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Fixed Point Results for Orthogonal G-F-Contraction mappings on O-complete G-metric spaces

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ABSTRACT: In this manuscript, we introduce the notion of an orthogonal $G-F$-contraction mapping and establish fixed points results for such contraction mappings in orthogonally G - metric spaces.

Key Words: G-metric space, self-map, Banach contraction, fixed point, orthogonally metric spaces.

Contents

1. Introduction

In 1922, Stephen Banach [1] proved the Banach's fixed point theorem. it has become a very popular tool in solving the existence problems in many branches of mathematical analysis. So, many authors have approved, extended and generated Banach's fixed point theorem in many direction. Especially, in 2004, Ran and Reurings [21] extended the Banach's fixed point theorem to the setting of partially ordered sets.

Recently, Gordji et al., [7] introduced the concept of an orthogonal set (briefly, O-set) and presented some fixed point theorem in orthogonal metric space. In 2012, Wardoski [28] introduced a new kind of contractions, called F-contractions, and proved some fixed point results using the family of F-contractions.

Very recently, Kanokwan Sawangsup et al.,[17] introduced a new concept of an orthogonal F - contractions mapping and establish some fixed point results for such contractions mappings on orthogonally metric spaces.

2. Preliminaries

Definition 2.1 [19,8,9,10,11] Let X be a non-empty set and $G : X \times X \times X \to \mathbb{R}^+$ be a function satisfying the following

1. $G(x, y, z) = 0$ if $x = y = z$,
2. $G(x, x, y) > 0$ for all $x, y \in X$, with $x \neq y$,
3. $G(x, x, y) \leq G(x, y, z)$, for all $x, y, z \in X$ with $y \neq z$,
4. $G(x, y, z) = G(y, z, x) = G(z, x, y) = \cdots$ (symmetry in all three variables),
5. $G(x, y, z) \leq G(x, a, a) + G(a, y, z)$, for all $x, y, z, a \in X$ (rectangular inequality).

Then the function G is called a generalized metric or more specifically a G-metric on X and the pair (X, G) is a G-metric space.

Example 2.1 *[19,12] If X is a non empty subset of $\mathbb{R}$, then the function $G : X \times X \times X \to [0, \infty)$, given by $G(x, y, z) = |x - y| + |y - z| + |z - x|$ for all $x, y, z \in X$, is a G-metric on X.*

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Example 2.2 *[19,13,14] Let $X = \{0,1,2\}$ and let $G : X \times X \times X \to [0, \infty)$ be the function given by the following table.*

(x,y,z)	G(x,y,z)
(0,0,0), (1,1,1), (2,2,2)	0
(0,0,1), (0,1,0), (1,0,0), (0,1,1), (1,0,1), (1,1,0)	1
(1,2,2), (2,1,2), (2,2,1)	2
(0,0,2), (0,2,0), (2,0,0), (0,2,2), (2,0,2), (2,2,0)	3
(1,1,2), (1,2,1), (2,1,1), (0,1,2), (0,2,1), (1,0,2)	4
(1,2,0), (2,0,1), (2,1,0)	4

Then G is a G-metric on X, but it is not symmetric because $G(1,1,2) = 4 \neq 2 = G(2,2,1)$.

Definition 2.2 [19,15] Let (X, G) be a G-metric space, let $\{x_n\}$ be sequence of points of X, a point $x \in X$ is said to be the limit of the sequence $\{x_n\}$ if $\lim_{n,m\to\infty} G(x, x_n, x_m) = 0$ and we say that the sequence $\{x_n\}$ is G-convergent to x.

Lemma 2.1 *[19] Let (X, G) be a G- metric space, then for a sequence $\{x_n\} \subseteq X$ and a point $x_0 \in X$ the following are equivalent:*

(1) $\{x_n\}$ is G-convergent to x.

(2) $G(x_n, x_n, x) \to 0$, as $n \to \infty$.

(3) $G(x_n, x, x) \to 0$, as $n \to \infty$.

(4) $G(x_m, x_n, x) \to 0$, as $m, n \to \infty$.

Definition 2.3 [19] Let (X, G) be a G-metric space. The sequence $\{x_n\} \subseteq X$ is said to be G-Cauchy if for every $\epsilon > 0$, there exists a positive integer $N \in \mathbb{N}$ such that $G(x_n, x_m, x_l) < \epsilon$ for all $n, m, l \geq \mathbb{N}$.

Definition 2.4 [19] A $G-$metric space (X, G) is said to be G-complete (or complete G-metric space) if every G-Cauchy sequence in (X, G) is G-convergent in (X, G).

Throughout this paper, we denote by $X, \mathbb{R}^+, \mathbb{N}$ and $\mathbb{N}_0$ the non-empty set, the set of positive real numbers, the set of positive integers and the set of non-negative integers respectively.

First, we recall the concept of a control function which is introduced by Wardoski [28]. Let $\mathfrak{F}$ denote the family of all functions $F : \mathbb{R}^+ \to \mathbb{R}$ satisfying the following properties:

(F1) F is strictly increasing;

(F2) for each sequence $\{\alpha_n\}$ of positive numbers we have $\lim_{n\to\infty} \alpha_n = 0$ if and only if $\lim_{n\to\infty} F(\alpha_n) = -\infty$;

(F3) there exists $k \in (0, 1)$ such that $\lim_{\alpha\to 0^+} \alpha^k . F(\alpha) = 0$.

We give some examples of functions belonging to F as follows:

Example [21], Let functions F_1, F_2, F_3, F_4, $\mathbb{R}^+ \to \mathbb{R}$,be defined by:

(1) $F_1(\alpha) = ln\alpha$ for all $\alpha > 0$;

(2) $F_2(\alpha) = \alpha + ln\alpha$ for all $\alpha > 0$;

(3) $F_3(\alpha) = \frac{-1}{\sqrt{\alpha}}$ for all $\alpha > 0$;

(4) $F_4(\alpha) = ln(\alpha^2 + \alpha)$ for all $\alpha > 0$.

Then $F_1, F_2, F_3, F_4 \in \mathfrak{F}$

In [28], Wardoski introduced the definition of an F-contraction mapping as follows:

Defination [28]: A mapping T from a metric space (X, d) into itself is called an F- contraction if there exist $\tau > 0$ and $F \in \mathfrak{F}$ such that $\forall x, y \in X$,

$$d(Tx, Ty) > 0 \Rightarrow \tau + F(d(Tx, Ty)) \leq F(d(x, y))$$

He also proved the following existence and uniqueness of a fixed point for F-contraction mappings.

Theorem 2.1 *[28] Let (X, d) be a complete metric space and $T : X \to X$ be an F-contraction mapping. Then, T has a unique fixed point in X. Moreover, for each $x_o \in X$, the Picard sequence $\{T^n_{x_0}\}$ converges to the fixed point of T*

On the other hand, Gordji et al.,[7] introduced the concept of an orthogonal set (or o-set), some examples and some properties of the orthogonal sets as follows:

Definition 2.5 [7,26,27] Let $X \neq \phi$ and $\perp \subseteq X \times X$ be a binary relation.
If $\perp$ satisfies the following condition:

$$\exists x_0[(\forall y \in X, y \perp x_0) or (\forall y \in X, x_0 \perp y)]$$

then it is called an orthogonal set (briefly, o- set) and x_0 is called an orthogonal element. We denote this O-set by $(X, \perp)$.

Example 2.3 *[7] Let X be the set of all peoples in the world. Define the binary relation $\perp$ on X by $x \perp y$ if x can give blood to y. According to the Table 1, if x_0 is a person such that his(her) blood type is O^-, then we have $x_0 \perp y$ for all $y \in X$. This means that $(X, \perp)$ is an O-set.In this O-set, x_0(in definition) is not unique.*

Note that, in the above example, x_0 may be a person with blood type AB^+. In the case we have $y \perp x_0$ for all $y \in X$.

Type	You can give blood to	You can receive blood from
A^+	A^+ AB^+	A^+ A^- O^+ O^-
O^+	O^+ A^+ B^+ AB^+	O^+ O^-
B^+	B^+ AB^+	B^+ B^- O^+ O^-
AB^+	AB^+	Everyone
A^-	A^+ A^- AB^+ AB^-	A^- O^-
O^-	Everyone	O^-
B^-	B^+ B^- AB^+ AB^-	B^- O^-
AB^-	AB^+ AB^-	AB^- B^- O^- A^-

Table 1: Blood donors and recipients.

Example 2.4 *[7] Let $X = Z$. Define the binary relation $\perp$ on X by $m \perp n$ if there exists $k \in Z$ such that $m = kn$. It is easy to see that $0 \perp n$ for all $n \in Z$. Hence $(X, \perp)$ is an o-set.*

Example 2.5 *[7] Let (X, d) be a metric space and $T : X \to X$ be a Picard operator, that is, T has a unique fixed point $x^* \in X$ and $\lim_{n\to\infty} T^n(y) = x^*$ for all $y \in X$. We define the binary relation $\perp$ on X by $(X, \perp)$ if $\lim_{n\to\infty}(x, T^n(y)) = 0$. Then, $(X, \perp)$ is an O-set.*

Example 2.6 *[7] Let X be an inner product space with the inner product $\langle .,. \rangle$. Define the binary relation $\perp$ on X by $x \perp y$ if $\langle x, y \rangle = 0$. It is easy to see that $O \perp x$ for all $x \in X$. Hence, $(X, \perp)$ is an O-set.*

Definition 2.6 [7] Let $(X, \perp)$ be an O-set. A sequence $\{x^n\}$ is called an orthogonal sequence (briefly, O-sequence) if $(\forall n \in \mathbb{N}, x_n \perp x_{n+1})$ or $(\forall n \in \mathbb{N}, x_{n+1} \perp x_n)$.

Definition 2.7 [7] The triplet $(X, \perp, d)$ is called an orthogonal metric space if $(X, \perp)$ is an O-set and (X, d) is a metric space.

Definition 2.8 [7] Let $(X, \perp, d)$ be an orthogonal metric space. Then, a mapping $f : X \to X$ is said to be orthogonally continuous(or $\perp$-continuous) in $x \in X$ if, for each O-sequence $\{x_n\}$ in X with $x_n \to x$ as $n \to \infty$, we have $f(x_n) \to f(x)$ as $n \to \infty$. Also, f is said to be $\perp$-continuous on X if f is $\perp$-continuous in each $x \in X$.

Remark 2.1 *[7] Every continuous mapping is $\perp$-continuous and the converse is not true.*

Definition 2.9 [7] Let $(X, \perp, d)$ be an orthogonal metric space. Then, X is said to be orthogonally complete(briefly, O-complete) if every Cauchy O-sequence is convergent.

Remark 2.2 *[7] Every complete metric space is O-complete and the converse is not true.*

Definition 2.10 [7] Let $(X, \perp)$ be an O-set. A mapping $f : X \to X$ is said to be $\perp$-preserving if $f(x) \perp f(y)$ whenever $x \perp y$. Also, $f : X \to X$ is said to be weakly $\perp$-preserving if $f(x) \perp f(y)$ or $f(y) \perp f(x)$ whenever $x \perp y$.

Kanokwan Sawangsup et al.,[17] have introduced the notion of an orthogonal F-contraction mapping and they proved some fixed point thermos for a new F-contractions mapping in an orthogonal metric space.

Definition 2.11 [17] Let $(X, \perp, d)$ be an orthogonal metric space. A mapping $T : X \to X$ is called orthogonal F-contraction(briefly, $F_\perp$-contraction) if there are $F \in \mathfrak{F}$ and $\tau < 0$ such that the following condition holds:

$$\forall x, y \in X \quad with \quad x \perp y [d(T_x, T_y) > 0 \Rightarrow \tau + F(d(T_x, T_y)) \leq F(d(x, y))]$$

Theorem 2.2 *[17]*

Let $(X, \perp, d)$ be an O-complete orthogonal metric space with an orthogonal element x_o and T be a self mapping on X satisfying the following conditions:

(1) T is $\perp$-preserving;

(2) T is an $F_\perp$-contraction mapping;

(3) T is $\perp$-continuous.

Then, T has a unique fixed point in X. Also, the Picard sequence $\{T^n_{x_0}\}$ converges to the fixed point of T.

3. Main Results

In this section, we introduce the new concepts of an orthogonal G- metric space, a $\perp$-G-continuous mapping, and orthogonal complete G-metric space and the notion of an orthogonal $G-F$-contraction mapping and prove some fixed point theorems for a new $G-F$-contraction mapping in an orthogonal G-metric space.

Definition 3.1 The triplet $(X, \perp, G)$ is called an orthogonal G-metric space if $(X, \perp)$ is an O-set and (X, G) is a G-metric space.

Definition 3.2 Let $(X, \perp, G)$ be an orthogonal G-metric space. Then, a mapping $f : X \to X$ is said to be orthogonally G-continuous (or $\perp$-G-continuous) in $x \in X$ if , for each O-sequence $\{x_n\}$ in X with $x_n \to x$ as $n \to \infty$, we have $f(x_n) \to f(x)$ as $n \to \infty$. Also, f is said to be $\perp$-G-continuous on X if f is $\perp$-G-continuous in each $x \in X$.

Remark 3.1 *Every G-continuous mapping is $\perp$-G-continuous and the converse is not true. Proof is similar to previous remark.*

Definition 3.3 Let $(X, \perp, G)$ be an orthogonal G-metric space. Then, X is said to be orthogonally complete (briefly, O-complete) if every G-Cauchy O-sequence is G-Convergent.

Remark 3.2 *Every complete G-metric space is O-complete and the converse is not true.*

Definition 3.4 Let $(X, \perp, G)$ be an orthogonal G-metric space. A mapping $T : X \to X$ is called an orthogonal $G - F$-contraction if there are $F \in \mathfrak{F}$ and $\tau > 0$ such that the following condition holds:

$\forall x, y, z \in X$ with $x \perp y$, $y \perp z$ and $z \perp x$,

$$[G(Tx, Ty, Tz) > 0 \Rightarrow \tau + F(G(Tx, Ty, Tz)) \leq F(G(x, y, z))] \tag{3.1}$$

Now, we give the first fixed point theorem for an orthogonal $G - F$-contraction mapping in an O-complete orthogonal G-metric space $(X, \perp, G)$.

Theorem 3.1 *Let $(X, \perp, G)$ be an orthogonally complete G-metric space with an orthogonal element x_o and T be a self-mapping on X satisfying the following conditions:*

(i) T is $\perp$-preserving;

(ii) T is an orthogonal $G - F$-contraction mapping;

(iii) T is an orthogonal G-continuous.

Then T has a unique fixed point in X. Also, the Picard sequence $\{T^n x_0\}$ converges to the fixed point of T.

Proof:

From the definition of the orthogonally, it follows that $x_0 \perp f(x_0)$ or $f(x_0) \perp x_0$.

Let $x_1 = Tx_0$, $x_2 = Tx_1 = T^2_{x_0}$,..., $x_n = Tx_{n-1} = T^n x_0$.

For all $n \in \mathbb{N} \cup \{0\}$. If $x_{n^*} = x_{n^*+1}$ for some $n^* \in \mathbb{N} \cup \{0\}$, then x_{n^*} is a fixed point of T and so the proof is completed. Now, we may assume that $x_n \neq x_{n+1}$ for all $n \in \mathbb{N} \cup \{0\}$. Thus, we have $G(Tx_n, Tx_{n+1}, Tx_{n+1}) > 0$ for all $n \in \mathbb{N} \cup \{0\}$. Since T is $\perp$-preserving, we have

$$x_n \perp x_{n+1} \quad or \quad x_{n+1} \perp x_n \tag{3.2}$$

For all $n \in \mathbb{N} \cup \{0\}$. This implies that $\{x_n\}$ is an O-sequence. Since T is orthogonally $G - F$-contraction mapping, we have

$$F(G(x_n, x_{n+1}, x_{n+1})) = F(G(Tx_{n-1}, Tx_n, Tx_n)) \leq F(G(x_{n-1}, x_n, x_n)) - \tau \tag{3.3}$$

For all $n \in \mathbb{N}$. Taking $\alpha_n = G(x_n, x_{n+1}, x_{n+1})$ for all $n \in \mathbb{N}$ and using (3.3), we have

$$F(\alpha_n) \leq F(\alpha_{n-1}) - \tau \leq F(\alpha_{n-2}) - 2\tau \leq ... \leq F(\alpha_0) - n\tau \tag{3.4}$$

For all $n \in \mathbb{N}$. Letting $n \to \infty$ in the inequality (3.4),we have

$$\lim_{n\to\infty} F(\alpha_n) = -\infty \tag{3.5}$$

By the property (F_2), we have

$$\lim_{n\to\infty} \alpha_n = 0 \tag{3.6}$$

By the property (F_3), there exists $k \in (0,1)$ such that

$$\lim_{n\to\infty} \alpha_n^k F(\alpha_n) = 0 \tag{3.7}$$

By (3.4), we have

$$\alpha_n^k F(\alpha_n) - \alpha_n^k F(\alpha_0) \leq -\alpha_n^k n\tau \leq 0 \tag{3.8}$$

For all $n \in \mathbb{N}$. Letting $n \to \infty$ in (3.8) and using (3.6) and (3.7), we obtain

$$\lim_{n\to\infty} n\alpha_n^k = 0 \tag{3.9}$$

From (3.9), there exists $n_1 \in \mathbb{N}$ such that $n\alpha_n^k \leq 1$ for all $n \geq n_1$ and so

$$\alpha_n \leq \frac{1}{\sqrt[k]{n}} \tag{3.10}$$

For all $n \geq n_1$. Now we claim that (x_n) is a G-Cauchy O-sequence. Using(3.10) and the rectangular, it follows that, for all $m > n \geq n_1$.

$$\begin{aligned} G(x_n, x_m, x_m) &\leq G(x_n, x_{n+1}, x_{n+1}) + G(x_{n+1}, x_{n+2}, x_{n+2} + ... + G(x_{m-1}, x_m, x_m) \\ &= \alpha_n + \alpha_{n+1} + ... + \alpha_{m-1} \\ &= \sum_{i=n}^{m-1} \alpha_i = \sum_{i=n}^{m-1} \frac{1}{\sqrt[k]{i}} \end{aligned}$$

Since $\sum_{i=n}^{\infty} \frac{1}{\sqrt[k]{i}} < \infty$, it follows that $\{x_n\}$ is a G-Cauchy O-sequence in X. Since X is O-complete, there exists $x^* \in X$ such that $x_n \to x^*$ as $n \to \infty$. Since T is $\perp - G$-continuous, we have $Tx^* = T(\lim_{n\to\infty} Tx_n) = \lim_{n\to\infty} x_{n+1} = x^*$ and so x^* is a fixed point of T.

Next,we show that x^* is unique fixed point of T.

Assume that y^* is an another fixed point of T. If $x_n \to y^*$ as $n \to \infty$, we have $x^* = y^*$. If x_n does not converge to y^* as $n \to \infty$ there is a subsequence $\{x_{n_k}\}$ such that $Tx_{n_k} \neq y^*$ for all $k \in \mathbb{N}$.

By the choice of x_0 on the first of the proof, we have

$$(x_0 \perp y^*) \quad or \quad (y^* \perp x_0)$$

Since T is $\perp$-preserving and $T^n y^* = y^*$ for all $n \in \mathbb{N}$, we have

$$(T^n x_0 \perp y^*) \quad or \quad (y^* \perp T^n x_0)$$

For all $n \in \mathbb{N}$. Since orthogonally $G - F$-contraction mapping,we have

$$F(G(T^{n_{k_{x_0}}}, y^*, y^*)) = F(G(T^{n_{k_{x_0}}}, T^{n_{k_{y^*}}}, T^{n_{k_{y^*}}})) \leq F(G(x_0, y^*, y^*)) - n_x\tau$$

for all $k \in \mathbb{N}$. This implies that $F(G(T^{n_{k_{x_0}}}, y^*, y^*)) \to -\infty$ as $k \to \infty$ and so it follows from the conditions (F_2) that $G(T^{n_{k_{x_0}}}, y^*, y^*) \to 0$ as $k \to \infty$. This yields that $x_n \to y^*$ as $n \to \infty$, which is a contradiction, hence, T has a unique fixed point. □

Example 3.1 *Let $X = [0, \infty)$ and $G : X \times X \times X \to [0, \infty)$ be a mapping defined $G(x, y, z) = |x - y| + |y - z| + |z - x|$ for all $x, y, x \in X$. Consider the sequence $\{x_n\}_{n \in \mathbb{N}}$ defined as $x_n = \frac{n}{6}(2n + 1)(n + 1)$ for all $n \in \mathbb{N} \cup \{0\}$. Define a relation $\perp$ on X by $x \perp y \Leftrightarrow xy \in \{x, y\} \subseteq \{x_n\}$. Thus $(X, \perp, G)$ is an O-complete G-metric space. Now, we define a mapping $T : X \to X$ by*

$$T_x = \begin{cases} x_0 & \text{if} \quad x_0 \le x \le x_1 \\ \frac{x_{n-1}(x_{n+1} - x) + x_n(x - x_n)}{x_{n+1} - x_n} & \text{if} \quad x_n \le x \le x_{n+1} \quad \text{for each} \quad n \ge 1 \end{cases}$$

It is easy to see that T is $\perp - G$-continuous and T is $\perp$-preserving. Next, let $F \in \mathfrak{F}$ be a function defined by $F(\alpha) = \alpha + \ln \alpha$ for all $\alpha > 0$. Now we show that T is an orthogonal $G-F$-contraction mapping with $\tau = 1$. Let $x, y, z \in X$ with $x \perp y, y \perp z$ and $z \perp x$ and $G(T_x, T_y, T_z) > 0$ without loss of generality, we may assume that $x < y < z$. this implies that $x \in \{x_0, x_1\}$ and $y = z = x_i$. For some $i \in \mathbb{N}\{1\}$. Then we obtain

$$\frac{G(T_x, T_y, T_z)}{G(x, y, z)} e^{[G(T_x, T_y, T_z) - G(x, y, z)]} \le [\frac{x_{i-1}}{x_i - 1} e^{[x_{i-1} - x_i + 1]} < e^{-1} \tag{3.11}$$

This means that T is an orthogonal $G-F$-contraction mapping with $\tau = 1$. Therefore all the properties of theorem 3.3 are satisfied and so T has a unique fixed point $x = x_0$.

Corollary 3.1 *Let $(X, \perp, G)$ be a O-complete G-metric space. Suppose that $T : X \to X$ is a $\perp - G-$continuous mapping such that the following conditions hold:*

(1) There exists $0 < k < 1$ such that $G(T_x, T_y, T_z) \le kG(x, y, z)$ for all $x, y, z \in X$ with $x \perp y$, $y \perp z$ and $z \perp x$

(2) T is $\perp$-preserving

Then, T has unique fixed point $x^* \in X$.

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