On almost b-continuous functions in a bitopological space

by

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Abstract: The aim of this paper is to investigate some properties of almost b-continuous function in a bitopological space. Relationships with some other types of functions are also investigated.

Mathematics Subject Classification (2010): 54E55.

Key words: Bitopological spaces, (i,j)-b-open, (i,j)-δ-closed, almost b-continuous function.

1. Introduction

The notion of a bitopological space (X,τ_1,τ_2), where X is a non-empty set and τ_1, τ_2 are topologies on X, was introduced by Kelly [7]. In 1996, Andrijevic [2] introduced the concept of b-open set in a topological space. Later Al-Hawary and Al-Omari [1] defined the notion b-open set and b-continuity in a bitopological space and established several fundamental properties. Sengul [11] defined the notion of almost b-continuous function in a topological space and established relationships between several properties of this notion with other known results. In addition to this, Duszynski et al.[6] introduced the concept of almost b-continuous function in a bitopological space. In the light of the above results, the purpose of this paper is to study almost b-continuity in a bitopological space and to obtain several characterizations and properties of this concept.

Bitopological space and its properties have many useful applications in real world. In 2010, Salama [10] used lower and upper approximations of Pawlak's rough sets by using a class of generalized closed set of bitopological space for data reduction of rheumatic fever data sets. Fuzzy topology integrated support vector machine (FTSVM)-classification method for remotely sensed images based on standard support vector machine (SVM) were introduced by using fuzzy topology by Zhang etal [16]. For some of recent applications of generalized forms of topological or bitopological space as fuzzy, rough version etc one may refer to [10,14,16]. Acharjee and Tripathy [20] used concept of (γ, δ)-BSC set of bitopology to determine poverty patterns and equilibria in mixed budget. Recently, Acharjee and Tripathy [21] investigated some fundamental results in soft bitopology, a new area of research created in 2014.

2. Preliminaries

Throughout this paper, bitopological spaces (X, τ_1, τ_2) and (Y, σ_1, σ_2) are represented by X and Y; on which no separation axiom is assumed and (i, j) means the topologies τ_i and τ_j; where $i, j \in \{1, 2\}, i \neq j$. For a subset A of (X, τ_1, τ_2), i-$int(A)$ (respectively, i-$cl(A)$) denotes interior (resp. closure) of A with respect to the topology τ_i, where $i \in \{1, 2\}$.

Now, we list some definitions and results those will be used throughout this article.

Definition 2.1. Let (X, τ_1, τ_2) be a bitopological space, then a subset A of X is said to be

(a) (i, j)-b-open ([1]) if $A \subseteq i\text{-}int(j\text{-}cl(A)) \cup j\text{-}cl(i\text{-}int(A))$.

(b) (i, j)-regular open ([3]) if $A = i\text{-}int(j\text{-}cl(A))$.

(c) (i, j)-regular closed ([4]) if $A = i\text{-}cl(j\text{-}int(A))$.

The complement of (i, j)-b-open set is said to be (i, j)-b-closed set.

Definition 2.2.([1]) Let (X, τ_1, τ_2) be a bitopological space and $A \subseteq X$. Then

(a) (i, j)-b-closure of A; denoted by (i, j)-$bcl(A)$, is defined as the intersection of all (i, j)-b-closed sets containing A.

(b) (i, j)-b-interior of A; denoted by (i, j)-$bint(A)$, is defined as the union of all (i, j)-b-open sets contained in A.

Lemma 2.1.([1]) Let (X, τ_1, τ_2) be a bitopological space and $A \subseteq X$. Then

(a) (i, j)-$bint(A)$ is (i, j)-b-open.

(b) (i, j)-$bcl(A)$ is (i, j)-b-closed.

(c) A is (i, j)-b-open if and only if $A = (i, j)$-$bint(A)$.

(d) A is (i,j)-b-closed if and only if $A = (i,j)$-$bcl(A)$.

Lemma 2.2.([9]) Let (X, τ_1, τ_2) be a bitopological space and $A \subseteq X$. Then

(a) $X \setminus (i,j)$-$bcl(A) = (i,j)$-$bint(X \setminus A)$

(b) $X \setminus (i,j)$-$bint(A) = (i,j)$-$bcl(X \setminus A)$

Lemma 2.3.([1]) Let (X, τ_1, τ_2) be a bitopological space and $A \subseteq X$. Then $x \in (i,j)$-$bcl(A)$, if and only if for every (i,j)-b-open set U containing x such that $U \cap A \neq \emptyset$.

Definition 2.3. A function $f : (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$ is said to be

(a) (i,j)-b-continuous ([1]) if $f^{-1}(A)$ is (i,j)-b-open in X, for each σ_i-open set A of Y.

(b) (i,j)-weakly b-continuous ([13]) if for each $x \in X$ and each σ_i-open set V of Y containing $f(x)$, there exists an (i,j)-b-open set U containing x such that $f(U) \subseteq j$-$cl(V)$.

Definition 2.4.([8]) Let (X, τ_1, τ_2) be a bitopological space. A point $x \in X$ is said to be an (i,j)-δ-cluster point of A if $A \cap U \neq \emptyset$; for every (i,j)-regular open set U containing x. The set of all (i,j)-δ-cluster points of A is called (i,j)-δ-closure of A and it is denoted by (i,j)-$cl_\delta(A)$. A subset A of X is said to be (i,j)-δ-closed if the set of all (i,j)-δ-cluster points of A is a subset of A. The complement of an (i,j)-δ-closed set is an (i,j)-δ-open. So, a subset of X is (i,j)-δ-open; if it is expressible as union of (i,j)-regular open sets.

3. (i,j)-almost b-continuous functions

Definition 3.1.([6]) A function $f : (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$ is said to be (i,j)-almost b-continuous at a point $x \in X$; if for each σ_i-open set V of Y containing $f(x)$, there exists an (i,j)-b-open set U of X containing x such that $f(U) \subseteq i$-$int(j$-$cl(V))$.

If f is (i,j)-almost b-continuous at every point x of X, then it is called (i,j)-almost b-continuous.

Theorem 3.1. The following statements are equivalent for a function $f : (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$.

(a) f is (i,j)-almost b-continuous.

(b) (i,j)-$bcl(f^{-1}(i$-$cl(j$-$int(i$-$cl(B))))) \subseteq f^{-1}(i$-$cl(B))$, for every subset B of Y.

(c) (i,j)-$bcl(f^{-1}(i\text{-}cl(j\text{-}int(G)))) \subseteq f^{-1}(G)$, for every (i,j)-regular closed set G of Y.

(d) (i,j)-$bcl(f^{-1}(i\text{-}cl(V))) \subseteq f^{-1}(i\text{-}cl(V))$, for every σ_j-open set V of Y.

(e) $f^{-1}(V) \subseteq (i,j)$-$bint(f^{-1}(i\text{-}int(j\text{-}cl(V))))$, for every σ_i-open set V of Y.

Proof. (a) ⇒(b) Let, $x \in X$ and B is any subset of Y. We assume that $x \in X \setminus f^{-1}(i\text{-}cl(B))$ and so, $f(x) \in Y \setminus i\text{-}cl(B)$. Then, there exists a σ_i-open set C of Y containing $f(x)$ such that $C \cap B = \emptyset$. Therefore $C \cap i\text{-}cl(j\text{-}int(i\text{-}cl(B))) = \emptyset$ and hence $i\text{-}int(j\text{-}cl(C)) \cap i\text{-}cl(j\text{-}int(i\text{-}cl(B))) = \emptyset$. By the given hypothesis, there exists an (i,j)-b-open set D such that $f(D) \subseteq i\text{-}int(j\text{-}cl(C))$. So, we have $D \cap f^{-1}(i\text{-}cl(j\text{-}int(i\text{-}cl(B)))) = \emptyset$. Therefore by Lemma 2.3, we have $x \in X \setminus (i,j)$-$bcl(f^{-1}(i\text{-}cl(j\text{-}int(i\text{-}cl(B)))))$. Hence, (i,j)-$bcl(f^{-1}(i\text{-}cl(j\text{-}int(i\text{-}cl(B))))) \subseteq f^{-1}(i\text{-}cl(B))$.

(b) ⇒(c) Let, G be an (i,j)-regular closed set in Y. Therefore, $G = i\text{-}cl(j\text{-}int(G))$. Now, (i,j)-$bcl(f^{-1}(i\text{-}cl(j\text{-}int(G)))) = (i,j)$-$bcl(f^{-1}(i\text{-}cl(j\text{-}int(i\text{-}cl(j\text{-}int(G)))))) \subseteq f^{-1}(i\text{-}cl(j\text{-}int(G))) = f^{-1}(G)$.

(c) ⇒(d) Let, V be σ_j-open in Y. Therefore, $i\text{-}cl(V)$ is (i,j)-regular closed in Y. Hence by (c) we have, (i,j)-$bcl(f^{-1}(i\text{-}cl(V))) \subseteq (i,j)$-$bcl(f^{-1}(i\text{-}cl(j\text{-}int(i\text{-}cl(V))) \subseteq f^{-1}(i\text{-}cl(V))$.

(d) ⇒(e) Let, V be σ_i-open in Y and so, $Y \setminus j\text{-}cl(V)$ is σ_j-open in Y.
Hence by (d) we have, (i,j)-$bcl(f^{-1}(i\text{-}cl(Y \setminus j\text{-}cl(V)))) \subseteq f^{-1}(i\text{-}cl(Y \setminus j\text{-}cl(V)))$.

$\Rightarrow (i,j)$-$bcl(f^{-1}(Y \setminus i\text{-}int(j\text{-}cl(V)))) \subseteq f^{-1}(Y \setminus i\text{-}int(j\text{-}cl(V)))$

$\Rightarrow (i,j)$-$bcl(X \setminus f^{-1}(i\text{-}int(j\text{-}cl(V)))) \subseteq X \setminus f^{-1}(i\text{-}int(j\text{-}cl(V)))$

$\Rightarrow X \setminus (i,j)$-$bint(f^{-1}(i\text{-}int(j\text{-}cl(V)))) \subseteq X \setminus f^{-1}(i\text{-}int(j\text{-}cl(V))) \subseteq X \setminus f^{-1}(V)$
Hence $f^{-1}(V) \subseteq (i,j)$-$bint(f^{-1}(i\text{-}int(j\text{-}cl(V))))$.

(e) ⇒(a) Let, $x \in X$ and V be a σ_i-open set in Y containing $f(x)$. Then, $x \in f^{-1}(V) \subseteq (i,j)$-$bint(f^{-1}(i\text{-}int(j\text{-}cl(V))))$. Putting $U = (i,j)$-$bint(f^{-1}(i\text{-}int(j\text{-}cl(V))))$ and by Lemma 2.1, we have U is (i,j)-b-open and $U \subseteq f^{-1}(i\text{-}int(j\text{-}cl(V)))$. So $f(U) \subseteq i\text{-}int(j\text{-}cl(V))$. Hence, f is (i,j)-almost b-continuous.

Theorem 3.2. The following statements are equivalent for a function $f : (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$.

(*a*) f is (i,j)-almost b-continuous.

(*b*) $f((i,j)\text{-}bcl(A)) \subseteq (i,j)\text{-}cl_\delta(f(A))$, for every subset A of X.

(c) (i,j)-$bcl(f^{-1}(B)) \subseteq f^{-1}((i,j)$-$cl_\delta(B))$, for every subset B of Y.

(d) $f^{-1}(C)$ is (i,j)-b-closed in X for every (i,j)-δ-closed subset C of Y.

(e) $f^{-1}(D)$ is (i,j)-b-open in X for every (i,j)-δ-open subset D of Y.

Proof. (a)⇒(b) Let, A be a subset of X containing x and V be a σ_i-open set of Y containing $f(x)$. Since, f is (i,j)-almost b-continuous, there exists an (i,j)-b-open set U containing x such that, $f(U) \subseteq i$-$int(j$-$cl(V))$. Let, $x \in (i,j)$-$bcl(A)$, then by Lemma 2.3, we have $U \cap A \neq \emptyset$; hence $\emptyset \neq f(U) \cap f(A) \subseteq i$-$int(j$-$cl(V)) \cap f(A)$. Since, V is σ_i-open in Y, $V \subseteq i$-$int(j$-$cl(V))$ and i-$int(j$-$cl(V))$ is (i,j)-regular open in Y. Hence, $f(x) \in (i,j)$-$cl_\delta f(A)$. Conseqently, (i,j)-$bcl(A) \subseteq f^{-1}((i,j)$-$cl_\delta(f(A)))$. This implies that $f((i,j)$-$bcl(A)) \subseteq (i,j)$-$cl_\delta(f(A))$.

(b)⇒(c) Suppose, B is any subset of Y. Then by (b), $f((i,j)$-$bcl(f^{-1}(B))) \subseteq (i,j)$-$cl_\delta(f(f^{-1}(B))) \subseteq (i,j)$-$cl_\delta(B)$. This implies; (i,j)-$bcl(f^{-1}(B)) \subseteq f^{-1}((i,j)$-$cl_\delta(B))$.

(c)⇒(d) Let, C be an (i,j)-δ-closed subset of Y. Then by (c), (i,j)-$bcl(f^{-1}(C)) \subseteq f^{-1}(C)$ and so, $f^{-1}(C)$ is (i,j)-b-closed in X.

(d)⇒(e) Let, D be an (i,j)-δ-open subset of Y. Then, $Y \setminus D$ is (i,j)-δ-closed in Y. By (d), $f^{-1}(Y \setminus D) = X \setminus f^{-1}(D)$ is (i,j)-b-closed in X. Hence, $f^{-1}(D)$ is (i,j)-b-open in X.

(e)⇒(a) Let, A be a σ_i-open subset of Y containing $f(x)$. Then, i-$int(j$-$cl(A))$ is (i,j)-regular open in Y containing $f(x)$. Since, i-$int(j$-$cl(A))$ is (i,j)-δ-open in Y, thus by (e), $f^{-1}(i$-$int(j$-$cl(A)))$ is (i,j)-b-open in X. Now, $A \subseteq i$-$int(j$-$cl(A))$. This implies that, $f^{-1}(A) \subseteq f^{-1}(i$-$int(j$-$cl(A)) = (i,j)$-$bint(f^{-1}(i$-$int(j$-$cl(A))))$. Hence, by theorem 3.1, f is (i,j)-almost b-continuous.

Definition 3.2.([15]) A function $f : (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$ is said to have (i,j)-b interiority condition, if (i,j)-$bint(f^{-1}(j$-$cl(V))) \subseteq f^{-1}(V)$, for every σ_i-open subset V of Y.

Theorem 3.3. Let, $f : (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$ be a function. If f is (i,j)-almost b-continuous and satisfies (i,j)-b interiority condition, then f is (i,j)-b-continuous.

Proof. Let, U be a σ_i-open subset of Y. By hypothesis, f is (i,j)-almost b-continuous. Therefore by theorem 3.1, we have $f^{-1}(U) \subseteq (i,j)$-$bint(f^{-1}(i$-$int(j$-$cl(U)))) \subseteq (i,j)$-$bint(f^{-1}(j$-$cl(U)))$. Again by the (i,j)-b interiority condition of f, we get (i,j)-$bint(f^{-1}(j$-$cl(U))) \subseteq f^{-1}(U)$. Thus we get $f^{-1}(U) = (i,j)$-$bint(f^{-1}(j$-$cl(U)))$ and so $f^{-1}(U)$ is (i,j)-b-open, by Lemma 2.1. Hence f is (i,j)-b-continuous.

Definition 3.3.([7]) A bitopological space (X, τ_1, τ_2) is said to be pairwise Hausdorff or pairwise T_2, if for each pair of distinct points x and y of X, there exist a τ_i-open set U containing x and a τ_j-open set V containing y such that $U \cap V = \emptyset$.

Definition 3.4.([15]) A bitopological space (X, τ_1, τ_2) is said to be pairwise b-T_2, if for each pair of distinct points x and y of X, there exist a (i, j)-b-open set U containing x and a (j, i)-b-open set V containing y such that $U \cap V = \emptyset$.

Theorem 3.4. Let $f : (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$ be a function such that, Y is pairwise T_2. If for any two distinct points x and y of X, following conditions are hold
(a) $f(x) \neq f(y)$
(b) f is (i, j)-weakly b-continuous at x,
(c) f is (j, i)-almost b-continuous at y,
then X is a pairwise b-T_2 space.

Proof. Let $x, y \in X$ such that $x \neq y$. Suppose, Y is pairwise T_2. Therefore, there exist a σ_i-open set U and a σ_j-open set V such that $f(x) \in U$, $f(y) \in V$ and $U \cap V = \emptyset$. Since $U \cap V = \emptyset$, so we have $j\text{-}cl(U) \cap (j\text{-}int(i\text{-}cl(V))) = \emptyset$. Again since f is (i, j)-weakly b-continuous at x and (j, i)-almost b-continuous at y, therefore there exists an (i, j)-b-open set F in X such that $x \in F$, $f(F) \subseteq j\text{-}cl(U)$ and there exists a (j, i)-b-open set G in X such that $y \in G$, $f(G) \subseteq j\text{-}int(i\text{-}cl(V))$. Thus, $F \cap G = \emptyset$. Hence, X is a pairwise b-T_2 space.

Definition 3.5.([4]) A bitopological space (X, τ_1, τ_2) is said to be pairwise Urysohn, if for each pair of distinct points x and y of X, there exist a τ_i-open set U containing x and a τ_j-open set V containing y such that $j\text{-}cl(U) \cap i\text{-}cl(V) = \emptyset$.

Theorem 3.5. Let $f : (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$ be a function, such that Y is a pairwise Urysohn space. If f is pairwise almost b-continuous, then X is pairwise b-T_2 space.

Proof. Let, $x, y \in X$ such that $x \neq y$. Therefore, $f(x) \neq f(y)$. Since, Y is pairwise Urysohn, therefore there exist a σ_i-open set U containing $f(x)$ and a σ_j-open set V containing $f(y)$ such that $j\text{-}cl(U) \cap i\text{-}cl(V) = \emptyset$. This implies $i\text{-}int(j\text{-}cl(U)) \cap j\text{-}int(i\text{-}cl(V)) = \emptyset$. Hence, $f^{-1}(i\text{-}int(j\text{-}cl(U))) \cap f^{-1}(j\text{-}int(i\text{-}cl(V))) = \emptyset$ and so, $(i, j)\text{-}bint(f^{-1}(i\text{-}int(j\text{-}cl(U)))) \cap (j, i)\text{-}bint(f^{-1}(j\text{-}int(i\text{-}cl(V)))) = \emptyset$. Again, since f is pairwise almost b-continuous, therefore by theorem 3.1, we have $x \in f^{-1}(U) \subseteq (i, j)\text{-}bint(f^{-1}(i\text{-}int(j\text{-}cl(U))))$ and $y \in f^{-1}(V) \subseteq (j, i)\text{-}bint(f^{-1}(j\text{-}int(i\text{-}cl(V))))$. Hence, X is pairwise b-T_2 space.

Theorem 3.6. Let $f : (X_1, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$ is (i, j)-weakly b-continuous, $g : (X_2, \psi_1, \psi_2) \longrightarrow (Y, \sigma_1, \sigma_2)$ is (i, j)-almost b-continuous and Y is pairwise Hausdorff, then the set $\{(x, y) \in X_1 \times X_2 : f(x) = g(y)\}$ is (i, j)-b-closed in $X_1 \times X_2$.

Proof. Let, $G = \{(x, y) \in X_1 \times X_2 : f(x) = g(y)\}$ and $(x, y) \in (X_1 \times X_2) \setminus G$. Thus, we get $f(x) \neq f(y)$. Since Y is pairwise Hausdorff, therefore there exist a σ_i-open set U_1 and a σ_j-open set U_2 of Y such that $f(x) \in U_1$, $g(y) \in U_2$ and $U_1 \cap U_2 = \emptyset$. Since, U_1 and U_2 are disjoint, hence $j\text{-}cl(U_1) \cap (i\text{-}int(j\text{-}cl(U_2)) = \emptyset$. Also, f is (i, j)-weakly b-continuous, so; there exists an (i, j)-b-open set V_1 containing x such that $f(V_1) \subseteq j\text{-}cl(U_1)$. Again g is (i, j)-almost b-continuous, thus, there exists an (i, j)-b-open set V_2 containing y such that $g(V_2) \subseteq i\text{-}int(j\text{-}cl(U_2))$. Thus, we obtain $(x, y) \in V_1 \times V_2 \subseteq (X_1 \times X_2) \setminus G$ and $V_1 \times V_2$ is (i, j)-b-open in $X_1 \times X_2$. It implies G is (i, j)-b-closed in $X_1 \times X_2$.

Definition 3.6.([13]) A bitopological space (X, τ_1, τ_2) is said to be (i, j)-almost regular, if for every $x \in X$ and for every τ_i-open set V of X, there exists a τ_i-open set U containing x such that $x \in U \subseteq j\text{-}cl(U) \subseteq i\text{-}int(j\text{-}cl(V))$.

Lemma 3.1.([6]) For a function $f : (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$, the following statements are equivalent :

(a) f is (i, j)-almost b-continuous.

(b) $f^{-1}(i\text{-}int(j\text{-}cl(V)))$ is (i, j)-b-open set in X, for each σ_i-open set V in Y.

(c) $f^{-1}(i\text{-}cl(j\text{-}int(F)))$ is (i, j)-b-closed set in X, for each σ_i-closed set F in Y.

(d) $f^{-1}(F)$ is (i, j)-b-closed set in X, for each (i, j)-regular closed set F of Y.

(e) $f^{-1}(V)$ is (i, j)-b-open set in X, for each (i, j)-regular open set V of Y.

Theorem 3.7. Let $f : (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$ be a function, such that Y is (i, j)-almost regular. Then, f is (i, j)-almost b-continuous if and only if f is (i, j)-weakly b-continuous.

Proof. Necessity : It is obvious that (i, j)-almost b-continuity implies (i, j)-weakly b-continuity.

Sufficiency : Assume that f is (i, j)-weakly b-continuous. Let, U be an (i, j)-regular open set in Y such that, $x \in f^{-1}(U)$. This implies $f(x) \in U$. Since Y is (i, j)-almost regular, therefore there exists a (i, j)-regular open set V in Y such that $f(x) \in V \subseteq j\text{-}cl(V) \subset U$. Again since f is (i, j)-weakly b-continuous, therefore there exists an (i, j)-b-open set W in X containing x such that $f(W) \subseteq j\text{-}cl(V) \subseteq U$. Thus, we get $W \subseteq f^{-1}(U)$. Thus, $x \in W = (i, j)\text{-}bint(W) \subseteq (i, j)\text{-}bint(f^{-1}(U))$. Hence $f^{-1}(U) \subseteq (i, j)\text{-}bint(f^{-1}(U))$. Consequently, $f^{-1}(U) = (i, j)\text{-}bint(f^{-1}(U))$ and so, $f^{-1}(U)$ is (i, j)-b-open. By Lemma 3.1, f is (i, j)-almost b-continuous.

Definition 3.7.([12]) A bitopological space (X, τ_1, τ_2) is said to be (i, j)-semi regular, if for every $x \in X$ and for every τ_i-open set V of X, there exists a τ_i-open set U containing x such that $x \in U \subseteq i\text{-}int(j\text{-}cl(U)) \subseteq V$.

Theorem 3.8. Let, $f : (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$ be a function, such that Y is (i, j)-semi regular. If f is (i, j)-almost b-continuous, then f is (i, j)-b-continuous.

Proof. Let, U be a σ_i-open set of Y containing $f(x)$. Therefore, $x \in f^{-1}(U)$. Since, Y is (i, j)-semi regular, thus there exists a σ_i-open set V such that $f(x) \in V \subset i\text{-}int(j\text{-}cl(V)) \subseteq U$. Again, f is (i, j)-almost b-continuous, so; there exists an (i, j)-b-open set W in X containing x such that $f(W) \subseteq i\text{-}int(j\text{-}cl(V)) \subset U$. So, $x \in W = (i, j)\text{-}bint(W) \subseteq (i, j)\text{-}bint(f^{-1}(U))$ and hence $f^{-1}(U) \subseteq (i, j)\text{-}bint(f^{-1}(U))$. Hence, $f^{-1}(U) = (i, j)\text{-}bint(f^{-1}(U))$. Now by Lemma 2.1, $f^{-1}(U)$ is (i, j)-b-open in X. Consequently, f is (i, j)-b-continuous.

Definition 3.8. A function $f : (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$ is said to be (i, j)-almost b -open if $f(U) \subseteq i\text{-}int(j\text{-}cl(f(U)))$, for every (i, j)-b-open set U of X.

Theorem 3.9. If a function $f : (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$ is (i, j)-almost b-open and (i, j)-weakly b-continuous, then f is (i, j)-almost b-continuous.

Proof. Let V be a σ_i-open set of Y containing $f(x)$. Since, f is (i, j)-weakly b-continuous, thus there exists an (i, j)-b-open set U in X containing x such that $f(U) \subseteq j\text{-}cl(V)$. Also, f is (i, j)-almost b-open, therefore $f(U) \subseteq i\text{-}int(j\text{-}cl(f(U))) \subseteq i\text{-}int(j\text{-}cl(V))$. Hence, f is (i, j)-almost b-continuous.

Lemma 3.2.([6]) For a function $f : (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$, the following statements are equivalent :

(a) f is (i, j)-almost b-continuous.

(b) For each $x \in X$ and each (i, j)-regular open set V of Y containing $f(x)$, there exists an (i, j)-b-open U in X containing x such that $f(U) \subseteq V$.

(c) For each $x \in X$ and each (i, j)-δ-open set V of Y containing $f(x)$, there exists an (i, j)-b-open U in X containing x such that $f(U) \subseteq V$.

Theorem 3.10. If $f : (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$ be a function and $g : X \longrightarrow X \times Y$ be the function defined by $g(x) = (x, f(x))$, for every $x \in X$, then g is (i, j)-almost b-continuous if and only if f is (i, j)-almost b-continuous.

Proof. Let, $x \in X$ and V be an (i, j)-regular open set of Y such that $f(x) \in V$. Then $g(x) = (x, f(x)) \in X \times V$ is (i, j)-regular open in $X \times Y$. Since, g is (i, j)-almost b-continuous, thus there exists an (i, j)-b-open set U containing x such that $g(U) \subseteq X \times Y$.

Thus we get $f(U) \subseteq V$. Hence by Lemma 3.2, we have f is (i,j)-almost b-continuous.

Conversely, let, $x \in X$ and W be an (i,j)-regular open set of $X \times Y$ such that $g(x) = (x, f(x)) \in X \times Y$. Then, there exists an (i,j)-regular open set V in Y such that $U \times V \subseteq W$. Since, f is (i,j)-almost b-continuous, hence there exists an (i,j)-b-open set A containing x such that $f(A) \subseteq V$. Let, $B = U \cap A$, then B is an (i,j)-b-open set containing x and so; $g(B) \subseteq U \times V \subseteq W$. Hence, g is (i,j)-almost b-continuous.

Theorem 3.11. If $g : (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$ is (i,j)-almost b-continuous and A is (i,j)-δ-closed set in $X \times Y$, then $P_X(A \cap G(g))$ is (i,j)-b-closed in X, where P_X denotes the projection of $X \times Y$ onto X and $G(g)$ denotes the graph of g.

Proof. Let, A be (i,j)-δ-closed set in $X \times Y$. Consider $x \in (i,j)\text{-}bcl(P_X(A \cap G(g)))$. Again, let U be a τ_i-open set of X containing x and V be a σ_i-open set of Y containing $g(x)$. Since, g is (i,j)-almost b-continuous, therefore by theorem 3.1, $x \in g^{-1}(V) \subseteq (i,j)\text{-}bint(g^{-1}(i\text{-}int(j\text{-}cl(V))))$ and $U \cap (i,j)\text{-}bint(g^{-1}(i\text{-}int(j\text{-}cl(V))))$ is (i,j)-b-open in X containing x. Since, $x \in (i,j)\text{-}bcl(P_X(A \cap G(g)))$, therefore $[U \cap (i,j)\text{-}bint(g^{-1}(i\text{-}int(j\text{-}cl(V))))] \cap P_X(A \cap G(g))$ containing some point y of X, which implies $(y, g(y)) \in A$ and $g(y) \in i\text{-}int(j\text{-}cl(V))$. Then, $\emptyset \neq (U \times (i\text{-}int(j\text{-}cl(V)))) \cap A \subseteq i\text{-}int(j\text{-}cl(U \times V)) \cap A$ and hence, $(x, g(x)) \in (i,j)\text{-}cl_\delta(A)$. Since, A is (i,j)-δ-closed, $(x, g(x)) \in A \cap G(g)$ and $x \in P_X(A \cap G(g))$. Therefore, $(i,j)\text{-}bcl(P_X(A \cap G(g))) \subseteq P_X(A \cap G(g))$. Hence, $P_X(A \cap G(g))$ is (i,j)-b-closed.

Definition 3.9.([3]) Let, (X, τ_1, τ_2) be a bitopological space and $A \subseteq X$, then A is said to be (i,j)-quasi H-closed relative to X; if for each cover $\{B_\alpha : \alpha \in \Delta\}$ of A by τ_i-open subsets of X, there exists a finite subset Δ_0 of Δ such that $A \subseteq \bigcup\{j\text{-}cl(B_\alpha) : \alpha \in \Delta_0\}$, wehere Δ is an index set.

Definition 3.10. Let (X, τ_1, τ_2) be a bitopological space and $A \subseteq X$, then A is said to be (i,j)-b-compact relative to X, if every cover of A by (i,j)-b-open sets of X has a finite subcover.

Theorem 3.12. If a function $f : (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$ is (i,j)-almost b-continuous and A is (i,j)-b-compact relative to X, then $f(A)$ is (i,j)-quasi H-closed relative to Y.

Proof. Let, A be (i,j)-b-compact relative to X and $\{B_\alpha : \alpha \in \Delta\}$ be any cover of $f(A)$ by σ_i-open sets of Y. Therefore, $f(A) \subseteq \bigcup\{B_\alpha : \alpha \in \Delta\}$ and so; $A \subseteq \bigcup\{f^{-1}(B_\alpha : \alpha \in \Delta\}$. Since, f is (i,j)-almost b-continuous, therefore by theorem 3.1, we have $f^{-1}(B_\alpha) \subseteq (i,j)\text{-}bint(f^{-1}(i\text{-}int(j\text{-}cl(B_\alpha)))) \subseteq (i,j)\text{-}bint(f^{-1}(j\text{-}cl(B_\alpha)))$. Then, $A \subseteq \bigcup\{(i,j)\text{-}bint(f^{-1}(j\text{-}cl(B_\alpha))) : \alpha \in \Delta\}$. Also, A is (i,j)-b-compact relative to X and $(i,j)\text{-}bint(f^{-1}(j\text{-}cl(B_\alpha)))$ is (i,j)-b-open for each $\alpha \in \Delta$, therefore there exists a finite subset Δ_0 of Δ such that $A \subseteq \bigcup\{(i,j)\text{-}bint(f^{-1}(j\text{-}cl(B_\alpha))) : \alpha \in \Delta_0\}$. This implies $f(A) \subseteq \bigcup\{f((i,j)\text{-}bint(f^{-1}(j\text{-}cl(B_\alpha)))) : \alpha \in \Delta_0\} \subseteq \bigcup\{f(f^{-1}(j\text{-}cl(B_\alpha))) : \alpha \in \Delta_0\} \subseteq \bigcup\{j\text{-}cl(B_\alpha) : \alpha \in \Delta_0\}$. Hence,

$f(A)$ is (i, j)-quasi H-closed relative to Y.

Conflict of interest: Authors declare that their is no conflict of interest.

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