_\emptyset$ , where  $F_D = F_B \tilde{\cap} F_C$ . The set of all  $(i, j)$ -soft nowhere dense sets at  $x$  is denoted by  $(i, j)\text{-SND}(F_A, x)$ .

**Definition 3.6.** Let  $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$  be a soft bitopological space. A soft subset  $F_B$  in  $F_A$  is said to be an  $(i, j)$ -soft boundary at a point  $x \in F_A$  if and only if there exists a  $\tilde{\tau}_i$ -soft open neighborhood  $F_C$  of  $x$  in  $F_A$  such that  $\tilde{\tau}_i-(\tilde{\tau}_j-F_D^o)^o = F_\emptyset$  where  $F_D = F_B \tilde{\cap} F_C$ . The set of all  $(i, j)$ -soft boundary sets at  $x$  is denoted by  $(i, j)\text{-SBD}(F_A, x)$ .

**Proposition 3.2.** Let  $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$  be a soft bitopological space with  $\tilde{\tau}_1 \subseteq \tilde{\tau}_2$ . Then the following results hold:

- (i)  $1\text{-SND}(F_A, x) \subseteq (1, 2)\text{-SND}(F_A, x)$ .
- (ii)  $(2, 1)\text{-SND}(F_A, x) \subseteq 2\text{-SND}(F_A, x)$ .
- (iii)  $2\text{-SBD}(F_A, x) \subseteq (2, 1)\text{-SBD}(F_A, x)$ .
- (iv)  $(1, 2)\text{-SBD}(F_A, x) \subseteq 1\text{-SBD}(F_A, x) \subseteq (2, 1)\text{-SBD}(F_A, x)$ .

**Proof:** The proofs of the parts (i) and (ii) are easy to obtain, so omitted.  $\square$

(iii) Let  $F_B \in 2\text{-SBD}(F_A, x)$  then there exists a  $\tilde{\tau}_2$ -soft open neighborhood  $F_C$  of  $x$  in  $F_A$  such that  $\tilde{\tau}_2-(\tilde{\tau}_2-F_D^o)^o = F_\emptyset$  where  $F_D = F_B \tilde{\cap} F_C$ . Since  $\tilde{\tau}_1 \subseteq \tilde{\tau}_2$ , we have  $\tilde{\tau}_1-F_D^o \subseteq \tilde{\tau}_2-F_D^o$ . Thus  $\tilde{\tau}_2-(\tilde{\tau}_1-F_D^o)^o = F_\emptyset$ . Thus  $F_B \in (2, 1)\text{-SBD}(F_A, x)$ . Hence the result.

**Theorem 3.4.** Let  $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$  be a soft bitopological space with  $\tilde{\tau}_1 \subseteq \tilde{\tau}_2$ . Let  $x$  be any point in  $F_A$ . Then  $F_B \in (1, 2)\text{-SND}(F_A, x) \Leftrightarrow \tilde{\tau}_2-\overline{F_B} \in 1\text{-SBD}(F_A, x)$ .

**Proof:** Suppose that  $F_B \notin (1, 2)\text{-SND}(F_A, x)$ , then there exists a  $\tilde{\tau}_1$ -soft open neighborhood  $F_C$  of  $x$  in  $F_A$  such that  $\tilde{\tau}_1-(\tilde{\tau}_2-\overline{F_D})^o \neq F_\emptyset$  where  $F_D = F_B \tilde{\cap} F_C$ .

Thus there exists a soft set  $F_M \in \tilde{\tau}_1 \setminus \{F_\emptyset\}$  such that  $F_M \subseteq \tilde{\tau}_2 - \overline{F_D}$ .

Clearly,  $F_M \cap F_C \subseteq \tilde{\tau}_2 - \overline{F_D} \cap F_C \subseteq \tilde{\tau}_2 - \overline{F_B} \cap F_C$ . Thus  $\tilde{\tau}_1 - F_N^o \subseteq \tilde{\tau}_1 - F_Y^o$ , where  $F_N = F_M \cap F_C$  and  $F_Y = \tilde{\tau}_2 - \overline{F_B} \cap F_C$ . This implies the fact that  $\tilde{\tau}_1 - F_Y^o \neq F_\emptyset$ . So,  $F_Y \notin 1\text{-}\mathcal{SBD}(F_A, x)$ .

As  $F_C$  is chosen arbitrarily from  $\tilde{\tau}_1$  as a soft neighborhood of  $x$ , so  $\tilde{\tau}_2 - \overline{F_B} \notin 1\text{-}\mathcal{SBD}(F_A, x)$ .

Conversely, suppose  $\tilde{\tau}_2 - \overline{F_B} \notin 1\text{-}\mathcal{SBD}(F_B, x)$ , then  $\tilde{\tau}_1 - F_K^o \neq F_\emptyset$ , where  $F_K = \tilde{\tau}_2 - \overline{F_B} \cap F_C$  and  $F_C$  be a  $\tilde{\tau}_1$ -neighborhood of  $x$ . Then there exists a soft set  $F_R \in \tilde{\tau}_1 \setminus \{F_\emptyset\}$  such that  $F_R \subseteq F_K \subseteq \tilde{\tau}_2 - \overline{F_Z}$ , where  $F_Z = F_B \cap F_C$  as  $\tilde{\tau}_1 \subseteq \tilde{\tau}_2$ . Thus  $\tilde{\tau}_1 - (\tilde{\tau}_2 - \overline{F_Z})^o \neq F_\emptyset$ . Since  $F_C$  is an arbitrary  $\tilde{\tau}_1$ -neighborhood of  $x$ , so  $F_B \notin (1, 2)\text{-}\mathcal{SND}(F_A, x)$ . Thus the proof.  $\square$

**Definition 3.7.** Let  $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$  be a soft bitopological space. A soft subset  $F_B$  of  $F_A$  is called  $(i, j)$ -soft first category or  $(i, j)$ -soft meager over  $F_A$  if  $F_B = \bigcup_{i \in \Delta} F_{A_i}$  where  $F_{A_i} \in (i, j)\text{-}\mathcal{SND}(F_A)$ . Otherwise it is said to be  $(i, j)$ -soft second category if it is not  $(i, j)$ -soft first category.

The collection of all soft sets in  $F_A$  which are of  $(i, j)$ -soft first category is denoted by  $(i, j)\text{-}\mathcal{SCI}(F_A)$  and the collection of all soft sets in  $F_A$ , those are of  $(i, j)$ -soft second category is denoted by  $(i, j)\text{-}\mathcal{SCII}(F_A)$ .

**Theorem 3.5.** Let  $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$  be a soft bitopological space, then  $F_A$  is of  $(i, j)$ -soft second category if and only if the soft intersection of any collection of  $i$ -soft dense  $j$ -soft open subset of  $F_A$  is not  $F_\emptyset$ .

**Proof:** Let  $F_A$  is of  $(i, j)$ -soft first category. Thus,  $F_A = \bigcup_{i \in \Delta} F_{A_i}$  where each  $F_{A_i}$  is  $\tau_j$ -soft closed set and belongs to  $i\text{-}\mathcal{SBD}(F_A)$ . Thus  $\bigcap_{i \in \Delta} F_{B_i} = \tilde{\emptyset}$ , where  $F_{B_i} = F_A \setminus F_{A_i}$ . Hence  $\{F_{B_i} | i \in \Delta\}$  is a collection of  $\tilde{\tau}_i$ -soft dense  $\tilde{\tau}_j$ -soft open subsets of  $F_A$ .

Conversely, let  $\{F_{A_i} | i \in \Delta\}$  be a collection of soft subsets of  $F_A$ , where  $F_{A_i}$ 's are  $i$ -soft dense  $j$ -soft open subsets with  $\bigcap_{i \in \Delta} F_{A_i} = F_\emptyset$ .

Thus  $F_A = \bigcup_{i \in \Delta} F_{D_i}$ , where  $F_{D_i} = F_A \setminus F_{A_i}$  are  $\tilde{\tau}_j$ -soft closed sets. Now after some steps we have  $\tilde{\tau}_i - (\tilde{\tau}_i - F_{D_i}^o)^o = F_\emptyset$ . Hence  $F_{D_i} \in i\text{-}\mathcal{SBD}(F_A)$ . So,  $\tilde{\tau}_i - F_{D_i}^o = F_\emptyset \Rightarrow \tilde{\tau}_i - (\tilde{\tau}_j - \overline{F_{D_i}})^o = F_\emptyset$ . Thus  $F_{D_i} \in (i, j)\text{-}\mathcal{SND}(F_A)$ . Hence  $F_A \in (i, j)\text{-}\mathcal{SCI}(F_A)$ . Hence the proof.  $\square$

**Theorem 3.6.** Let  $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$  be a soft bitopological space with  $\tilde{\tau}_1 \subseteq \tilde{\tau}_2$ . If  $F_A$  is of  $(1, 2)$ -soft second category, then  $2\text{-}\mathcal{SG}_\delta(F_A) \cap 1\text{-}\mathcal{SD}(F_A) \subseteq (1, 2)\text{-}\mathcal{SCII}(F_A)$ .

**Proof:** Let  $F_B \in 2-SG_\delta(F_A) \cap 1-SD(F_A)$ , then  $F_B = \bigcap_{i \in \Delta} F_{A_i}$ , where each  $F_{A_i}$  is  $\tilde{\tau}_2$ -soft open and belongs to  $1-\mathcal{SD}(F_A)$ . Let  $F_D = F_A \setminus F_B = \bigcup_{i \in \Delta} F_{C_i} \in (1, 2)-SC_I(F_A)$ , where  $F_{C_i} = F_A \setminus F_{A_i}$  as  $F_{C_i} \in (1, 2)-\mathcal{SD}(F_A)$ .

If  $F_B \in (1, 2)-SC_I(F_A)$ , then  $F_A = F_B \cup F_D$  which implies  $F_A$  is of  $(1, 2)$ -soft first category; which is a contradiction. Thus  $F_A \in (1, 2)-SC_{II}(F_A)$ .  $\square$

#### 4. Conclusion

This paper can be considered as an introductory paper on various fundamental notions viz. soft nowhere dense, soft boundary of a soft set, soft boundary at a point which are important for development of soft bitopological space as defined by Şenel and Çağman [17]. Special emphasis is given to contribute fundamental structures for soft bitopological space under soft finer topologies. We studied various results between concepts of soft nowhere dense, soft boundary of a soft set, soft boundary with respect to a point under soft bitopological space and so on. Some basic properties have investigated. Further systematic study on this area is necessary for development of this area and its possible applications in different aspects of science for betterment of mankind.

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