Some results on soft bitopology

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Abstract. In this paper we study some properties under soft bitopology. We have studied properties introducing nowhere dense, boundary of a soft set, boundary with respect to a point, first category etc from the point of view of soft bitopological space. In this paper we contribute some results in "soft bitopology" which was initially introduced by Şenel and Çağman in the year 2014.

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1. Introduction and motivation: It seems easy to understand that a mathematical theory is based on various abstract thoughts. In these cases one has full freedom to establish certain environment depending on many neglecting facts; for example in physics we often neglect the frictional effect of air on a free falling body, but this fact is fully impossible in real life. Similarly other branches like medicine, economics, engineering, social sciences etc are full of uncertainties. Before to 1999, we had only four mathematical tools to deal with uncertainties, namely probability theory, fuzzy set theory, rough set theory and the theory of interval mathematics. To overcome the choice of degree of membership in fuzzy set theory when the facts are concerned with uncertainties, Molodtsov [14] introduced the concept of soft set theory in the year 1999 and investigated various applications in game theory, smoothness of functions, operation researches, Perron integration, probability theory, theory of measurement and so on.

Later Maji et al.[25] defined various operations on soft sets to study some of the fundamental properties. Pei-Miao [13] and Chen [12] pointed out errors in some of the results of the paper of Maji et al.[25] and introduced some new notions and properties. At present; investigations of different properties and applications of soft set theory have attracted many researchers from various backgrounds. It has been used in fuzzy set theory too (one may refer to [10,11]).

In the year 2011, Shabir and Naz [22] introduced soft topology and studied some introductory results. In the same year Çağman et al.[23] introduced soft topology in a different approach. Till then various researchers have studied various foundational results in soft topology (one may refer to [21,22,27,28,32]). Kandil etal [2,3,4] introduced concept of soft ideal and studied some other weaker notations and properties. Recently Yuksel et al. [30] applied soft set theory to determine prostate cancer risk. Recently various forms of fuzzy soft topologies have came in existence from the view point of fuzzy set theory viz Intuitionistic Fuzzy Soft Topology, fuzzy soft topology etc. More recently Sęnel and Çağman [18] introduced soft topological subspace and studied some properties.

It is well known to us that both general topology and fuzzy topology play crucial roles in mathematics, economics, data reduction, image processing, information sciences, genotype-phenotype mapping of DNA etc. One may refer to [7,24,9] for applications of topology in biology, where modified fundamental topological results play crucial role to resolve various difficulties with DNA, mRNA etc. Hence in this paper we are interested to contribute some results in soft bitopology which is a newly developed area by Şenel and Çağman in the year 2014 [17]. In this paper we mainly focus on soft finer topology via soft bitopology as it is well known that coarser topology (resp. bitopology), coarser fuzzy topology (resp. fuzzy bitopology) etc play crucial role in various information sciences related to computer, biology, remote sensing, signalling etc.

In this paper we will follow Çağman's notion of soft topology.

2.Basic definitions:

In this section we discuss some basic definitions and notions those are defined by various authors.

Definition 2.1. *(see [23])* A soft set F_A on the universe U is defined by the set of ordered pairs $F_A = \{(x, f_A(x) : x \in E, f_A(x) \in P(U)\}$ where $f_A : E \to P(U)$ such that $f_A(x) = \emptyset$ if $x \notin A$.

Here f_A is called an *approximate function* of the soft set F_A. The value of $f_A(x)$ may be arbitrary. Some of them may be empty, some may have nonempty intersection. we will denote the set of all soft sets over U as $S(U)$.

Definition 2.2. *(see [23])* Let $F_A \in S(U)$. If $f_A(x) = \emptyset$ for all $x \in E$, then F_A is called a *soft empty set*, denoted by $F_\emptyset$.

$f_A(x) = \emptyset$ means there is no element in U related to the parameter $x \in E$. Therefore, we do not display such elements in the soft sets, as it is meaningless to consider such parameters.

Definition 2.3. *(see [23])* Let $F_A \in S(U)$. If $f_A(x) = U$ for all $x \in E$, then F_A is called an *A-universal soft set*, denoted by $F_{\widetilde{A}}$.

If $A = E$ then the A-universal soft set is denoted by $F_{\widetilde{E}}$.

Definition 2.4. *(see [23])* Let $F_A, F_B \in S(U)$. Then F_A is a *soft subset* of F_B, denoted by $F_A \widetilde{\subseteq} F_B$, if $f_A(x) \subseteq f_B(x)$ for all $x \in E$.

Definition 2.5. *(see [23])* Let $F_A, F_B \in S(U)$. Then F_A and F_B are soft equal, denoted by $F_A \widetilde{=} F_B$, if and only if $f_A(x) = f_B(x)$ for all $x \in E$.

Definition 2.6. *(see [23])* Let $F_A, F_B \in S(U)$. Then, the *soft union* $F_A \widetilde{\cup} F_B$, *the soft intersection* $F_A \widetilde{\cap} F_B$ and the *soft difference* $F_A \widetilde{\setminus} F_B$ of F_A and F_B are defined by the approximation function

$f_{A\widetilde{\cup}B}(x) = f_A(x) \cup f_B(x)$, $f_{A\widetilde{\cap}B}(x) = f_A(x) \cap f_B(x)$ and $f_{A\widetilde{\setminus}B}(x) = f_A(x) \setminus f_B(x)$ respectively, and the soft complement $F_A^{\widetilde{c}}$ of F_A is defined by the approximate function, $f_A^{\widetilde{c}}(x) = f_A^c(x)$, where $f_A^c(x)$ is the compliment of the set $f_A(x)$; that is $f_A^c(x) = U \setminus f_A(x)$ for all $x \in E$.

It is easy to see that $(F_A^{\widetilde{c}})^{\widetilde{c}} = F_A$ and $F_{\emptyset}^{\widetilde{c}} = F_{\widetilde{E}}$.

Definition 2.7. *(see [23])* Let $F_A \in S(U)$. Then, the *soft power set* of F_A is defined by $\widetilde{P}(F_A) = \{F_{A_i} : F_{A_i} \widetilde{\subseteq} F_A, i \in I \subseteq \mathcal{N}\}$ and its cardinality is defined by $|\widetilde{P}(F_A)| = 2^{\sum_{x \in E} |f_A(x)|}$ where$|f_A(x)|$ is the cardinality of $f_A(x)$

Definition 2.8. *(see[23,17])* Let $F_A \in S(X)$. A *soft topology* on F_A, denoted by $\widetilde{\tau}$, is a collection of soft subsets of F_A having the following properties:

(i) $F_{\emptyset}, F_A \in \widetilde{\tau}$.

(ii) Union of any number of soft sets in $\widetilde{\tau}$ belongs to $\widetilde{\tau}$.

(iii) Intersection of two soft sets in $\widetilde{\tau}$ belongs to $\widetilde{\tau}$.

Then the pair $(F_A, \widetilde{\tau})$ is called a *soft topological space.*

Definition 2.9. *(see [23])* Let $(F_A, \widetilde{\tau_1})$ and $(F_A, \widetilde{\tau_2})$ be soft topological spaces. Then, the following hold.

If $\widetilde{\tau_2} \supseteq \widetilde{\tau_1}$, then $\widetilde{\tau_2}$ is *soft finer* than $\widetilde{\tau_1}$.

If $\widetilde{\tau_2} \supset \widetilde{\tau_1}$, then $\widetilde{\tau_2}$ is *soft strictly finer* than $\widetilde{\tau_1}$.

If either $\widetilde{\tau}_2 \supseteq \widetilde{\tau}_1$ or $\widetilde{\tau}_2 \subseteq \widetilde{\tau}_1$, then $\widetilde{\tau}_1$ is *comparable* with $\widetilde{\tau}_2$.

Definition 2.10. *(see [23])* Let $(F_A, \widetilde{\tau})$ be a soft topological space, then every element of $\widetilde{\tau}$ is called a *soft open set*. Clearly, $F_\emptyset$ and F_A are soft open sets.

Definition 2.11. *(see [23])* Let $(F_A, \widetilde{\tau})$ be a soft topological space and $F_B \widetilde{\subseteq} F_A$. Then F_B is said to be *soft closed* if the soft set $F_B^{\tilde{c}}$ is soft open.

Definition 2.12. *(see [17])* Let F_A be a non-empty soft set on the universe U. $\widetilde{\tau}_1$ and $\widetilde{\tau}_2$ be two different soft topologies on F_A. Then, $(F_A, \widetilde{\tau}_1, \widetilde{\tau}_2)$ is called a *soft bitopological space*.

The following definition of soft closed subset in a soft topological space (in Çağman's sense) is due to Renukadevi and Shanthi [32]. They modified the definition of soft closed subset based on some notional errors in [22]. Throughout this paper we follow their definitions.

Definition 2.13. *(see[33])* Let $(F_A, \widetilde{\tau})$ be a soft topological space and $F_B \widetilde{\subseteq} F_A$. Then F_B is said to be a *soft closed set* in F_A as the soft set $F_{B/A}^{\tilde{c}}$ is soft open in F_A where $F_{B/A}^{\tilde{c}} = F_B^{\tilde{c}} \cap F_A$.

3. On soft bitopological space.

In this section we define nowhere dense set and boundary etc with respect to a soft point and proved some results.

At first let us recall the following definition:

Definition 3.1. *(see [17])* Let F_A be a non-empty soft set on the universe U. $\widetilde{\tau}_1$ and $\widetilde{\tau}_2$ be two different soft topologies on F_A, Then $(F_A, \widetilde{\tau}_1, \widetilde{\tau}_2)$ is called a *soft bitopological space*.

It is to be noted that τ_i-F_B indicates the τ_i- *soft open set* F_B, τ_i-F_B^o indicates the *soft interior* of F_B with respect to τ_i and τ_i-$\overline{F_B}$ indicates the *soft closure* of F_B with respect to τ_i where $i \in \{1, 2\}$.

Note 1: Throughout this paper we write F_B, F_C etc and F_{A_i} etc and these mean that all are soft subsets of F_A where $F_A \in S(U)$.

Definition 3.2. Let $(F_A, \widetilde{\tau}_1, \widetilde{\tau}_2)$ be a soft bitopological space. Then a soft subset F_B is said to be (i, j)-*soft nowhere dense set* in F_A if and only if τ_i-$(\tau_j$-$\overline{F_B})^o = F_\emptyset$ where $i, j \in \{1, 2\}$. The family of all (i, j)-soft nowhere dense sets is denoted by (i, j)-$\mathcal{SND}(F_A)$.

Lemma 3.1. *Let $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$ be a soft bitopological space. Then $F_B \in (i, j)$-$\mathcal{SND}(F_A) \Leftrightarrow$ τ_j-$\overline{F_B} \tilde{\subseteq} \tau_i$-$\overline{((\tau_j - \overline{F_B})^{\tilde{c}}_{/A})}$.*

Proof. Necessity.

Let $F_B \in (i, j)$-$\mathcal{SND}(F_A)$ then τ_i-$(\tau_j$-$\overline{F_B})^o = F_\emptyset$. Clearly τ_i-$(\tau_j$-$\overline{F_B})^o \tilde{\cap}$ τ_j-$\overline{F_B}$=$F_\emptyset$. Thus τ_j-$\overline{F_B} \tilde{\subseteq}$ $F_A \setminus$ τ_i-$(\tau_j$-$\overline{F_B})^o$. Hence τ_j-$\overline{F_B} \tilde{\subseteq} \tau_i$-$(\overline{F_A \setminus \tau_j - \overline{F_B}})$.

Sufficiency.

Let τ_j-$\overline{F_B} \tilde{\subseteq} \tau_i$-$(\overline{F_A \setminus \tau_j - \overline{F_B}})$. Then τ_i-$(\tau_j$-$\overline{F_B})^o \tilde{\cap}$ τ_j-$\overline{F_B}$=$F_\emptyset$ implies τ_i-$(\tau_j$-$\overline{F_B})^o$=$F_\emptyset$. Hence the result.

Theorem 3.1. *Let $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$ be a soft bitopological space and $F_B \in (i, j)$-$\mathcal{SND}(F_A)$, then for every soft subset $F_C \in (\tilde{\tau}_1 \cap \tilde{\tau}_2) \setminus \{F_\emptyset\}$; there is a soft set $F_D \in \tilde{\tau}_j \setminus \{F_\emptyset\}$ such that $F_D \tilde{\subseteq} F_C$ and $F_D \tilde{\cap} F_B = F_\emptyset$. Conversely, if for every soft set $F_c \in \tilde{\tau}_i \setminus \{F_\emptyset\}$; there is a soft subset $F_D \in \tilde{\tau}_j \setminus \{F_\emptyset\}$ such that $F_D \tilde{\subseteq} F_C$ and $F_D \tilde{\cap} F_B = F_\emptyset$, then $F_B \in (i, j)$-$\mathcal{SND}(F_A)$ where $i, j \in \{1, 2\}$.*

Proof. Let $F_B \in (i, j)$-$\mathcal{SND}(F_A)$ and $F_C \in (\tilde{\tau}_1 \cap \tilde{\tau}_2) \setminus \{F_\emptyset\}$ be any soft set. Let $F_D = F_C \setminus \tilde{\tau}_j$-$\overline{F_B}$. Then $F_D \in \tilde{\tau}_j$.

If $F_D = F_\emptyset$ then $F_\emptyset = F_C \tilde{\cap} \tilde{\tau}_i$-$\overline{(\tau_j - F_K^o)}$=$F_C \tilde{\cap}(F_A \setminus \tilde{\tau}_i$-$(\tilde{\tau}_j$-$\overline{F_B})^o) = F_C$ where $F_K = F_A \setminus F_B$. This contradicts to the fact $F_C \neq F_\emptyset$. Hence $F_D \in \tilde{\tau}_j \setminus \{F_\emptyset\}$ i.e. $F_D \neq F_\emptyset$.

Conversely, suppose that $F_B \notin (i, j)$-$\mathcal{SND}(F_A)$ and all other conditions in the statement of converse part hold. Then $\tilde{\tau}_i$-$(\tilde{\tau}_j$-$\overline{F_B})^o \neq F_\emptyset$. Consider an arbitrary soft set $F_D \in \tilde{\tau}_j \setminus \{F_\emptyset\}$ satisfying $F_D \tilde{\subseteq} \tilde{\tau}_i$-$(\tilde{\tau}_j$-$\overline{F_B})^o$. Then $F_D \tilde{\subseteq} \tilde{\tau}_j$-$\overline{F_B}$. Hence $F_D \tilde{\cap} \tilde{\tau}_j$-$\overline{F_B} \neq F_\emptyset$; which implies $F_D \tilde{\cap} F_B \neq F_\emptyset$. Thus we arrive at a contradiction. So; $F_B \in (i, j)$-$\mathcal{SND}(F_A)$.

Corollary 3.1. *Let $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$ be a soft bitopological space with $\tilde{\tau}_1 \subseteq \tilde{\tau}_2$, then $F_B \in (1, 2)$-$\mathcal{SND}(F_A)$ if and only if for every $F_C \in \tilde{\tau}_1 \setminus \{F_\emptyset\}$, there is a soft set $F_D \in \tilde{\tau}_2 \setminus \{F_\emptyset\}$ such that $F_D \tilde{\subseteq} F_C$ and $F_D \tilde{\cap} F_B = F_\emptyset$.*

Proposition 3.1. *Let $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$ be a soft bitopological space with $\tilde{\tau}_1 \subseteq \tilde{\tau}_2$. Then the following results hold:*

(i) $(2, 1)$-$\mathcal{SND}(F_A) \subseteq$ 1-$\mathcal{SND}(F_A) \subseteq$ $(1, 2)$-$\mathcal{SND}(F_A)$.

(ii) $(2, 1)$-$\mathcal{SND}(F_A) \subseteq$ 2-$\mathcal{SND}(F_A) \subseteq$ $(1, 2)$-$\mathcal{SND}(F_A)$.

Proof. (i) Let $F_B \in (2,1)$-$\mathcal{SND}(F_A)$ then by Lemma 3.1; we have $\tilde{\tau}_1$-$\overline{F_B}\tilde{\subseteq}\tilde{\tau}_2$-$\overline{(F_A \setminus \tilde{\tau}_1 - \overline{F_B})}\Rightarrow \tilde{\tau}_1$-$\overline{F_B}\tilde{\subseteq}$ $F_A \setminus \tilde{\tau}_2$-$(\tilde{\tau}_1$-$\overline{F_B})^O\tilde{\subseteq}F_A \setminus \tilde{\tau}_1$-$(\tilde{\tau}_1$-$\overline{F_B})^O$. This implies $\tilde{\tau}_1$-$\overline{F_B}\tilde{\cap}\tilde{\tau}_1$-$(\tilde{\tau}_1$-$\overline{F_B})^O = F_\emptyset \Rightarrow \tilde{\tau}_1$-$(\tilde{\tau}_1$- $\overline{F_B})^O = F_\emptyset$. Thus $F_B \in 1$-$\mathcal{SND}(F_A)$.

Since $\tilde{\tau}_1 \subseteq \tilde{\tau}_2$, we have $\tilde{\tau}_2$-$\overline{F_B}\tilde{\subseteq}$ $\tilde{\tau}_1$-$\overline{F_B}$. Thus $\tilde{\tau}_1$-$(\tilde{\tau}_2$-$\overline{F_B})^o\tilde{\subseteq}$ $\tilde{\tau}_1$-$(\tilde{\tau}_1$-$\overline{F_B})^o = F_\emptyset$. This implies $\tilde{\tau}_1$-$(\tilde{\tau}_2$-$\overline{F_B})^o$=$F_\emptyset$. Hence $F_B \in(1,2)$-$\mathcal{SND}(F_A)$.

(ii) Let $F_B \in (2,1)$-$\mathcal{SND}(F_A)$. This implies $\tilde{\tau}_2$-$(\tilde{\tau}_1$-$\overline{F_B})^o$=$F_\emptyset$. Since $\tilde{\tau}_1 \subseteq \tilde{\tau}_2$, it can be easily established that $\tilde{\tau}_2$-$(\tilde{\tau}_2$-$\overline{F_B})^o$=$F_\emptyset$. Thus $F_B \in 2$-$\mathcal{SND}(F_A)$.

Further more $\tilde{\tau}_1 \subseteq \tilde{\tau}_2$, we have $\tilde{\tau}_2$-$(\tilde{\tau}_2$-$\overline{F_B})^o$=$F_\emptyset \Rightarrow \tilde{\tau}_1$-$(\tilde{\tau}_2$-$\overline{F_B}))^o$=$F_\emptyset$. Thus $F_B \in$ (1,2)-$\mathcal{SND}(F_A)$.

Corollary 3.2. *Let $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$ be a soft bitopological space with $\tilde{\tau}_1 \subseteq \tilde{\tau}_2$. Then $F_B \in (2,1)$-$\mathcal{SND}(F_A)$ if for every $F_C \in \tilde{\tau}_2$ such that $\tilde{\tau}_1$-$F_B^o \neq F_\emptyset$; there is a soft set $F_D \in \tilde{\tau}_1 \setminus \{F_\emptyset\}$ such that $F_D\tilde{\subseteq}F_C$ and $F_D\tilde{\cap}F_B = F_\emptyset$.*

Conversely; if for every soft subset $F_C \in \tilde{\tau}_2 \setminus \{F_\emptyset\}$ there is a soft set $F_D \in \tilde{\tau}_1 \setminus \{F_\emptyset\}$ such that $F_D\tilde{\subseteq}F_C$ and $F_D\tilde{\cap}F_B = F_\emptyset$, then $F_B \in (2,1)$-$\mathcal{SND}(F_A)$.

Proof. Using Proposition 3.1. and previous results we can easily prove this theorem.

Definition 3.3. Let $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$ be a soft bitopological space. A soft subset F_B of F_A is said to be (i,j)-*soft dense* in F_A if and only if $\tilde{\tau}_i$-$\overline{(\tilde{\tau}_j - \overline{F_B})} = F_A$. The set of (i,j)-soft dense sets in F_A is denoted by (i,j)-$\mathcal{SD}(F_A)$.

Definition 3.4. Let $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$ be a soft bitopological space. A soft subset F_B of F_A is said to be (i,j)-*soft boundary* in F_A if and only if $\tilde{\tau}_i$-$(\tilde{\tau}_j - F_B^o)^o = F_\emptyset$. The set of (i,j)-soft boundary in F_A is denoted by (i,j)-$\mathcal{SBD}(F_A)$.

Theorem 3.2. *Let $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$ be a soft bitopological space. If $F_B \in j$-$\mathcal{SBD}(F_A)$ and $F_C \in (i,j)$-$\mathcal{SND}(F_A)$ then $F_D \in (i,j)$-$\mathcal{SBD}(F_A)$ where $F_D = F_B\tilde{\cup}F_C$.*

Proof. We have to show that $\tilde{\tau}_i$-$(\tilde{\tau}_j - F_D^o)^o = F_\emptyset$.

If $F_B \in j$-$SBD(F_A)$ $\Leftrightarrow\tilde{\tau}_j$-$(\tilde{\tau}_j - F_B^o)^o = F_\emptyset \Leftrightarrow \tilde{\tau}_j$-$F_B^o = F_\emptyset \Leftrightarrow \tilde{\tau}_j$-$\overline{F_R} = F_A$; where $F_R = F_A \setminus F_B$.

Now $F_A\setminus$ $\tilde{\tau}_j$-$\overline{F_C}$ =$\tilde{\tau}_j$-$\overline{F_R}\setminus\tilde{\tau}_j$-$\overline{F_C}\tilde{\subseteq}\tilde{\tau}_j$- $\overline{F_R \setminus F_C} = \tilde{\tau}_j$-$\overline{F_K}$, where $F_K = F_A \setminus (F_B\tilde{\cup}F_C)$.

Now F_A =$\tilde{\tau}_i$-$\overline{F_S}$, where $F_S = F_A\setminus$ $\tilde{\tau}_j$-$\overline{F_C}$. Thus F_A =$\tilde{\tau}_i$-$(\overline{F_S})\tilde{\subseteq}$ $\tilde{\tau}_i$-$(\overline{\tilde{\tau}_j - \overline{F_K}})$. Thus $F_A = \tau_i$-$(\overline{\tau_j - \overline{(F_K)}})$, which implies τ_i-$(\tau_j - F_D^o)^o = F_\emptyset$, where $F_D = F_B\tilde{\cup}F_C$. Hence the proof

Theorem 3.3. *Let $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$ be a soft bitopological space with $\tilde{\tau}_1 \subseteq \tilde{\tau}_2$. If $F_B, F_C \in (2,1)$-$\mathcal{SND}(F_A)$. Then $F_D \in (2,1)$-$\mathcal{SND}(F_A)$ where $F_D = F_B \tilde{\cup} F_C$.*

Proof. If $F_B, F_C \in (2,1)$-$\mathcal{SND}(F_A)$, then $\tilde{\tau}_2$-$(\tilde{\tau}_1$-$\overline{F_B})^o = F_\emptyset$ and $\tilde{\tau}_2$-$(\tilde{\tau}_1$-$\overline{F_C})^o = F_\emptyset$.

So, $\tilde{\tau}_2$-$(\tilde{\tau}_2$-$\overline{(\tilde{\tau}_1\text{-}\overline{F_B})^o})^o = F_\emptyset$. Hence $\tilde{\tau}_1$-$\overline{F_B} \in 2$-$\mathcal{SBD}(F_A)$.
Also $\tilde{\tau}_2$-$(\tilde{\tau}_2 - \overline{(\tilde{\tau}_1 - \overline{F_C})})^o \tilde{\subseteq}\ \tilde{\tau}_2$-$(\tilde{\tau}_1$-$\overline{F_C})^o = F_\emptyset$. Thus $\tilde{\tau}_1$-$\overline{F_C} \in 2$-$\mathcal{SND}(F_A)$. Hence by Theorem 3.2, $\tilde{\tau}_1$-$\overline{F_D} \in 2$-$\mathcal{SBD}(F_A)$, where $F_D = F_B \tilde{\cup} F_C$.

Then $\tilde{\tau}_2$-$(\tilde{\tau}_2 - (\tilde{\tau}_1 - F_D)^o)^o = F_\emptyset$, Thus $\tilde{\tau}_2$-$(\tilde{\tau}_1 - F_D)^o = F_\emptyset \Rightarrow F_D \in (2,1)$-$\mathcal{SND}(F_A)$.

Definition 3.5. Let $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$ be a soft bitopological space. A soft subset F_B of F_A is said to be (i,j)*-soft nowhere dense set at a point* x if and only if there exists a $\tilde{\tau}_i$-soft open neighborhood F_C of x in F_A such that $\tilde{\tau}_i$-$(\tilde{\tau}_j - \overline{F_D})^o = F_\emptyset$, where $F_D = F_B \tilde{\cap} F_C$. The set of all (i,j)-soft nowhere dense at x is denoted by (i,j)-$\mathcal{SND}(F_A, x)$.

Definition 3.6. Let $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$ be a soft bitopological space. A soft subset F_B in F_A is said to be (i,j)*-soft boundary at a point* x iff there exists a $\tilde{\tau}_i$-soft open neighborhood F_C of x over F_A such that $\tilde{\tau}_i$-$(\tilde{\tau}_j - F_D^o)^o = F_\emptyset$ where $F_D = F_B \tilde{\cap} F_C$. The set of all (i,j)-soft boundary sets at x is denoted by (i,j)-$\mathcal{SBD}(F_A, x)$.

Proposition 3.2. *Let $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$ be a soft bitopological space with $\tilde{\tau}_1 \subseteq \tilde{\tau}_2$. Then the following results hold:*

(i) 1-$\mathcal{SND}(F_A, x) \subseteq (1,2)$-$\mathcal{SND}(F_A, x)$.

(ii) $(2,1)$-$\mathcal{SND}(F_A, x) \subseteq 2$-$\mathcal{SND}(F_A, x)$.

(iii) 2-$\mathcal{SBD}(F_A, x) \subseteq (2,1)$-$\mathcal{SBD}(F_A, x)$.

(iv) (1,2)-$\mathcal{SBD}(F_A, x) = 1$-$\mathcal{SBD}(F_A, x) \subseteq (2,1)$-$\mathcal{SBD}(F_A, x)$.

Proof: The proofs of the parts (i) and (ii) are easy to obtain, so omitted.

(iii) Let $F_B \in 2$-$\mathcal{SBD}(F_A, x)$ then there exists a $\tilde{\tau}_2$-soft open neighborhood F_C of x in F_A such that $\tilde{\tau}_2$-$(\tilde{\tau}_2 - F_D^o)^o = F_\emptyset$ where $F_D = F_B \tilde{\cap} F_C$. Since $\tilde{\tau}_1 \subseteq \tilde{\tau}_2$, we have $\tilde{\tau}_1 - {F_D}^o \tilde{\subseteq} \tilde{\tau}_2 - {F_D}^o$. Thus $\tilde{\tau}_2$-$(\tilde{\tau}_1 - F_D^o)^o = F_\emptyset$. Thus $F_B \in (2,1)$-$\mathcal{SBD}(F_A, x)$. Hence the result.

Theorem 3.4. *Let $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$ be a soft bitopological space with $\tilde{\tau}_1 \subseteq \tilde{\tau}_2$. Let x be any point in F_A. Then $F_B \in (1,2)$-$\mathcal{SND}(F_A, x) \Leftrightarrow \tilde{\tau}_2$-$\overline{F_B} \in 1$-$\mathcal{SBD}(F_A, x)$.*

Proof. Suppose that $F_B \notin (1,2)$-$\mathcal{SND}(F_A,x)$, then there exists a $\tilde{\tau}_1$-soft open neighborhood F_C of x in F_A such that $\tilde{\tau}_1$-$(\tilde{\tau}_2 - \overline{F_D})^o \neq F_\emptyset$ where $F_D = F_B \tilde{\cap} F_C$. Thus there exists a soft set $F_M \in \tilde{\tau}_1 \setminus \{F_\emptyset\}$ such that $F_M \tilde{\subseteq} \tilde{\tau}_2 - \overline{F_D}$.

Clearly, $F_M \tilde{\cap} F_C \tilde{\subseteq} \tilde{\tau}_2 - \overline{F_D} \tilde{\cap} F_C \tilde{\subseteq} \tilde{\tau}_2 - \overline{F_B} \tilde{\cap} F_C$.Thus $\tilde{\tau}_1$-$F_N^o \tilde{\subseteq} \tilde{\tau}_1$-$F_Y^o$, where $F_N = F_M \tilde{\cap} F_C$ and $F_Y = \tilde{\tau}_2 - \overline{F_B} \tilde{\cap} F_C$. This implies the fact that $\tilde{\tau}_1$-$F_Y^o \neq F_\emptyset$. So, $F_Y \notin 1$-$\mathcal{SBD}(F_A,x)$.

As F_C is chosen arbitrarily from $\tilde{\tau}_1$ as a soft neighborhood of x, so $\tilde{\tau}_2$-$\overline{F_B} \notin 1$-$\mathcal{SBD}(F_A,x)$.

Conversely, suppose $\tilde{\tau}_2$-$\overline{F_B} \notin 1$-$\mathcal{SBD}(F_B,x)$, then $\tilde{\tau}_1$-$F_K^o \neq F_\emptyset$, where $F_K = \tilde{\tau}_2$-$\overline{F_B} \tilde{\cap} F_C$ and F_C be a $\tilde{\tau}_1$-neighborhood of x. Then there exists a soft set $F_R \in \tilde{\tau}_1 \setminus \{F_\emptyset\}$ such that $F_R \tilde{\subseteq} F_K \tilde{\subseteq} \tilde{\tau}_2$-$\overline{F_Z}$, where $F_Z = F_B \tilde{\cap} F_C$ as $\tilde{\tau}_1 \subseteq \tilde{\tau}_2$. Thus $\tilde{\tau}_1$-$(\tilde{\tau}_2$-$\overline{F_Z})^o \neq F_\emptyset$. Since F_C is an arbitrary $\tilde{\tau}_1$-neighborhood of x, so $F_B \notin (1,2)$-$\mathcal{SND}(F_A,x)$. Thus the proof.

Definition 3.7. Let $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$ be a soft bitopological space. A soft subset F_B of F_A is called *(i,j)-soft first category or (i,j)-soft meager* over F_A if $F_B = \tilde{\bigcup}_{i\in\Delta} F_{A_i}$ where $F_{A_i} \in (i,j)$-$\mathcal{SND}(F_A)$. Otherwise it is said to be *(i,j)-soft second category* if it is not (i,j)-soft first category.

The collection of all soft sets over F_A which are of (i,j)-soft first category is denoted by (i,j)-$SC_I(F_A)$ and the collection of all soft sets over F_A, those are of (i,j)-soft second category is denoted by (i,j)-$SC_{II}(F_A)$.

Theorem 3.5. *Let $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$ be a soft bitopological space, then F_A is of (i,j)-soft second category if and only if the soft intersection of any collection of i-soft dense j-soft open subset of F_A is not $F_\emptyset$.*

Proof. Let F_A is of (i,j)-soft first category. Thus, $F_A = \tilde{\bigcup}_{i\in\Delta} F_{A_i}$ where each F_{A_i} is τ_j- soft closed set and belongs to i-$\mathcal{SBD}(F_A)$. Thus $\tilde{\bigcap}_{i\in\Delta} F_{B_i} = \tilde{\emptyset}$, where $F_{B_i} = F_A \setminus F_{A_i}$. Hence $\{F_{B_i} | i \in \Delta\}$ is a collection of $\tilde{\tau}_i$-soft dense $\tilde{\tau}_j$-soft open subsets of F_A.

Conversely, let $\{F_{A_i} | i \in \Delta\}$ be a collection of soft subsets of F_A, where F_{A_i}'s are i-soft dense j-soft open subsets with $\tilde{\bigcap}_{i\in\Delta} F_{A_i} = F_\emptyset$.
Thus $F_A = \tilde{\bigcup}_{i\in\Delta} F_{D_i}$, where $F_{D_i} = F_A \setminus F_{A_i}$ are $\tilde{\tau}_j$-soft closed sets. Now after some steps we have $\tilde{\tau}_i$-$(\tilde{\tau}_i - F_{D_i}^o)^o = F_\emptyset$. Hence $F_{D_i} \in i$-$\mathcal{SBD}(F_A)$. So, $\tilde{\tau}_i - F_{D_i}^o = F_\emptyset \Rightarrow \tilde{\tau}_i$-$(\tilde{\tau}_j - \overline{F_{D_i}})^o = F_\emptyset$. Thus $F_{D_i} \in (i,j)$-$\mathcal{SND}(F_A)$. Hence $F_A \in (i,j)$-$SC_I(F_A)$. Hence the proof.

Theorem 3.6. *Let $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$ be a soft bitopological space with $\tilde{\tau}_1 \subseteq \tilde{\tau}_2$. If F_A is of $(1,2)$-soft second category, then 2-$SG_\delta(F_A) \cap 1$-$SD(F_A) \subseteq (1,2)$-$SC_{II}(F_A)$.*

Proof. Let $F_B \in 2\text{-}SG_\delta(F_A) \cap 1\text{-}SD(F_A)$, then $F_B = \widetilde{\bigcap\limits_{i\in\Delta}} F_{A_i}$, where each F_{A_i} is $\tilde{\tau}_2$-soft open and belongs to 1-$\mathcal{SD}(F_A)$. Let $F_D = F_A \setminus F_B = \widetilde{\bigcup\limits_{i\in\Delta}} F_{C_i} \in (1,2)\text{-}SC_I(F_A)$, where $F_{C_i} = F_A \setminus F_{A_i}$ as $F_{C_i} \in (1,2)\text{-}\mathcal{SND}(F_A)$.

If $F_B \in (1,2)\text{-}SC_I(F_A)$, then $F_A = F_B \tilde{\cup} F_D$ which implies F_A is of $(1,2)$-soft first category; which is a contradiction. Thus $F_A \in (1,2)\text{-}SC_{II}(F_A)$.

4. Conclusion.

This paper can be considered as an introductory paper on various fundamental notions viz. soft nowhere dense, soft boundary of a soft set, soft boundary at a point which are important for development of soft bitopological space as defined by Şenel and N. Çağman [17]. Special emphasis is given to contribute fundamental structures for soft bitopological space under soft finer topologies. We studied various results between concepts of nowhere dense, boundary of a soft set, boundary with respect to a point under soft bitopological space and so on. Some basic properties have investigated. Further systematic study on this area is necessary for development of this area and its possible applications in different aspects of science for betterment of mankind.

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