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On Euler Type Integrals Involving Extended Mittag-Leffler Functions

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Abstract: The object of the present paper is to establish some interested theorems on Euler type integral involving extended Mittag-Leffler function. Further, we reduce some special cases involving various known functions like Wiman function, Prabhakar function, exponential and Binomial functions.

Key Words: Mittag-Leffler function, Hypergeometric function, Exponential function.

Contents

1 Introduction **125**

2 Main Result **127**

3 Special Cases **130**

4 Conclusion **133**

References **133**

1. Introduction

Swedish mathematician Gosta Mittag-Leffler (1903) introduced the function

$$E_\alpha(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\alpha n+1}, \Re(\alpha) > 0, \alpha \in C. \tag{1.1}$$

where z is a complex variable and Γ is a Gamma function $\alpha \geq 0$. The Mittag-Leffler function is a direct generalization of the exponential function to which it reduces for $\alpha = 1$. For $0 < \alpha < 1$, it interpolates between the pure exponential and hyper geometric function $\frac{z}{z-1}$. Its importance realized during the last two decades due to its involvement in the problems of physics, chemistry, biology, engineering and applied sciences. For obtaining solutions of fractional differential and integral equations Mittag-Leffler function plays an important role. Mittag-Leffler function is connected with an extensive variety of problem in diverse areas of mathematics and mathematical physics. In addition, from exponential behavior, the deviations of physical phenomena could also be represented by physical laws via Mittag-Leffler functions. Therefore, the uses of Mittag-Leffler functions are constantly increasing, especially in physics.

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The generalization of $E_\alpha(z)$ was studied by Wiman in 1905 and he defined the function as

$$E_{\alpha,\beta}(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\alpha n + \beta)}, (\alpha, \beta, \in C, \Re(\alpha) > 0, \Re(\beta) > 0). \quad (1.2)$$

Which is known as Wiman function.
Later In 1971, Prabhakar introduced the function $E^{\gamma}_{\alpha,\beta}(z)$ in the form of

$$E^{\gamma}_{\alpha,\beta}(z) = \sum_{n=0}^{\infty} \frac{(\gamma)_n z^n}{\Gamma(\alpha n + \beta) n!}, (\alpha, \beta, \gamma \in C, \Re(\alpha) > 0, \Re(\beta) > 0, \Re(\gamma) > 0), \quad (1.3)$$

where$(\gamma)_n$ is the Pochhammer symbol (Rainville (1960)).

$$(\gamma)_n = \frac{\Gamma(\gamma + n)}{\Gamma(\gamma)}, (\gamma)_0 = 1, (\gamma)_n = \gamma(\gamma+1)(\gamma+2)\ldots(\gamma+n+1), \quad for \;\; n \geq 1.$$

In the year 2007, Shukla and Prajapati introduced the function $E^{\gamma,q}_{\alpha,\beta}(z)$ and in the year 2009 Tariq O. Salim introduced the function $E^{\gamma,\delta}_{\alpha,\beta}(z)$ which are defined for $\alpha, \beta, \gamma \in C; \Re(\alpha) > 0, \Re(\beta) > 0, \Re(\gamma) > 0, q \in (0,1) \cup N$ and for $\alpha, \beta, \gamma, \delta \in C; \Re(\alpha) > 0, \Re(\beta) > 0, \Re(\gamma) > 0, \Re(\delta) > 0$, are as follows

$$E^{\gamma,q}_{\alpha,\beta} = \sum_{n=0}^{\infty} \frac{(\gamma)_{qn} z^n}{\Gamma(\alpha n + \beta) n!} \quad (1.4)$$

$$E^{\gamma,\delta}_{\alpha,\beta} = \sum_{n=0}^{\infty} \frac{(\gamma)_n z^n}{\Gamma(\alpha n + \beta)(\delta)_n} \quad (1.5)$$

Finally in 2012, a new generalization of Mittag-Leffler function was defined by Salim as

$$E^{\gamma,\delta,q}_{\alpha,\beta,p}(z) = \sum_{n=0}^{\infty} \frac{(\gamma)_{qn} z^n}{\Gamma(\alpha n + \beta)(\delta)_{pn}}, \quad (1.6)$$

where $\alpha, \beta, \gamma, \delta \in C; min[\Re(\alpha) > 0, \Re(\beta) > 0, \Re(\gamma) > 0, \Re(\delta) > 0]$ and $p, q > 0$.
In 2013, a new generalized form of Mittag-Leffler function is defined by Mumtaz Ahmad Khan et al. [10] as

$$E^{\mu,\rho,\gamma,q}_{\alpha,\beta,\nu,\sigma,\delta,p} = \sum_{n=0}^{\infty} \xi z^n, \quad (1.7)$$

where

$$\xi = \frac{(\mu)_{\rho n}(\gamma)_{qn}}{\Gamma(\alpha n + \beta)(\nu)_{\sigma n}(\delta)_{pn}}$$

and $\alpha, \beta, \gamma, \delta, \rho, \sigma, \mu, \nu \in C; p, q > 0, q < \Re(\alpha) + p$, and $min\{\Re(\alpha), \Re(\beta), \Re(\gamma), \Re(\delta), \Re(\rho), \Re(\sigma), \Re(\mu), \Re(\nu)\} > 0$.

2. Main Result

Theorem 2.1. *If* $\alpha, \beta, \gamma, \delta, \rho, \sigma, \mu, \nu, \eta \in C, \{\Re(\alpha), \Re(\beta), \Re(\gamma), \Re(\delta), \Re(\rho), \Re(\sigma), \Re(\mu), \Re(\nu), \Re(\eta)\} > 0, q < \Re(\alpha) + p,$ *then*

$$\int_0^1 u^{\beta-1}(1-u)^{\eta-1} Exp[\frac{-A}{u(1-u)}] E^{\mu,\rho,\gamma,q}_{\alpha,\beta,\nu,\sigma,\delta,p}(zu^{\alpha})du$$
$$= \sum_{n=0}^{\infty} \frac{(\mu)_{\rho} n(\gamma)_{qn}}{\Gamma(\alpha n+\beta)(\nu)_{\sigma n}(\delta)_{pn}}(z)^n B(\alpha n+\beta, \eta, A)$$

Proof: We consider L.H.S. of Theorem (2.1) by I_1

$$I_1 = \int_0^1 u^{\beta-1}(1-u)^{\eta-1} Exp[\frac{-A}{u(u-1)}] \sum_{n=0}^{\infty} \xi(zu^{\alpha})^n du$$

using the change of order of integration, we get

$$= \sum_{n=0}^{\infty} \xi(z)^n \int_0^1 u^{\beta+\alpha n-1}(1-u)^{\eta-1} Exp[\frac{-A}{u(u-1)}]du$$
$$= \sum_{n=0}^{\infty} \xi(z)^n B(\alpha n+\beta, \eta, A)]$$

which is required result. □

Corollary 2.2. *For* $A = 0$*, then theorem* (2.1) *reduces the following result*

$$I_1 = \frac{1}{\Gamma(n)} \int_0^1 u^{\beta-1}(1-u)^{\eta-1} E^{\mu,\rho,\gamma,q}_{\alpha,\beta,\nu,\sigma,\delta,p}(zu^{\alpha})du = E^{\mu,\rho,\gamma,q}_{\alpha,\beta+n,\nu,\sigma,\delta,p}(z).$$

Theorem 2.3. *If* $\alpha, \beta+n, \gamma, \delta, \rho, \sigma, \mu, \nu, \eta, \lambda, k, A \in C$,
$\{\Re(\alpha), \Re(\beta), \Re(\gamma), \Re(\delta), \Re(\rho), \Re(\sigma), \Re(\mu), \Re(\nu), \Re(\eta), \Re(k), \Re(\lambda), \Re(A)\} > 0$,
$q < \Re(\alpha) + p$*, and* $arg|\frac{bc+d}{ac+d}| < \pi$*, then*

$$\int_a^b (u-a)^{k-1}(b-u)^{\eta-1}(cu+d)^{\lambda} Exp[\frac{-A}{(u-a)(b-u)}] E^{\mu,\rho,\gamma,q}_{\alpha,\beta,\nu,\sigma,\delta,p}[z(b-u)^{\omega})du$$
$$= \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \zeta(z)^n (b-a)^{-2m+k+\eta+\omega n-1}$$
$$\times B(k-m, \eta+\omega n-m)(ac+d)^{\lambda} {}_2F_1\left[\begin{matrix} k-m, & -\lambda; & \frac{-(b-a)c}{ac+d} \\ k+\eta+\omega n-2m, & & \end{matrix}\right]$$

where

$$\zeta = \frac{(-A)^m(\mu)_{\rho n}(\gamma)_{qn}}{m!\Gamma(\alpha n+\beta)(\nu)_{\sigma n}(\delta)_{pn}}.$$

Proof:

$$\int_a^b (u-a)^{k-1}(b-u)^{\eta-1}(cu+d)^{\lambda}.\sum_{m=0}^{\infty}\frac{(-A)^m}{m!(u-a)^m(b-u)^m}\sum_{n=0}^{\infty}\xi[z(b-u)^{\omega}]^n du$$

$$= \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\zeta(z)^n\int_a^b (u-a)^{-m+k-1}(b-u)^{\omega n+\eta-m-1}(cu+d)^{\lambda}du \tag{2.1}$$

Using the integral [4, p.263], we have

$$\int_a^b (t-a)^{\alpha-1}(b-t)^{\beta-1}(ut+v)^{\gamma}dt = B(\alpha,\beta)(b-a)^{\alpha+\beta-1}(au+v)^{\gamma}$$

$$_2F_1\left[\alpha, \quad -\lambda; \quad \alpha+\beta; \tfrac{-(b-a)u}{au+\nu}\right],$$

$$\Re(\alpha)>0, \Re(\beta)>0; arg|\frac{bu+\nu}{au+\nu}|<\pi$$

Again, using above result in (2.1), we have

$$= \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\zeta(z)^n(b-a)^{-2m+k+\eta+\omega n-1}.B(k-m,\eta+\omega n-m)(ac+d)^{\lambda}$$

$$_2F_1\left[\begin{matrix} k-m, & -\lambda; & \frac{-(b-a)c}{ac+d} \\ k+\eta+\omega n-2m & & \end{matrix}\right]$$

□

Corollary 2.4. *Put $A=0$ then Theorem (2.3) reduces to the following interesting result*

$$\int_a^b (u-a)^{k-1}(b-u)^{\eta-1}(cu+d)^{\lambda}E^{\mu,\rho,\gamma,q}_{\alpha,\beta,\nu,\sigma,\delta,p}[z(b-u)^{\omega}]du$$

$$= (ac+d)^{\lambda}\sum_{n=0}^{\infty}\xi(z)^n B(k,\eta+\omega n)(b-a)^{k+\eta+\omega n-1}$$

$$_2F_1\left[\begin{matrix} k, & -\lambda; & \frac{-(b-a)c}{ac+d} \\ k+\eta+\omega n; & & \end{matrix}\right]$$

Corollary 2.5. *For $a=0, b=1$, then theorem (2.3) reduces to*

$$\int_0^1 (u)^{k-1}(1-u)^{\eta-1}(cu+d)^{\lambda}Exp[\frac{-A}{u(1-u)}]E^{\mu,\rho,\gamma,q}_{\alpha,\beta,\nu,\sigma,\delta,p}[z(1-u)^{\omega}]du$$

$$= (d)^{\lambda}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\zeta(z)^n B(k-m,\eta+\omega n-m)$$

$$_2F_1\left[\begin{matrix} k-m, & -\lambda; & \frac{-c}{d} \\ k+\eta+\omega n-2m; & & \end{matrix}\right]$$

Theorem 2.6. *If* $\alpha,\beta,\gamma,\delta,\rho,\sigma,\mu,\nu,k,\eta,A\in C$, $\{\Re(\lambda),\Re(k),\Re(A)\}>0$, $\lambda_1,\lambda_2\geq 0$, $q\leq\Re(\alpha)+p$ *and* $\{\Re(\alpha),\Re(\beta),\Re(\gamma),\Re(\delta),\Re(\rho),\Re(\sigma),\Re(\mu)\Re(\nu),\Re(\eta)\}>0$, *then*

$$\int_0^1 (u)^{k-1}(1-u)^{\eta-\lambda-1}[1-tu^{\lambda_1}(1-u)^{\lambda_2}]^{-a}Exp[\frac{-A}{u(1-u)}]E^{\mu,\rho,\gamma,q}_{\alpha,\beta,\nu,\sigma,\delta,p}(zu^{\alpha})du$$
$$=\sum_{r=0}^{\infty}\sum_{n=0}^{\infty}\xi(z)^n(a)_rB(k+\alpha n+\lambda_1 r,\eta-k+\lambda_2 r;A)\frac{t^r}{r!}$$

Proof: We starts from L.H. S. of theorem (2.6),

$$\int_0^1 (u)^{k-1}(1-u)^{\eta-\lambda-1}[1-tu^{\lambda_1}(1-u)^{\lambda_2}]^{-\alpha}Exp[\frac{-A}{u(1-u)}]E^{\mu,\rho,\gamma,q}_{\alpha,\beta,\nu,\sigma,\delta,p}(zu^{\alpha}))du$$

and using the definition of the generalized Mittage- Leffler function, we have

$$\int_0^1 (u)^{k-1}(1-u)^{\eta-\lambda-1}[1-tu^{\lambda_1}(1-u)^{\lambda_2}]^{-\alpha}Exp[\frac{-A}{u(1-u)}]\sum_{n=0}^{\infty}\xi(z)^n(u^{\alpha n})du$$
$$=\sum_{n=0}^{\infty}\xi(z)^n\int_0^1 (u)^{k+\alpha n-1}(1-u)^{\eta-k-1}[1-tu^{\lambda_1}(1-u)^{\lambda_2}]^{-\alpha}Exp[\frac{-A}{u(1-u)}]du \tag{2.2}$$

By using the integral [2, Eq.(3.5)]

$$\int_0^1 (u)^{\lambda-1}(1-u)^{\mu-\lambda-1}[1-tu^{\rho}(1-u)^{\sigma}]^{-\alpha}Exp[\frac{-A}{u(1-u)}]du$$
$$=\sum_{n=0}^{\infty}(\alpha)_n.B(\lambda+\rho n,\mu-\lambda+\sigma n,A)\frac{t^n}{n!}$$

Using above result in (2.2), we get

$$\sum_{r=0}^{\infty}\sum_{n=0}^{\infty}\xi(z)^n(a_r)B(k+\alpha n+\lambda_1 r,\eta-k+\lambda_2 r;A)\frac{t^r}{r!}$$

□

Corollary 2.7. *For* $a=0$, *the theorem* (2.6) *reduces to the following result*

$$\int_0^1 (u)^{k-1}(1-u)^{\eta-k-1}Exp[\frac{-A}{u(1-u)}]E^{\mu,\rho,\gamma,q}_{\alpha,\beta,\nu,\sigma,\delta,p}(zu^{\alpha}))du$$
$$=\sum_{n=0}^{\infty}\xi B(k+\alpha n,\eta-k;A)$$

Corollary 2.8. *For $a = A = 0$, then theorem (2.6) reduces to*

$$\int_0^1 (u)^{k-1}(1-u)^{\eta-k-1} E^{\mu,\rho,\gamma,q}_{\alpha,\beta,\nu,\sigma,\delta,p}(zu^{\alpha}))du$$
$$= \sum_{n=0}^{\infty} \xi B(k+\alpha n, \eta - k)$$

3. Special Cases

Case-1 (i) Put $\mu = \nu, \rho = \sigma$ and $p = 1$ in Theorem (2.1), we get

$$\int_0^1 (u)^{\beta-1}(1-u)^{\eta-1} Exp[\frac{-A}{u(1-u)}] E^{\gamma,q}_{\alpha,\beta,\delta}(zu^{\alpha})du$$
$$= \sum_{n=0}^{\infty} \frac{(\gamma)_{qn}}{\Gamma(\alpha n+\beta)} \frac{z^n}{(\delta)_n} B(\alpha n+\beta, \eta; A)$$

(ii) Put $\mu = \nu, \rho = \sigma$ and $p = \delta = 1$ in theorem (2.1), we get

$$\int_0^1 (u)^{\beta-1}(1-u)^{\eta-1} Exp[\frac{-A}{u(1-u)}] E^{\gamma,q}_{\alpha,\beta}(zu^{\alpha})du$$
$$= \sum_{n=0}^{\infty} \frac{(\gamma)_{qn}}{\Gamma(\alpha n+\beta)} \frac{z^n}{n!} B(\alpha n+\beta, \eta; A)$$

(iii) Put $\mu = \nu, \rho = \sigma$ and $p = \delta = 1, q = 1$ in theorem (2.1), we get

$$\int_0^1 (u)^{\beta-1}(1-u)^{\eta-1} Exp[\frac{-A}{u(1-u)}] E^{\gamma}_{\alpha,\beta}(zu^{\alpha})du$$
$$= \sum_{n=0}^{\infty} \frac{(\gamma)_{qn}}{\Gamma(\alpha n+\beta)} \frac{z^n}{n!} B(\alpha n+\beta, \eta; A),$$

where $E^{\gamma}_{\alpha,\beta}(z)$ is Prabhakar function.

(iv) Put $\mu = \nu, \rho = \sigma$ and $p = \delta = \gamma = q = 1$ in theorem (2.1), we get

$$\int_0^1 (u)^{\beta-1}(1-u)^{\eta-1} Exp[\frac{-A}{u(1-u)}] E_{\alpha,\beta}(zu^{\alpha})du$$
$$= \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\alpha n+\beta)} B(\alpha n+\beta, \eta; A),$$

where $E_{\alpha,\beta}[z]$ is Wiman function.

Case-2 (i) Put $\mu=\nu, \rho=\sigma$ and $p=1$ in theorem (2.3), we get

$$\int_a^b (u-a)^{k-1}(b-u)^{\eta-1}(cu+d)^{\lambda}$$
$$\times Exp[\frac{-A}{(u-a)(b-u)}]E^{\gamma,\sigma}_{\alpha,\beta,\delta,}[z(b-u)^{\omega}]du$$
$$=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-A)^m(\gamma)_{qn}}{m!\Gamma(\alpha n+\beta)(\delta)_n}(z)^n(b-a)^{-2m+k+\eta+\omega n-1}$$
$$\times B(k-m,\eta+\omega n-m)(ac+d)^{\lambda}_2 F_1\left[\begin{matrix} k-m, & -\lambda; & \frac{-(b-a)c}{ac+d} \\ k+\eta+\omega n-2m & & \end{matrix}\right]$$

(ii) Put $\mu=\nu, \rho=\sigma$ and $p=\delta=1$ in theorem (2.3), we get

$$\int_a^b (u-a)^{k-1}(b-u)^{\eta-1}(cu+d)^{\lambda}$$
$$\times Exp[\frac{-A}{(u-a)(b-u)}]E^{\gamma,\sigma}_{\alpha,\beta,}[z(b-u)^{\omega}]du$$
$$=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-A)^m(\gamma)_{qn}}{m!\Gamma(\alpha n+\beta)n!}(z)^n(b-a)^{-2m+k+\eta+\omega n-1}$$
$$\times B(k-m,\eta+\omega n-m)(ac+d)^{\lambda}\ {}_2F_1\left[\begin{matrix} k-m, & -\lambda; & \frac{-(b-a)c}{ac+d} \\ k+\eta+\omega n-2m & & \end{matrix}\right]$$

(iii) Put $\mu=\nu, \rho=\sigma$ and $p=\delta=1, q=1$ in theorem (2.3), we get

$$\int_a^b (u-a)^{k-1}(b-u)^{\eta-1}(cu+d)^{\lambda}$$
$$\times Exp[\frac{-A}{(u-a)(b-u)}]E^{\gamma}_{\alpha,\beta,}[z(b-u)^{\omega}]du$$
$$=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-A)^m(\gamma)_{n}}{m!\Gamma(\alpha n+\beta)n!}(z)^n(b-a)^{-2m+k+\eta+\omega n-1}$$
$$\times B(k-m,\eta+\omega n-m)(ac+d)^{\lambda}\ {}_2F_1\left[\begin{matrix} k-m, & -\lambda; & \frac{-(b-a)c}{ac+d} \\ k+\eta+\omega n-2m & & \end{matrix}\right]$$

where $E^{\gamma}_{\alpha,\beta}$ is Prabhakar function.

(iv) Put $\mu=\nu, \rho=\sigma$ and $p=\delta=\gamma=q=1$ and $\alpha=0, \beta=1$ in theorem

(2.3), we get

$$\int_a^b (u-a)^{k-1}(b-u)^{\eta-1}(cu+d)^{\lambda}$$
$$\times Exp[\frac{-A}{(u-a)(b-u)}][1-z(b-u)^{\omega}]^{-1}du$$
$$= \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-A)^m}{m!}(z)^n(b-a)^{-2m+k+\eta+\omega n-1}$$
$$\times B(k-m,\eta+\omega n-m)(ac+d)^{\lambda}\ {}_2F_1\left[\begin{matrix} k-m, & -\lambda; & \frac{-(b-a)c}{ac+d} \\ k+\eta+\omega n-2m & & \end{matrix}\right]$$

where $[1-z(b-u)^{\omega}]^{-1}$ is a binomial function.

Case-3 (i) Put $\mu=\nu, \rho=\sigma$ and $p=1$ in theorem (2.6), we get

$$\int_0^1 (u)^{k-1}(1-u)^{\eta-\lambda-1}[1-tu^{\lambda_1}(1-u)^{\lambda_2}]^{-a}$$
$$\times Exp[\frac{-A}{u(1-u)}]E^{\gamma,q}_{\alpha,\beta,\delta}(zu^{\alpha})du$$
$$= \sum_{r=0}^{\infty}\sum_{n=0}^{\infty}\frac{(\gamma)_{qn}}{\Gamma(\alpha n+\beta)(\delta)_n}(z)^n(a)_r B(k+\alpha n+\lambda_1 r, \eta-k+\lambda_2 r; A)\frac{t^r}{r!}$$

(ii) Put $\mu=\nu, \rho=\sigma$ and $p=\delta=1$ in theorem (2.6), we get

$$\int_0^1 (u)^{k-1}(1-u)^{\eta-\lambda-1}[1-tu^{\lambda_1}(1-u)^{\lambda_2}]^{-a}$$
$$\times Exp[\frac{-A}{u(1-u)}]E^{\gamma,q}_{\alpha,\beta}(zu^{\alpha})du$$
$$= \sum_{r=0}^{\infty}\sum_{n=0}^{\infty}\frac{(\gamma)_{qn}}{\Gamma(\alpha n+\beta)n!}(z)^n(a)_r B(k+\alpha n+\lambda_1 r, \eta-k+\lambda_2 r; A)\frac{t^r}{r!}$$

(iii) Put $\mu=\nu, \rho=\sigma$ and $p=\delta=1, q=1$ in theorem (2.6), we get

$$\int_0^1 (u)^{k-1}(1-u)^{\eta-k-1}[1-tu^{\lambda_1}(1-u)^{\lambda_2}]^{-a}$$
$$\times Exp[\frac{-A}{u(1-u)}]E^{\gamma}_{\alpha,\beta}(zu^{\alpha})du$$
$$= \sum_{r=0}^{\infty}\sum_{n=0}^{\infty}\frac{(\gamma)_{n}}{\Gamma(\alpha n+\beta)n!}(z)^n(a)_r B(k+\alpha n+\lambda_1 r, \eta-k+\lambda_2 r; A)\frac{t^r}{r!}$$

where $E^{\gamma}_{\alpha,\beta}$ is Prabhakar function.

(iv) Put $\mu = \nu, \rho = \sigma$ and $\gamma = \alpha = \beta = p = \delta = q = 1$ in theorem (2.6), we get

$$\int_0^1 (u-a)^{k-1}(b-u)^{\eta-\lambda-1}[1-tu^{\lambda_1}(1-u)^{\lambda_2}]^{-a} Exp[\frac{-A}{u(1-u)} + zu]du$$
$$= \sum_{r=0}^{\infty}\sum_{n=0}^{\infty} \frac{z^n}{n!}(a)_r B(k+\lambda_1 r + n, \eta - k + \lambda_2 r; A)\frac{t^r}{r!}$$

where Exp(zu) is an exponential function.

(v) Put $\mu = \nu, \rho = \sigma$ and $\gamma = \alpha = \beta = p = \delta = q = 1$ in theorem (2.6), we get

$$\int_a^b (u-a)^{k-1}(b-u)^{\eta-\lambda-1}[1-tu^{\lambda_1}(1-u)^{\lambda_2}]^{-a}$$
$$\times Exp[\frac{-A}{u(1-u)}][1-z]^{-\gamma}du$$
$$= \sum_{r=0}^{\infty}\sum_{n=0}^{\infty} \frac{z^n(\gamma)_n}{n!}(a)_r B(k+\lambda_1 r, \eta - k + \lambda_2 r; A)\frac{t^r}{r!}$$

where $E_{0,1}^{\gamma}$ is a binomial function.

4. Conclusion

In this paper we presented Euler type integrals involving generalized Mittag-Leffler function defined by [6]. The obtained results provides an extension of known results [10]. Our paper concludes with the remark that the reported result are significant and can lead to yield the number of other Euler type integral involving different kinds of Mittag-Leffler functions.

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