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On Quasi Focal Curves with Quasi Frame in Space

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ABSTRACT: In this study, we firstly characterize focal curves by considering quasi frame in the ordinary space. Then, we obtain the relation of each quasi curvatures of curve in terms of focal curvatures. Finally, we give some new conditions with constant quasi curvatures in the ordinary space.

Key Words: Quasi frame, focal curve, focal curvatures.

Contents

1. Backround on Quasi Frame

By way of design and style, this is model to kind of a moving frame with regards to a particle. In the quick stages of regular differential geometry, the Frenet-Serret frame was applied to create a curve in location. After that, Frenet-Serret frame is established by way of subsequent equations for a presented framework [1-18],

$$\begin{bmatrix} \nabla_{\mathbf{t}}\mathbf{t} \\ \nabla_{\mathbf{t}}\mathbf{n} \\ \nabla_{\mathbf{t}}\mathbf{b} \end{bmatrix} = \begin{bmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & \tau \\ 0 & -\tau & 0 \end{bmatrix} \begin{bmatrix} \mathbf{t} \\ \mathbf{n} \\ \mathbf{b} \end{bmatrix},$$

where $\kappa = \|\mathbf{t}\|$ and τ are the curvature and torsion of γ, respectively.

The quasi frame of a regular curve γ is given by

$$\mathbf{t_q}= \mathbf{t}, \mathbf{n_q} = \frac{\mathbf{t} \wedge \mathbf{k}}{\|\mathbf{t} \wedge \mathbf{k}\|}, \mathbf{b_q} = \mathbf{t_q} \wedge \mathbf{n_q},$$

where $\mathbf{k}$ is the projection vector [4].

For simplicity, we have chosen the projection vector $\mathbf{k} = (0, 0, 1)$ in this paper. However, the q-frame is singular in all cases where $\mathbf{t}$ and $\mathbf{k}$ are parallel. Thus, in those cases where $\mathbf{t}$ and $\mathbf{k}$ are parallel the projection vector $\mathbf{k}$ can be chosen as $\mathbf{k} = (0, 1, 0)$ or $\mathbf{k} = (1, 0, 0)$.

If the angle between the quasi normal vecctor $\mathbf{n_q}$ and the normal vector $\mathbf{n}$ is choosen as ψ, then following relation is obtained between the quasi and FS frame.

$$\begin{aligned} \mathbf{t_q} &= \mathbf{t}, \\ \mathbf{n_q} &= \cos\psi\mathbf{n} + \sin\psi\mathbf{b}, \\ \mathbf{b_q} &= -\sin\psi\mathbf{n} + \cos\psi\mathbf{b}, \end{aligned}$$

such that short computation by using Eqs. $(1-3)$ yields that the variation of parallel adapted quasi frame is given by

$$\begin{aligned} \nabla_{\mathbf{t_q}}\mathbf{t_q} &= \varkappa_1\mathbf{n_q} + \varkappa_2\mathbf{b_q}, \\ \nabla_{\mathbf{t_q}}\mathbf{n_q} &= -\varkappa_1\mathbf{t_q} + \varkappa_3\mathbf{b_q}, \\ \nabla_{\mathbf{t_q}}\mathbf{b_q} &= -\varkappa_2\mathbf{t_q} - \varkappa_3\mathbf{n_q}, \end{aligned}$$

where

$$\varkappa_1 = \kappa\cos\psi, \quad \varkappa_2 = -\kappa\sin\psi, \quad \varkappa_3 = \psi' + \tau,$$

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and

$$\mathbf{t_q} \times \mathbf{n_q} = \mathbf{b_q},\ \mathbf{n_q} \times \mathbf{b_q} = \mathbf{t_q},\ \mathbf{b_q} \times \mathbf{t_q} = \mathbf{n_q}.$$

In this paper, we study quasi focal curves in the Euclidean 3-space. We characterize quasi focal curves in terms of their focal curvatures.

2. Quasi Focal Curves with Quasi Frame In $\mathbb{E}^3$

The focal curve of α is given by

$$\beta = \alpha + \phi_1 \mathbf{n_q} + \phi_2 \mathbf{b_q}, \tag{2.1}$$

where the coefficients ϕ_1, ϕ_2 are smooth functions of the parameter of the curve γ, called the first and second focal curvatures of γ, respectively.

Theorem 2.1. *Let $\gamma : I \longrightarrow \mathbb{E}^3$ be a unit speed curve and β its focal curve on $\mathbb{E}^3$. Then,*

$$\beta = \alpha + e^{-\int \frac{\varkappa_1\varkappa_3}{\varkappa_2}ds}\left(\int e^{\int \frac{\varkappa_1\varkappa_3}{\varkappa_2}ds}\frac{\varkappa_3}{\varkappa_2}ds + C\right)\mathbf{n_q} + \left(\frac{1}{\varkappa_2} - \frac{\varkappa_1}{\varkappa_2}e^{-\int \frac{\varkappa_1\varkappa_3}{\varkappa_2}ds}\left(\int e^{\int \frac{\varkappa_1\varkappa_3}{\varkappa_2}ds}\frac{\varkappa_3}{\varkappa_2}ds + C\right)\right)\mathbf{b_q}, \tag{2.2}$$

where C is a constant of integration.

Proof. Assume that α is a unit speed curve and β its focal curve in $\mathbb{E}^3$.
So, by differentiating of the formula (2.1), we get

$$\beta' = (1 - \varkappa_1\phi_1 - \varkappa_2\phi_2)\mathbf{t_q} + (\phi_1' - \varkappa_3\phi_2)\mathbf{n_q} + (\phi_2' + \varkappa_3\phi_1)\mathbf{b_q}$$

From above equation, the first 2 components vanish, we get

$$\begin{aligned} 1 - \varkappa_1\phi_1 - \varkappa_2\phi_2 &= 0, \\ \phi_1' - \varkappa_3\phi_2 &= 0. \end{aligned}$$

Using the above equations, we obtain

$$\phi_1' - \frac{\varkappa_3}{\varkappa_2}(1 - \varkappa_1\phi_1) = 0,$$

$$\phi_1' + \frac{\varkappa_1\varkappa_3}{\varkappa_2}\phi_1 = \frac{\varkappa_3}{\varkappa_2}.$$

By integrating this equation, we find

$$\phi_1 = e^{-\int \frac{\varkappa_1\varkappa_3}{\varkappa_2}ds}\left(\int e^{\int \frac{\varkappa_1\varkappa_3}{\varkappa_2}ds}\frac{\varkappa_3}{\varkappa_2}ds + C\right),$$

$$\phi_2 = \frac{1}{\varkappa_2} - \frac{\varkappa_1}{\varkappa_2}e^{-\int \frac{\varkappa_1\varkappa_3}{\varkappa_2}ds}\left(\int e^{\int \frac{\varkappa_1\varkappa_3}{\varkappa_2}ds}\frac{\varkappa_3}{\varkappa_2}ds + C\right).$$

By means of obtained equations, we express (2.2). This completes the proof of the theorem.

As an immediate consequence of the above theorem, we have:

Corollary 2.2. *Let $\alpha : I \longrightarrow \mathbb{E}^3$ be a unit speed curve and β its focal curve on $\mathbb{E}^3$. Then, the focal curvatures of β are*

$$\phi_1 = e^{-\int \frac{\varkappa_1\varkappa_3}{\varkappa_2}ds}\left(\int e^{\int \frac{\varkappa_1\varkappa_3}{\varkappa_2}ds}\frac{\varkappa_3}{\varkappa_2}ds + C\right),$$

$$\phi_2 = \frac{1}{\varkappa_2} - \frac{\varkappa_1}{\varkappa_2} e^{-\int \frac{\varkappa_1 \varkappa_3}{\varkappa_2} ds} \left(\int e^{\int \frac{\varkappa_1 \varkappa_3}{\varkappa_2} ds} \frac{\varkappa_3}{\varkappa_2} ds + C \right).$$

Proof. From above theorem, we have above system, which completes the proof.

In the light of Theorem 2.1, we express the following corollary without proof:

Corollary 2.3. *Let* $\gamma : I \longrightarrow \mathbb{E}^3$ *be a unit speed curve and* β *its focal curve on* $\mathbb{E}^3$*. If* $\varkappa_1, \varkappa_2, \varkappa_3$ *are constant then, the focal curvatures of* β *are*

$$\phi_1 = \left(\frac{1}{\varkappa_1} + e^{-\frac{\varkappa_1 \varkappa_3}{\varkappa_2} s} C \right),$$

$$\phi_2 = \frac{1}{\varkappa_2} - \frac{\varkappa_1}{\varkappa_2} \left(\frac{1}{\varkappa_1} + C e^{-\frac{\varkappa_1 \varkappa_3}{\varkappa_2} s} \right).$$

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