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On Parallel p-equidistant Ruled Surfaces by Using Mofied Orthogonal Frame with Curvature in $\mathbb{E}^3$

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ABSTRACT: In this paper, it is investigated Ruled surfaces according to modified orthogonal frame with curvature in 3-dimensional Euclidean space. Firstly, we give apex angle, pitch and drall of closed ruled surface in $\mathbb{E}^3$. Then, it is characterized the relationship between these invariant of parallel p-equidistant ruled surfaces.

Key Words: Modified Orthogonal Frame, Ruled Surface, Drall.

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1. Introduction

The Frenet-Serret frame which has an importance place in Euclidean space is obtained continuously differentiable non-degenerate curves, [3,7,20]. For this reason, the studies related to the frenet frame are often used in differential geometry. One of most important of them is Karacan's study. Karacan described new frames, called modified orthogonal frame, with the help of the frenet frame [5,6]. These frames are created both curvature and torsion of a space curve in $\mathbb{E}^3$. Karacan also described these frames in Minkowski 3-space. Because these frames are newly defined, there is not study in the literature.

Modern surface modeling systems are contain the ruled surface, because this surface frequently used many areas suct that simulation of rigid body, design, production, motion analysis. For this reason, it has an important place in kinematical geometry and positional mechanisms in Euclidean 3-space. For instance, Brosius classifyed rank 2-vector bundels on a ruled surface and Onder and other authors viewed ruled surfaces minkowski space, [4, 20,25].

In this paper, we obtain new characteristic properties parallel p-equidistant ruled surfaces according to modified orthogonal frame frame in Euclidean 3-space. Firstly, we summarize properties modified orthogonal frame and the basic concepts on curves and ruled surfaces. Finally, we give new theorem to take the relationship between the differential and integral invariants of parallel p-equidistant ruled surfaces according to modified orthogonal frame in Euclidean 3-space.

2. Preliminaries

Given a spatial curve $\xi : s \to \xi(s)$, which is parameterized by arc-length parameter s. Derivative of the Frenet frame according to arc-length parameter is governed by the relations [20];

$$\begin{aligned} \dot{\mathbf{T}} &= \kappa\mathbf{N}, \\ \dot{\mathbf{N}} &= -\kappa\mathbf{T}+\tau\mathbf{B}, \\ \dot{\mathbf{B}} &= -\tau\mathbf{N}, \end{aligned}$$

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where κ is the curvature and τ is torsion of the curve α. Now we define an orthogonal frame $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ as follows:

$$\begin{aligned} \mathbf{e}_1 &= \mathbf{T}, \\ \mathbf{e}_2 &= \kappa \mathbf{N}, \\ \mathbf{e}_3 &= \kappa \mathbf{B}. \end{aligned} \tag{2.1}$$

Thus, $\mathbf{e}_2(s_0) = \mathbf{e}_3(s_0) = 0$ when $\kappa(s_0) = 0$ and squares of the length of $\mathbf{e}_2$ and $\mathbf{e}_3$ vary analitically in s. By the definition of $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ or eq (2.1), a simple calculation show that

$$\frac{d}{ds}\begin{pmatrix} \mathbf{e}_1(s) \\ \mathbf{e}_2(s) \\ \mathbf{e}_3(s) \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ -\kappa^2(s) & \frac{\dot{\kappa}}{\kappa} & \tau \\ 0 & -\tau & \frac{\dot{\kappa}}{\kappa} \end{pmatrix}\begin{pmatrix} \mathbf{e}_1(s) \\ \mathbf{e}_2(s) \\ \mathbf{e}_3(s) \end{pmatrix}, \tag{2.2}$$

where a dash denotes the differentation with respect to arc length s and

$$\tau(s) = \frac{(\dot{\alpha}(s),\ \ddot{\alpha}(s),\ \dddot{\alpha}(s))}{\kappa^2(s)}$$

is torsion of α. Moreover, $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ satisfies:

$$\begin{aligned} \langle \mathbf{e}_1, \mathbf{e}_1 \rangle &= 1, \langle \mathbf{e}_2, \mathbf{e}_2 \rangle = \langle \mathbf{e}_3, \mathbf{e}_3 \rangle = \kappa^2, \\ \langle \mathbf{e}_1, \mathbf{e}_2 \rangle &= \langle \mathbf{e}_1, \mathbf{e}_3 \rangle = \langle \mathbf{e}_2, \mathbf{e}_3 \rangle = 0, \end{aligned}$$

where $\langle , \rangle$ denotes the inner product of $\mathbb{E}^3$, [5,6].

On the other hand, a ruled surface is a surface generated by the motion of a straight line δ along α. Furthermore, if α is a closed curve, then this surfaces is called closed ruled surface [20]. Moreover, the drall P_X, the striction γ, the apex angle λ_X and the pitch I_X of the closed ruled surface are defined by

$$\begin{aligned} P_X &= \frac{\det(\alpha', X, X')}{\|X'\|^2}, \\ \gamma &= \alpha - \frac{\langle X', \alpha' \rangle}{\|X'\|^2} X, \\ \lambda_X &= \langle D, X \rangle, \\ I_X &= \langle V, X \rangle. \end{aligned} \tag{2.2}$$

3. On Parallel p-Equidistant Surfaces in $\mathbb{E}^3$

Definition 3.1. *Let α and $\tilde{\alpha}$ be two regular curves and let $\{\mathbf{e}_1,\ \mathbf{e}_2,\ \mathbf{e}_3\}$ and $\{\tilde{\mathbf{e}}_1,\ \tilde{\mathbf{e}}_2,\ \tilde{\mathbf{e}}_3\}$ be the modified orthogonal frames with curcature of α and $\tilde{\alpha}$ at the points $\alpha(s)$ and $\tilde{\alpha}(s)$, respectively, in $\mathbb{E}^3$. Then, the ruled surfaces are*

$$\mathcal{R}(s,t) = \alpha(s) + t\mathbf{e}_1(s) \text{ and } \tilde{\mathcal{R}}(s,t) = \tilde{\alpha}(s) + t\tilde{\mathbf{e}}_1(s).$$

For these surfaces, if the $\mathbf{e}$ and $\tilde{\mathbf{e}}$ vectors are parallel and the distance p between central planes in suitable points are constant, then this couple ruled surface are called parallel p-equidistant ruled surfaces according to modified orthogonalp frame.

Theorem 3.2. *Let γ, $\tilde{\gamma}$ and κ, τ, $\tilde{\kappa}$, $\tilde{\tau}$ be striction curves and curvatures of $\mathcal{R}$ and $\tilde{\mathcal{R}}$ ruled surfaces, respectively. Then, equations of striction curves according to Bishop frame are*

$$\gamma = \alpha(s), \tag{3.1}$$
$$\tilde{\gamma} = \tilde{\alpha}(s). \tag{3.2}$$

Proof. Considering the definition of striction curve in equation (2.2), we can write

$$\gamma(s) = \alpha(s) - \frac{\langle \mathbf{e}_1'(s), \alpha'(s) \rangle}{\|\mathbf{e}_1'(s)\|^2}\mathbf{e}_1(s).$$

From (2.1), we have

$$\gamma = \alpha.$$

Similarly, if we apply the same equations for $\tilde{\gamma}(s)$, $\tilde{\gamma}(s)$ can obtained as simple calculations by

$$\tilde{\gamma} = \tilde{\alpha}.$$

Theorem 3.3. *Let $\alpha(s)$, $\tilde{\alpha}(s)$ and $\gamma(s)$, $\tilde{\gamma}(s)$ be anchor curves and striction curves of $\mathcal{R}(s,t)$ and $\tilde{\mathcal{R}}(s,t)$ parallel p-equidistant ruled surfaces and $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ and $\{\tilde{\mathbf{e}}_1, \tilde{\mathbf{e}}_2, \tilde{\mathbf{e}}_3\}$ be the modified orthogonal frames of α and $\tilde{\alpha}$ at the points $\alpha(s)$ and $\tilde{\alpha}(s)$ in $\mathbb{E}^3$, respectively. Then, the anchor curve $\tilde{\alpha}$ of $\tilde{\mathcal{R}}$ is given by*

$$\tilde{\alpha} = \gamma + p\mathbf{e}_1 + z\mathbf{e}_2 + q\mathbf{e}_3. \tag{3.3}$$

Proof. Assume that $\gamma\tilde{\gamma}$ be

$$\tilde{\gamma} - \gamma = p\mathbf{e}_1 + z\mathbf{e}_2 + q\mathbf{e}_3 \tag{3.4}$$

vector is written related to frame $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$, where $|p| = |\langle \gamma\tilde{\gamma}, \mathbf{e}_1 \rangle|$ is distance between polar planes. From (3.4), we have

$$\tilde{\gamma} = \gamma + p\mathbf{e}_1 + z\mathbf{e}_2 + q\mathbf{e}_3.$$

If we write the (3.2) equation instead of $\tilde{\gamma}$, then we can write

$$\tilde{\alpha} = \gamma + p\mathbf{e}_1 + z\mathbf{e}_2 + q\mathbf{e}_3.$$

Theorem 3.4. *Let $\gamma(s)$, $\tilde{\gamma}(s)$ be striction curves of $\mathcal{R}(s,t)$ and $\tilde{\mathcal{R}}(s,t)$ parallel p-equidistant ruled surfaces, respectively. Then, the relation between striction curves are given by*

$$\tilde{\gamma} = \gamma + \left(\tau q - z' - z\frac{\kappa'}{\kappa}\right)\mathbf{e}_1 + z\mathbf{e}_2 + q\mathbf{e}_3. \tag{3.5}$$

Proof. If we take the derivative of equation (3.3), we obtain

$$\begin{aligned} \tilde{\alpha}' &= \left[\|\alpha'\| + p' - z\kappa^2\right]\mathbf{e}_1 \\ &\quad + \left[p + z' + z\frac{\kappa'}{\kappa} - q\tau\right]\mathbf{e}_2 \\ &\quad + \left[z\tau + q' + q\frac{\kappa'}{\kappa}\right]\mathbf{e}_3. \end{aligned} \tag{3.6}$$

Now, if we take the inner product of (3.6) and $\mathbf{e}_1'$, one can calculate by

$$\left\langle (\tilde{\alpha})', \mathbf{e}_1' \right\rangle = \kappa^2\left(p + z' + z\frac{\kappa'}{\kappa} - q\tau\right). \tag{3.7}$$

Since $\tilde{\gamma}(s) = \tilde{\alpha}(s) - \frac{\langle \tilde{\mathbf{e}}_1'(s), \tilde{\alpha}'(s)\rangle}{\|\tilde{\mathbf{e}}_1'(s)\|^2}\tilde{\mathbf{e}}_1(s)$ and $\mathbf{e}_1$, $\tilde{\mathbf{e}}_1$ vectors are parallel vectors, we have

$$\tilde{\gamma}(s) = \tilde{\alpha}(s) - \frac{\langle \mathbf{e}_1'(s), \tilde{\alpha}'(s)\rangle}{\|\mathbf{e}_1'(s)\|^2}\mathbf{e}_1(s)$$

From (3.3) and (3.7), we take

$$\tilde{\gamma} = \gamma + \left(\tau q - z' - z\frac{\kappa'}{\kappa}\right)\mathbf{e}_1 + z\mathbf{e}_2 + q\mathbf{e}_3.$$

Corollary 3.5. *The distance between central planes of $\mathcal{R}(s,t)$ and $\tilde{\mathcal{R}}(s,t)$ parallel p-equidistant ruled surfaces is*

$$p = \tau q - z' - z\frac{\kappa'}{\kappa}.$$

Theorem 3.6. *Let* $\{\mathbf{e}_1,\ \mathbf{e}_2,\ \mathbf{e}_3\}$ *and* $\{\tilde{\mathbf{e}}_1,\ \tilde{\mathbf{e}}_2,\ \tilde{\mathbf{e}}_3\}$ *be the modified orthogonal frames of the points* $\alpha(s)$ *and* $\tilde{\alpha}(s)$ *of anchor curves of* $\mathcal{R}(s,t)$ *and* $\tilde{\mathcal{R}}(s,t)$ *parallel p-equidistant ruled surfaces in* $\mathbb{E}^3$*. Then, the relation between modified orthogonal frames are given by*

$$\begin{aligned}\tilde{\mathbf{e}}_1 &= \mathbf{e}_1,\\ \tilde{\mathbf{e}}_2 &= \cos\phi\mathbf{e}_2 - \sin\phi\mathbf{e}_3,\\ \tilde{\mathbf{e}}_3 &= \sin\phi\mathbf{e}_2 + \cos\phi\mathbf{e}_3,\end{aligned}$$

where ϕ *is angle between the vector* $\mathbf{e}_2$ *and the vector* $\tilde{\mathbf{e}}_2$*.*

Remark 3.7. *Let* κ*,* τ *and* $\tilde{\kappa}$*,* $\tilde{\tau}$ *be curvatures of* $\mathcal{R}$ *and* $\tilde{\mathcal{R}}$ *parallel p-equidistant ruled surfaces in* $\mathbb{E}^3$*. Then, the relation between curvatures are given by*

$$\tilde{\kappa} = \kappa\cos\phi\frac{ds}{ds^*}, \tag{3.8}$$

$$\tilde{\tau} = \tau\frac{ds}{ds^*}. \tag{3.9}$$

On the other hand, we calculate apex angle, pitch and drall of closed ruled surface $\mathcal{R}(s,t)$ as

$$P_{\mathbf{e}_1} = 0,\ P_{\mathbf{e}_2} = \frac{\tau\|\alpha'\|}{\kappa^4 + (\kappa')^2 + \kappa^2\tau^2},\ \text{and}\ P_{\mathbf{e}_3} = \frac{\tau\|\alpha'\|}{(\kappa')^2 + \kappa^2\tau^2}, \tag{3.10}$$

$$\lambda_{\mathbf{e}_1} = \oint_{(\alpha)}\tau ds,\ \lambda_{\mathbf{e}_2} = 0,\ \text{and}\ \lambda_{\mathbf{e}_3} = \oint_{(\alpha)} ds, \tag{3.11}$$

$$I_{\mathbf{e}_1} = \oint_{(\alpha)} ds\ \text{and}\ I_{\mathbf{e}_2} = I_{\mathbf{e}_3} = 0. \tag{3.12}$$

Now, we can write the relationship between the differential and integral invariants of parallel $p-$equidistant ruled surfaces.

Theorem 3.8. *Let* $\lambda_{\mathbf{e}_1},\ \lambda_{\mathbf{e}_2},\ \lambda_{\mathbf{e}_3}$ *and* $\lambda_{\tilde{\mathbf{e}}_1},\ \lambda_{\tilde{\mathbf{e}}_2},\ \lambda_{\tilde{\mathbf{e}}_3}$ *be apex angles of* $\mathcal{R}$ *and* $\tilde{\mathcal{R}}$ *parallel p-equidistant ruled surfaces in* $\mathbb{E}^3$*. Then, the relation between apex angles are given by*

$$\begin{aligned}\lambda_{\tilde{\mathbf{e}}_1} &= \lambda_{\mathbf{e}_1} + \oint_{(p\mathbf{e}_1+z\mathbf{e}_2+q\mathbf{e}_3)}\tilde{\tau}d\tilde{s},\\ \lambda_{\tilde{\mathbf{e}}_2} &= \lambda_{\mathbf{e}_2} = 0,\\ \lambda_{\tilde{\mathbf{e}}_3} &= \cos^4\phi\lambda_{\mathbf{e}_3} + \oint_{(p\mathbf{e}_1+z\mathbf{e}_2+q\mathbf{e}_3)}\tilde{\kappa}^4 d\tilde{s}.\end{aligned} \tag{3.13}$$

Proof. From (3.11), we can write following equation

$$\lambda_{\tilde{\mathbf{e}}_1} = \oint_{(\tilde{\alpha})}\tilde{\tau}d\tilde{s}. \tag{3.14}$$

If we consider the eq. (3.3), the we obtain

$$\begin{aligned}\lambda_{\tilde{\mathbf{e}}_1} &= \oint_{(\alpha+p\mathbf{e}_1+z\mathbf{e}_2+q\mathbf{e}_3)}\tilde{\tau}d\tilde{s},\\ &= \oint_{(\alpha)}\tau ds + \oint_{(p\mathbf{e}_1+z\mathbf{e}_2+q\mathbf{e}_3)}\tilde{\tau}d\tilde{s},\\ &= \lambda_{\mathbf{e}_1} + \oint_{(p\mathbf{e}_1+z\mathbf{e}_2+q\mathbf{e}_3)}\tilde{\tau}d\tilde{s}.\end{aligned}$$

Similarly, we have

$$\begin{aligned}
\lambda_{\tilde{\mathbf{e}}_3} &= \oint_{(\tilde{\alpha})} \tilde{\kappa}^4 d\tilde{s}, \\
&= \oint_{(\alpha+p\mathbf{e}_1+z\mathbf{e}_2+q\mathbf{e}_3)} \tilde{\kappa}^4 d\tilde{s}, \\
&= \oint_{(\alpha)} \kappa^4 \cos^4 \phi ds + \oint_{(p\mathbf{e}_1+z\mathbf{e}_2+q\mathbf{e}_3)} \tilde{\kappa}^4 d\tilde{s} \\
&= \cos^4 \phi \lambda_{\mathbf{e}_3} + \oint_{(p\mathbf{e}_1+z\mathbf{e}_2+q\mathbf{e}_3)} \tilde{\kappa}^4 d\tilde{s}.
\end{aligned}$$

Theorem 3.9. *Let $I_{\mathbf{e}_1}$, $I_{\tilde{\mathbf{e}}_1}$ be pitchs of $\mathcal{R}$ and $\tilde{\mathcal{R}}$ parallel p-equidistant closed ruled surfaces in $\mathbb{E}^3$. If we specially take helix curve instead of anchor curve of parallel p-equidistant closed ruled surface, then the relation between pitchs are given by*

$$I_{\tilde{\mathbf{e}}_1} = \frac{\kappa}{\tilde{\kappa}} \cos \phi I_{\mathbf{e}_1} + \oint_{(p\mathbf{e}_1+z\mathbf{e}_2+q\mathbf{e}_3)} ds^*.$$

Proof. From (3.12), then we can write

$$I_{\tilde{\mathbf{e}}_1} = \oint_{(\alpha^*)} ds^*.$$

Thus, we get

$$\begin{aligned}
I_{\tilde{\mathbf{e}}_1} &= \oint_{(\alpha+p\mathbf{e}_1+z\mathbf{e}_2+q\mathbf{e}_3)} ds^*, \\
&= \oint_{(\alpha)} ds^* + \oint_{(p\mathbf{e}_1+z\mathbf{e}_2+q\mathbf{e}_3)} ds^*, \\
&= \frac{\kappa}{\tilde{\kappa}} \cos \phi \oint_{(\alpha)} ds + \oint_{(p\mathbf{e}_1+z\mathbf{e}_2+q\mathbf{e}_3)} ds^*, \\
&= \frac{\kappa}{\tilde{\kappa}} \cos \phi I_{\mathbf{e}_1} + \oint_{(p\mathbf{e}_1+z\mathbf{e}_2+q\mathbf{e}_3)} ds^*.
\end{aligned}$$

Let us consider two unit speed curve in $\mathbb{E}^3$ by

$$\alpha(s) = (\frac{4}{5} \cos s, 1 - \sin s, -\frac{3}{5} \cos s) \text{ and } \alpha(s) = (\cos s, \sin s, 0).$$

Then, we can easily draw the following graphics;

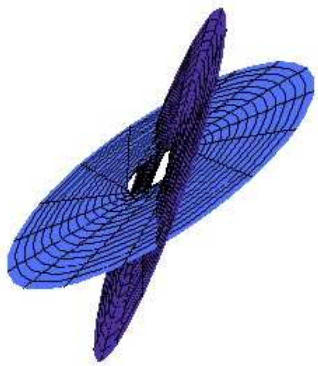

Figure 1: The ruled surface of curves α and $\tilde{\alpha}$.

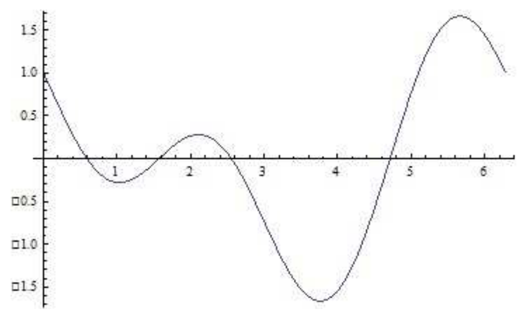

Figure 2: Gaph of function p.

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